Lecture 6: Electric Potential

Note Title 9/27/2009

Let's take a set of pt. charges, all held stationary. This makes a <u>static $\vec{E}$.</u>

Electrostatics

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots\ldots$$

What is the $\nabla \times \vec{E}$? $\nabla \times \vec{E} = \sum_{i=1}^{n} (\nabla \times \vec{E}_i)$

Now take a pt charge at the origin...

$$\vec{E}_{origin} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

Invoke Stoke's Theorem: $\oint \vec{E} \cdot d\vec{l} = \int_s (\nabla \times \vec{E}) \cdot d\vec{a}$

$$d\vec{l} = \hat{r}\,dr + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{E} \cdot d\vec{l} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr$$

$$\oint \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\epsilon_0} \oint r^2 dr = 0\,!$$

closed loop implies $r_i = r_f$

So $\boxed{\nabla \times \vec{E} = 0}$

Now if $\nabla \times \vec{E} = 0$, we can write $\boxed{\vec{E} = -\nabla V}$

for the scalar potential V.

Also $\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \nabla V \cdot d\vec{l} = -\left(V(b) - V(a)\right)$

implies

$$\boxed{\begin{aligned} V(b) - V(a) &= -\int_a^b \vec{E} \cdot d\vec{l} \\ \text{or} \quad V(\vec{r}) &= -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{l} \end{aligned}}$$

where $\mathcal{O}$ is a fixed place where $V = 0$.

So if you know $\vec{E}$, you can compute V, but more importantly, if you can compute V somehow, $\vec{E}$ is easy to get.

(Read 2.3.2, especially iii ← last paragraph.)

Now $\vec{E} = -\nabla V$ + $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

so $\nabla \cdot \vec{E} = \nabla \cdot (-\nabla V) = \frac{\rho}{\epsilon_0}$

or $\boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}}$

$$\nabla^2 V = \frac{-\rho}{\epsilon_0} \quad \text{— Poisson's equation}$$

$$\nabla^2 V = 0 \quad \text{— Laplace's eqn}$$ for region containing no charge.

You might not immediately realize it, but Laplace's eqn is much more common and useful. Most of the time you want to find V + $\vec{E}$ around the charges, not in the spot where the charge is.

Chapter 3 gives special techniques.

Direct method: (1) pt charge $V = \frac{q}{4\pi\epsilon_0 r}$

(2) $V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$ ← lots of pt charges

(3) $$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')\,d\tau'}{\mathscr{r}}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(r')\,da'}{\mathscr{r}}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(r')\,dl'}{\mathscr{r}}$$

HW: Ch. 2: 20 - 27