Chapter 2 Homework, Part 1

Physics 459: Nonlinear Dynamics

In the next three problems, interpret $\dot{x} = \sin x$ *as a flow on the line.*

1. Find all the fixed points of the flow.
2. At which points x does the flow have the greatest velocity to the right?
3. Find the flow's acceleration as a function of x. Find the points where the flow has a maximum positive acceleration.

Analyze the following equations graphically. In each case, sketch the vector field on the real line, find all the fixed points, classify their stability, and sketch the graph of $x(t)$ *for different initial conditions. The try for a few minutes to obtain the analytic solution for* $x(t)$*; if you get stuck, don't try for too long since in several cases it's impossible to solve the equation in closed form!*

1. $\dot{x} = 4x^2 - 16$
2. $\dot{x} = 1 - x^{14}$
3. $\dot{x} = x - x^3$
4. $\dot{x} = e^{-x} \sin x$
5. $\dot{x} = 1 + \frac{1}{2} \cos x$
6. $\dot{x} = e^x - \cos x$ *Hint: Sketch both* e^x *and* $\cos x$ *on the same graph and look for intersections.*