Lecture 15: Magnetization

Note Title 11/24/2009

There are 3 ways to make a material magnetic.

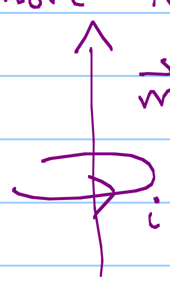
(1) Ferromagnetism: permanent magnets – read 6.4.2 for an interesting account

(2) paramagnetism: due to electron <u>spin</u> – atoms w/ odd # of electrons – aligned to field

(3) Diamagnetism: due to changing the orbital shape – atoms w/ even # of electrons – aligned opposite field

<u>Basics:</u> Magnetic dipole moment $\vec{m} = i\vec{a}$ ... think of either the electron spin as a little spinning charged shell (paramagnetism) or an electron in a circular orbit (diamagnetism)

(Ignore that both of these are wrong!)



Ampere model

Also wrong is the <u>Gilbert</u> model think of a N + S pole's

Now, like the electric dipole, $\vec{m}$ feels a torque

$$\boxed{\vec{N} = \vec{m} \times \vec{B}}$$

and a force in a non-uniform field

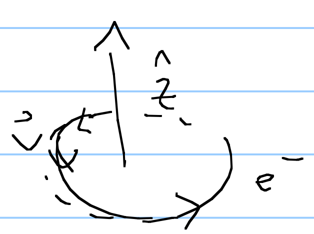
$$\boxed{\vec{F} = \nabla (\vec{m} \cdot \vec{B})}$$

If you apply an external $\vec{B}$ field to a material w/ an odd # of electrons, the torque causes the electron spins to line up w/ $\vec{m}$ // to $\vec{B}$. (Paramagnetism.)

But because of Pauli exclusion principle, all the e^- can't line their spins up the same way – hence paramagnetism only works if the # of e^- is odd – it's just the last e^- that lines up its spin.

(So paramagnetism is <u>weak</u>!)

Think now of an e^- orbiting a nucleus

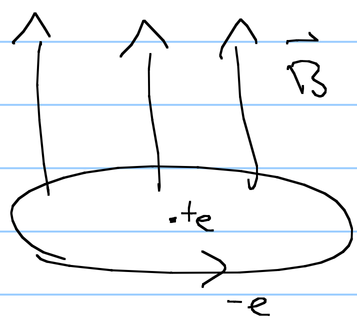


$\sim$ current $I = \frac{e}{T} = \frac{ev}{2\pi R}$

is roughly constant.

So $\vec{m} = i\vec{a} = -\frac{ev}{2\pi R}(\pi R^2) = -\frac{1}{2} evR\hat{z}$

w/ $-\hat{z}$ b/c e^- has neg. charge.



Usually $F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}$

electric force holds e^- in orbit

But if we apply $\vec{B} \Rightarrow F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = m_e\frac{\bar{v}^2}{R}$

for $\bar{v}$ the new speed of e^- in orbit.

$$\boxed{\Delta v = \bar{v} - v = \frac{eRB}{2m_e}}$$

So when you turn on $\vec{B}$, the e^- speeds up.

And $\Delta\vec{m} = -\frac{1}{2}e\,\Delta v R\hat{z} = -\frac{e^2R^2}{4m_e}\vec{B}$

- Diamagnetism – very weak & the induced $\vec{m}$ is opposite the $\vec{B}$.

Now, like electric case, define $\vec{M}$ = magnetic dipole moment per unit volume.

Since $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$ for 1 dipole

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'$$

Play same game as for V due to $\vec{P}$...

define $\vec{K}_B = \vec{M} \times \hat{n}$ – surface bound current density

$\vec{J}_b = \nabla \times \vec{M}$ – bound volume current density

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}} \, d\tau' + \frac{\mu_0}{4\pi} \oint_S \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}} \, da'}$$

Section 6.2.2, particularly figures 6.15 + 6.16 are interesting

The "real" current

Now write $\vec{J} = \vec{J}_b + \vec{J}_{free}$

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} = \vec{J}_b + \vec{J}_{free} = \vec{J}_{free} + \nabla \times \vec{M}$$

or $$\boxed{\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_{free}}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}, \quad \boxed{\nabla \times \vec{H} = \vec{J}_{free}}$$

or $$\boxed{\oint \vec{H} \cdot d\vec{\ell} = I_{free,\, enc}}$$

Great! If symmetry.

As w/ $\vec{E}$ & $\vec{D}$, there are boundary conditions...

Linear Media if $\vec{M} = \chi_m \vec{H}$, a material is Linear

χ_m - magnetic susceptibility

Then $\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H}$

or $\vec{B} = \mu \vec{H}$ for $\mu = \mu_0 (1 + \chi_m)$

$\mu \equiv$ permeability.

And you can do fun problems like 6.16...

Also $\vec{J}_b = \nabla \times \vec{M} = \nabla \times (\chi_m \vec{H}) = \underline{\chi_m \vec{J}_{free}}$

HW: 1, 6, 7, 12, 16