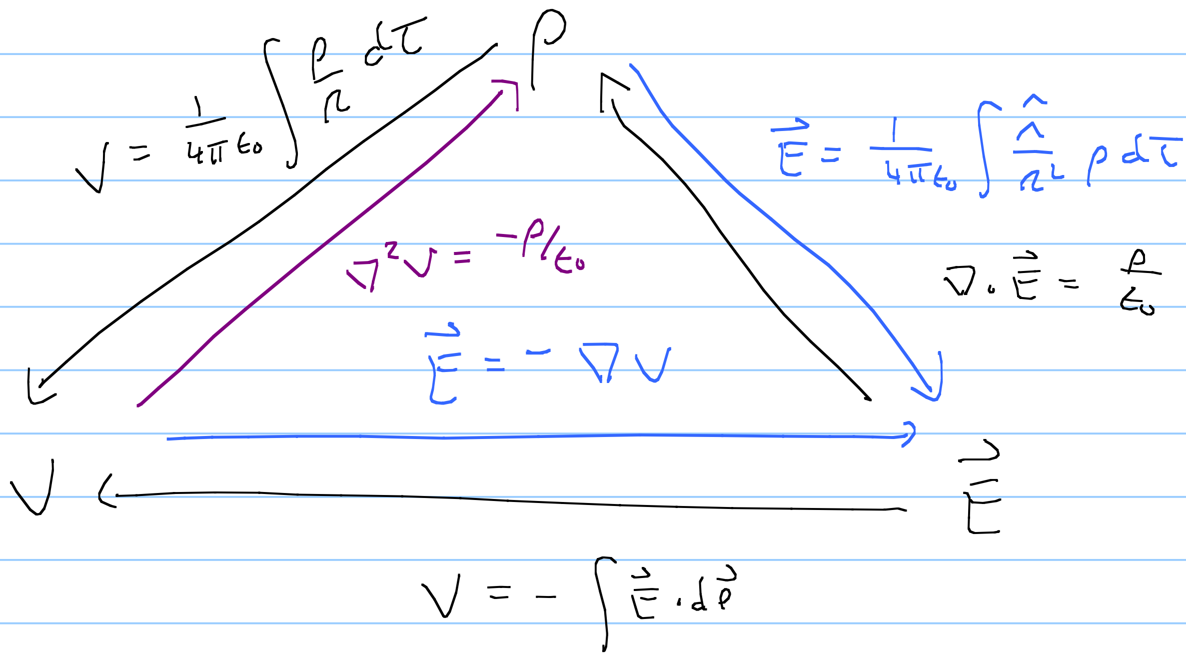
Lecture 7: Final Issues of Electrostatics.

Note Title 10/2/2009

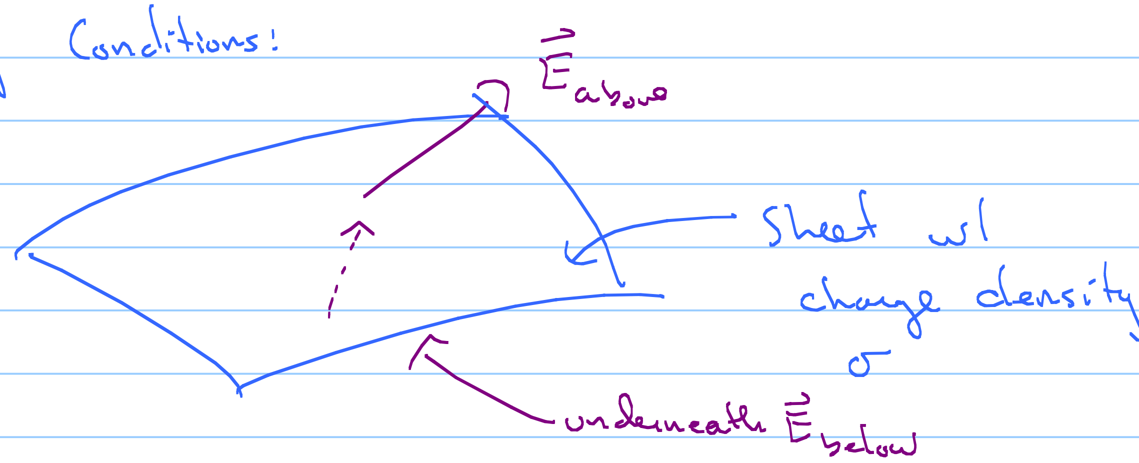
HW: 31, 36, 39

(1) Connection of ρ, V, $\vec{E}$: Figure 2.35



Here, the point is that if you know any one of the three items, you can compute the remaining two.

