Chapter 3 Homework, Part 2
Physics 459: Nonlinear Dynamics

You should complete this assignment by February 14.

Sketch all the qualitatively different vector fields that occur as r is varied. Show that a pitchfork bifurcation occurs at a critical value of r to be determined and classify the bifurcation as supercritical or subcritical. Sketch the bifurcation diagram of x^* vs. r.

1. $\dot{x} = rx + 4x^3$
2. $\dot{x} = rx - \sinh x$
3. $\dot{x} = r - x + \frac{rx}{1+x^2}$

The systems in the next exercises undergo some sort of bifurcation. Determine the value of r where the bifurcations occur, sketch the bifurcation diagrams, and classify the bifurcations.

1. $\dot{x} = r - 3x^2$
2. $\dot{x} = rx - \frac{x}{1+x^2}$
3. $\dot{x} = x + \tanh(rx)$
4. $\dot{x} = rx + \frac{x^3}{1+x^2}$

Consider the system $\dot{x} = rx + x^3 - x^5$, which exhibits a subcritical pitchfork bifurcation.

1. Find algebraic expressions for all the fixed points as r varies.
2. Sketch the vector fields as r varies. Be sure to indicate all the fixed points and their stability.
3. Calculate r_s, the parameter value at which nonzero fixed points are born in a saddle-node bifurcation.