Lecture 12 Electric Displacement

Note Title 11/3/2009

Ch.4 HW Part 2: 15, 18, 19, 21, 28

In a dielectric, write $\rho = \rho_b + \rho_f$

Then $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \iff \nabla \cdot \epsilon_0 \vec{E} = \rho_b + \rho_f$

But $\rho_b = -\nabla \cdot \vec{P}$, so Gauss's law is

$$\boxed{\nabla \cdot \vec{D} = \rho_f} \quad \text{for} \quad \boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}}$$

$\vec{D}$ is the electric displacement.

In integral form

$$\boxed{\oint \vec{D} \cdot d\vec{a} = Q_{free}}$$

$\oint \vec{D} \cdot d\vec{a} = Q_{free}$ is very useful if there is symmetry, for instance problem 15.

On the other hand, $\nabla \times \vec{D} = \nabla \times \vec{P} \neq 0$

so $\nabla \cdot \vec{D} = \rho_f$ isn't sufficient in finding $\vec{D}$ if there isn't symmetry.

And $D(\vec{r}) \neq \frac{1}{4\pi} \int \frac{\hat{r}}{r^2} \rho_f(\vec{r}') \, d\tau'$

- At an interface $D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$

  and $D^{\parallel}_{above} - D^{\parallel}_{below} = P^{\parallel}_{above} - P^{\parallel}_{below}$.

The most common, and useful, application are <u>linear</u> dielectrics, where

$$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$$

$\chi_e \leftarrow$ Electric susceptibility

Then $\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 (1 + \chi_e) \vec{E}$

or $\vec{D} = \epsilon \vec{E}$ $\quad \epsilon \leftarrow$ permittivity

and define $\epsilon_r = \dfrac{\epsilon}{\epsilon_0} = 1 + \chi_e$

relative permittivity
or <u>dielectric constant</u>.

If a space is entirely filled w/ a homogeneous dielectric, then $\vec{E}$ is reduced by ϵ_r.

$\hookrightarrow$ <u>or</u> pretty much replace ϵ_0 w/ ϵ!

In particular, you can use $\oint \vec{D} \cdot d\vec{a} = Q_{free}$ to solve capacitor problems, and $C = \epsilon_r C_{vac}$.

How about energy? To charge cap: $W = \frac{1}{2} C V^2$

so if $C \uparrow$, $W \uparrow$

$$W = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, d\tau \quad \ldots \text{ energy in the fields.}$$

What about forces on dielectrics?

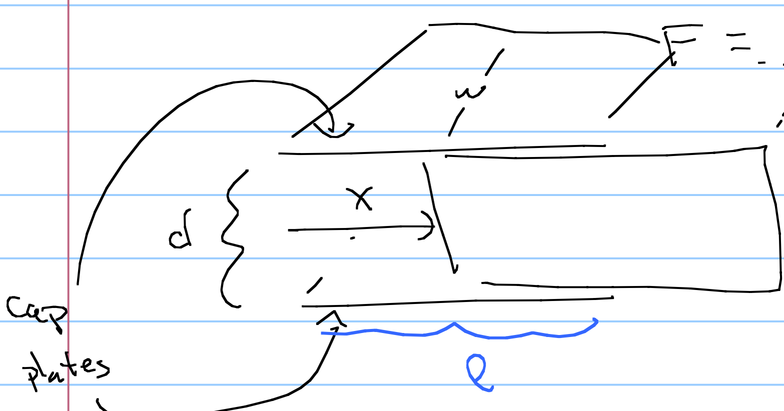
Dielectrics are pulled into the $\vec{E}$ field.

- Suppose you have a dielectric slab inside a capacitor and apply a force, F_{me}, to pull it out a distance dx.

  ↳ The work you do is $dW = F_{me} \, dx$,

  working against the electrical force on the slab

$$F = -\frac{dW}{dx}, \quad W = \frac{1}{2} C V^2$$



← This is 2 parallel caps, 1 w/ dielectric, one without

$$C = C_{air} + C_{die} = \frac{\epsilon_0 A_{air}}{d} + \frac{\epsilon_0 \epsilon_r A_{diel}}{d}$$

$$C = \frac{\epsilon_0 \, x w}{d} + \frac{\epsilon_0 (1+\chi_e)(\ell - x) w}{d}$$

$$C = \cancel{\frac{\epsilon_0 \, x w}{d}} + \frac{\epsilon_0 w \ell (1+\chi_e)}{d} - \frac{\epsilon_0 (\cancel{1} + \chi_e) x w}{d}$$

$$\boxed{C = \frac{\epsilon_0 w}{d}\left(\ell \epsilon_r - \chi_e x\right)}$$ eqn 4.62

w/ <u>Q</u> held fixed, $Q = VC \Rightarrow W = \frac{Q^2}{2C}$

$$F = -\frac{dW}{dx} = \frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{dx} = \frac{1}{2}V^2\frac{dC}{dx}$$

$$\boxed{F = -\frac{\epsilon_0 \chi_e w}{2d} V^2}$$ ... ok