(2) Boundary Conditions:

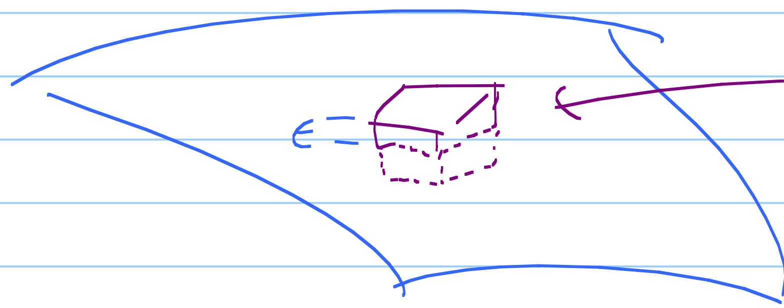


General question: how is $\vec{E}_{above}$ related to $\vec{E}_{below}$ and the charge density.

Decompose $\vec{E} = E_\perp \hat{n} + E_{\parallel} \hat{\parallel}$

$\perp$ to surface

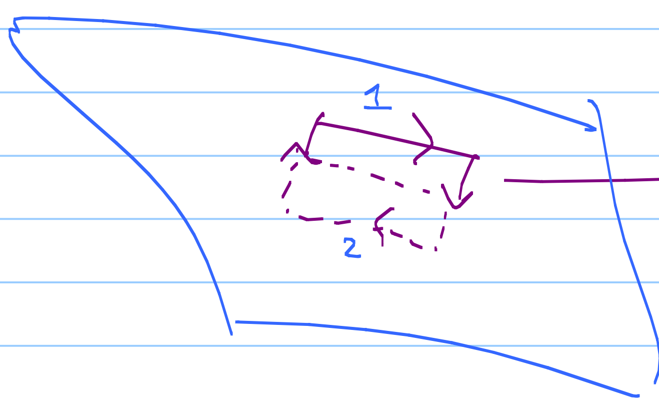
2-D vector in surface



small box through surface.

$$\lim_{\epsilon \to 0} \oint \vec{E} \cdot d\vec{a} = E^{\perp}_{top}(A) - E^{\perp}_{bottom}(A) = \frac{q_{enc}}{\epsilon_0}$$

$$\boxed{E^{\perp}_{Top} - E^{\perp}_{Bottom} = \frac{\sigma}{\epsilon_0}}$$



loop over, through, under through

$$\oint \vec{E} \cdot d\vec{l} \Rightarrow E^{1}_{\parallel}(l) - E^{2}_{\parallel}(l) = 0$$

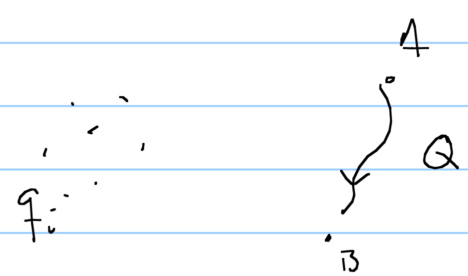
or $\boxed{E^{Top}_{\parallel} = E^{bottom}_{\parallel}}$

Also $\quad V_{above} = V_{below}$ (for V to be a fnc)

but $\quad \nabla V_{above} - \nabla V_{below} = \dfrac{-\sigma\,\hat{n}}{\epsilon_0}$

b/c of $\vec{E}$ $\perp$ components.

(3) What is the work needed to move a charge?



$$W = \int_a^b \vec{F}\cdot d\vec{l} = -Q \int_a^b \vec{E}\cdot d\vec{l}$$

Force you exert is opposite force due to $\vec{E}$ of the q_i

so $\quad \underline{W = Q\left[V(b) - V(a)\right]}$

if $\vec{a}$ is at ∞, $\quad \underline{W = Q\,V}$

$\Rightarrow$ Page 92: proof that the work to <u>assemble</u> a set of pt charges is

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i\, V(\vec{r}_i)$$

• If charges are continuous $W = \frac{1}{2}\int \rho V \, d\tau$

but $\rho = \epsilon_0 \nabla \cdot E$

$$W = \frac{\epsilon_0}{2}\int (\nabla \cdot E) V \, d\tau = \frac{\epsilon_0}{2}\Big[-\int \vec{E} \cdot \underbrace{\nabla V}_{-\vec{E}!} \, d\tau + \oint V \vec{E} \cdot d\vec{a} \Big]$$

$$W = \frac{\epsilon_0}{2}\Big[\int_V E^2 \, d\tau + \oint_S V \vec{E} \cdot d\vec{a} \Big]$$ ← Always True

$$V \sim \frac{1}{r} \qquad E \sim \frac{1}{r^2} \qquad da \sim r^2$$

$$V E \, da \sim \frac{1}{r}$$

So if you do the surface integral at infinity + integrate over all space...

$$\boxed{W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau}$$

(4) Conductors: For electrostatics

(i) $\vec{E} = 0$ in conductor

(ii) $\rho = 0$ inside

(iii) all charge is on surface

(iv) entire conductor is at 1 potential
(or else there would be a gradient, $\vec{E} = -\nabla V$)

(v) $\vec{E}$ is $\perp$ to surface just outside

See example 2.9

(5) Capacitors: By definition, any 2 metal objects w/ charge $+q$ + $-q$.

Define capacitance $C = \dfrac{Q}{V}$

for $V = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{l}$