Chapter 4 Homework
Physics 459: Nonlinear Dynamics

You should complete this assignment by Thursday, February 23.

For each of the following vector fields, find and classify all the fixed points, and sketch the phase portrait on the circle.

1. $\dot{\theta} = 1 + 2 \cos \theta$
2. $\dot{\theta} = \sin \theta + \cos \theta$
3. $\dot{\theta} = 3 + \cos 2\theta$

For each of the following questions, draw the phase portrait as functions of the control parameter $\mu$. Classify the bifurcations that occur as $\mu$ varies, and find all the bifurcation values of $\mu$.

1. $\dot{\theta} = \mu \sin \theta - \sin 2\theta$
2. $\dot{\theta} = \frac{\sin \theta}{\mu + \cos \theta}$
3. $\dot{\theta} = \mu + \sin \theta + \cos 2\theta$

Consider a Josephson junction in the overdamped limit $\beta = 0$.

1. Sketch the supercurrent $I_c \sin \phi(t)$ as a function of $t$, assuming first that $I/I_c$ is slightly greater than 1, and then assuming that $I/I_c >> 1$. (Hint: In each case, visualize the flow on the circle as given by Equation 4.6.7)
2. Sketch the instantaneous voltage $V(t)$ for each of the cases considered above.