Lecture 13: Magnetostatics

Note Title 11/6/2009

- Charged particles feel forces from both $\vec{E}$ & $\vec{B}$. The Lorentz force Law

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

is one of the central tenets of E&M.

- You get interesting motion from Lorentz force Law – see examples 5.1, 5.2, 5.3.

- Since $\vec{F}_{mag} = q\,\vec{v} \times \vec{B}$, $\vec{F}_{mag}$ is $\perp$ to $\vec{v}$ & the motion.

(Magnetic fields do no work – even if they are critical in getting the work done – see page 211.)

- A group of moving charges make a <u>current</u> I, so a current-carrying wire feels a force.

Let $\vec{I} = \lambda \vec{v}$ ← charge density λ moving at $\vec{v}$.

$$\vec{F} = \int (\vec{v} \times \vec{B})\, dq = \int (\vec{v} \times \vec{B})\, \lambda\, dl = \int (\vec{I} \times \vec{B})\, dl$$

or $\vec{I} = I\, d\vec{l}$, $\vec{F} = \int I\,(d\vec{l} \times \vec{B})$

More about currents: If a current flows along a line, we just call it I.

- If a current flows down a 2-D ribbon, or along some 2-D surface, we say there is a surface current density

$$\vec{K} = \frac{d\vec{I}}{d\ell_\perp} \quad \text{or} \quad \vec{K} = \sigma\vec{v}$$

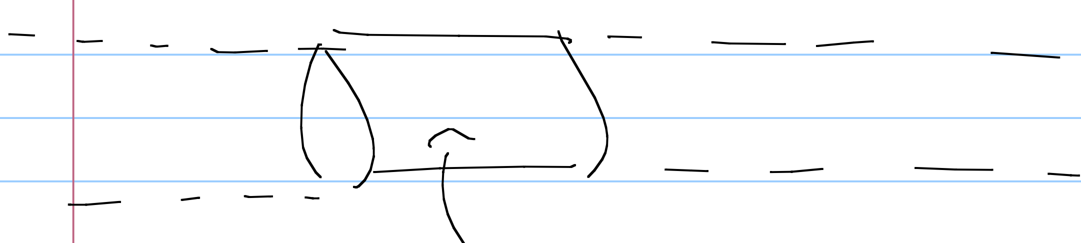
$$\& \quad \vec{F}_{mag} = \int (\vec{K} \times \vec{B})\, da$$

- If a current flows through some volume (3-D), we say there is a current density $\vec{J}$

$$\vec{J} = \frac{d\vec{I}}{da_\perp} \quad \text{or} \quad \vec{J} = \rho\vec{v}$$

$$\& \quad \vec{F}_{mag} = \int (\vec{J} \times \vec{B})\, d\tau$$

Particularly important → derive the continuity eqn



Some volume of a wire

Think about the net charge entering and leaving the vol

$\oint_S \vec{J}\cdot d\vec{a}$ ← measures the flux of charge leaving the wire per unit time

$$\oint_S \vec{J}\cdot d\vec{a} = \int_V (\nabla\cdot\vec{J})\,d\tau$$

Now any change in charge in the volume leads to

$$-\frac{d}{dt}\int_V \rho \, d\tau$$

so $$\int_V (\vec{\nabla}\cdot\vec{J})\,d\tau = -\int_V \frac{\partial\rho}{\partial t}\,d\tau$$

or $$\boxed{\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}}$$ ← The continuity eqn.

<u>Biot-Savart Law:</u> if $\nabla\cdot\vec{J} = 0$ ← in magnetostatics

and

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,dl' = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

∘ Examples are critical!

$\rightarrow$ For $\vec{K}$: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, da'$

$\rightarrow$ For $\vec{J}$: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, d\tau'$

HW: Ch. 5 $\rightarrow$ 2, 3, 4, 5, 8, 9, 13, 15