Lecture 5: Gauss's Law

Note Title 9/22/2009

We know that the flux of a vector field through a surface is the count of the field lines passing through the surface.

$$\Phi_E = \int_S \vec{E} \cdot d\vec{a}$$ is the flux of an electric field.

→ Take 1 pt charge + a sphere - closed surface

$$d\vec{a} = \hat{r}\, r^2 \sin\theta \, d\theta \, d\phi$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\oint \vec{E} \cdot d\vec{a} = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \, \frac{q}{4\pi\epsilon_0} \sin\theta = \underline{\frac{q}{\epsilon_0}}$$

<u>Now:</u> Griffiths makes a non-trivial statement:

"Any closed surface traps the same number of field lines."

You can prove that. The fact that Griffiths does <u>not</u> tells you something.

<u>Continuing:</u>

Since for pt. q's $\vec{E} = \sum_{i=1}^{n} \vec{E}_i$

and since $\oint \vec{E}_i \cdot d\vec{a} = \frac{q_i}{\epsilon_0}$ for any surface and any I q_i

$$\oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^{n} \oint \vec{E}_i \cdot d\vec{a} = \sum_{i=1}^{n} \frac{q_i}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

Now replace: $Q_{enc} = \sum_{i=1}^{n} q_i \Rightarrow Q_i = \int_V \rho \, d\tau$

Use Divergence Theorem $\oint_S \vec{E} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{E}) \, d\tau$

$$\int_V (\nabla \cdot \vec{E}) \, d\tau = \int_V \frac{\rho}{\epsilon_0} \, d\tau$$

or $\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$ Gauss's Law in Diff. form

$\boxed{\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}}$ Gauss's Law in Int. form

In practice: integral form is useful to compute $\vec{E}$ if very symmetric charge distributions.

For theory: differential form is very important in calculations and proofs, etc.

Examples 2.2 – 2.5 are critical for us.
As is the list on page 71.

HW: 2.11 – 2.16.