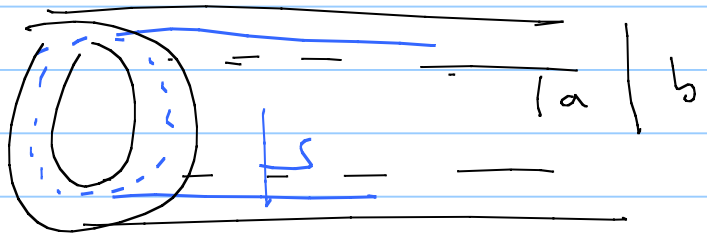
Quiz 2 Solutions

Note Title 2/16/2011

(1)



$\rho = \alpha s$

for $a < s < b$

for s in region $a < s < b$, getting Field...

$$\oint \vec{E} \cdot d\vec{a} = E(2\pi s)L = \frac{q_{enc}}{\epsilon_0}$$

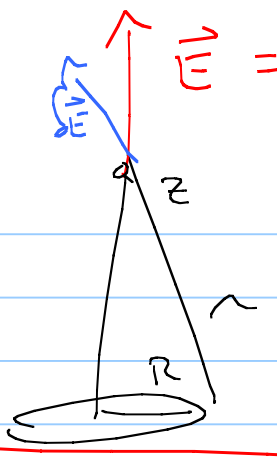
$$q_{enc} = \int \rho \, d\tau = \int_0^L \int_0^{2\pi} \int_a^s (\alpha s') \, s' \, ds' \, d\phi \, dz$$

$$q_{enc} = \frac{2\pi L \alpha}{3}\left(s^3 - a^3\right)$$

$$E(2\pi s)L = \frac{2\pi L \alpha}{3\epsilon_0}\left(s^3 - a^3\right)$$

$$\boxed{E = \left(\frac{\alpha}{3\epsilon_0 s}\right)\left(s^3 - a^3\right)}$$

(2)



$\vec{E} = E_z \hat{z}$

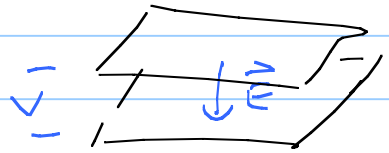
$E_z = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \left(\frac{z}{r}\right)$

$r = \sqrt{R^2 + z^2}$

$dq = \lambda dl = \lambda R \, d\theta$

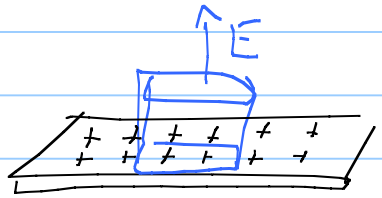
$$E_z = \frac{\lambda z R}{4\pi\epsilon_0} \frac{1}{r^3} \int_0^{2\pi} d\theta = \frac{\lambda z R}{2\epsilon_0 (R^2+z^2)^{3/2}}$$

(3)



$\vec{E}$ = constant between plates, so $V = Ed$

Use Gauss's Law to find E_{out}



$\oint \vec{E} \cdot d\vec{A} = EA = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{1}{\epsilon_0} \frac{Q}{A}$

— single metal plate

$$E = \frac{\sigma}{\epsilon_0}$$

$V = Ed = \frac{\sigma d}{\epsilon_0}$ ooo

$Q = VC \Rightarrow C = \frac{Q}{V}$

def of σ!

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\epsilon_0} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}}$$