Quiz 1 Solutions

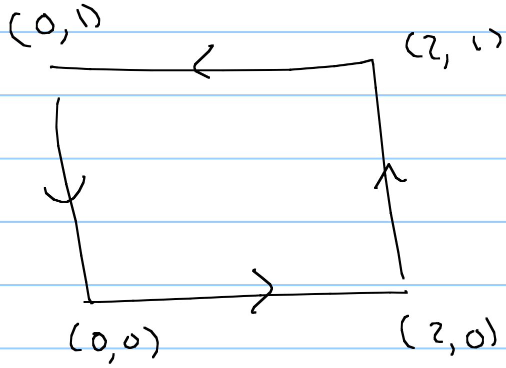
Note Title 1/28/2011

① $\vec{v} = y\,\hat{x} + 2x\,\hat{y}$

$$\nabla \cdot \vec{v} = \partial_x (y) + \partial_y (2x) = 0$$

$$\nabla \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ y & 2x & 0 \end{vmatrix} = \hat{x}\left(-\partial_z (2x)\right) - \hat{y}\left(-\partial_z y\right) + \hat{z}\left(\partial_x (2x) - \partial_y y\right)$$

$$\boxed{\nabla \times \vec{v} = 1\,\hat{z}}$$

② 

Do in 4 parts, order matters...

$$\oint \vec{v} \cdot d\vec{l} = \int_0^2 v_x\Big|_{y=0} dx + \int_0^1 v_y\Big|_{x=2} dy + \int_2^0 v_x\Big|_{y=1} dx + \int_1^0 v_y\Big|_{x=0} dy$$

$$\oint \vec{v} \cdot d\vec{l} = \int_0^2 \cancelto{0}{0}\, dx + \int_0^1 (2 \cdot 2)\, dy + \int_2^0 (1)\, dx + \int_1^0 \cancelto{0}{2(0)}\, dy$$

$$\oint \vec{v} \cdot d\vec{l} = 4\, y\Big|_0^1 + x\Big|_2^0 = 4 + (-2) = \underline{2}$$

Bonus

$$\int (\nabla \times \vec{v}) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l} \qquad \text{(stoke's theorem)}$$

$$\nabla \times \vec{v} = 1\hat{z} \qquad d\vec{a} = dx\,dy\,\hat{z}$$

$$(\nabla \times \vec{v}) \cdot d\vec{a} = dx\,dy$$

$$\int (\nabla \times \vec{v}) \cdot d\vec{a} = \int_0^2 dx \int_0^1 dy = 2$$