

Lecture 11 : Polarization

Note Title

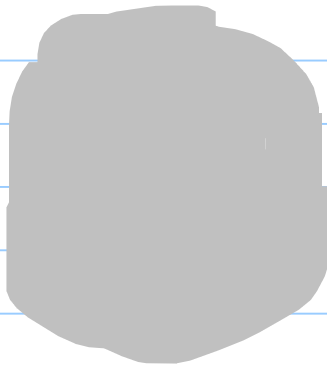
11/3/2009

2 classes of materials in E+M:

conductors: electrons free to move

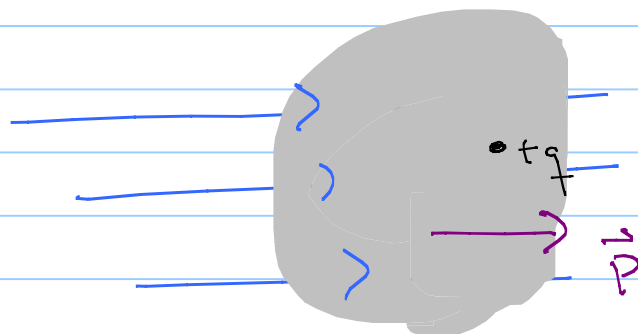
insulators or dielectrics: charge is bound

→ Dielectrics are not impervious to externally applied fields. At the molecular level, the atoms or molecules will stretch or rotate, and this can alter an \vec{E} field.



← "Spherical" cloud of e^- about a positive nucleus at center

Now apply an \vec{E}_{ext}



The \vec{E} pulls the cloud leftwards and pushes the nucleus to the right, polarizing the atom, making a dipole

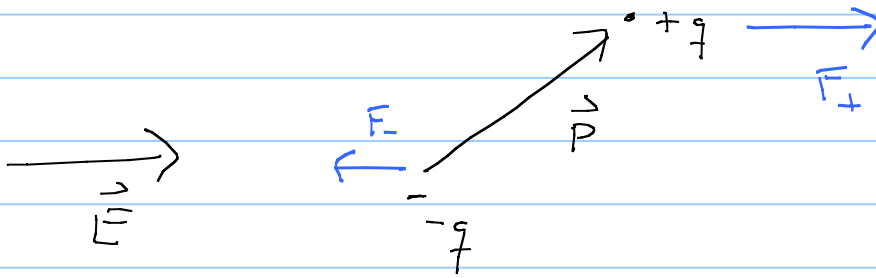
$$\vec{P} = \alpha \vec{E} \quad \alpha: \text{atomic polarizability}$$

The situation for molecules is slightly more complex since they polarize more in some directions, but the idea is the same.

↳ An applied \vec{E} produces \vec{p} in the material and the resulting \vec{E} is the sum of the initially applied \vec{E} + the dipole \vec{E} .

What happens to a dipole in a field?

(1) It gets a torque.



The torque will be $\vec{\tau} = \vec{p} \times \vec{E}$

(2) If the field is not uniform, then there can be a net force.

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

$\vec{p} \cdot \nabla$ is a directional derivative. Think of it as the scalar operator
 $p_x \frac{\partial}{\partial x} + p_y \frac{\partial}{\partial y} + p_z \frac{\partial}{\partial z}$

A material will have lots of dipoles, but it is easiest to work w/ an average

$\vec{P} \equiv$ dipole moment per unit volume

or Polarization

Dipole Potential $V = \frac{\hat{n} \cdot \vec{P}}{4\pi\epsilon_0 r^2}$

so integrate the polarization over the volume

$$V = \frac{1}{4\pi\epsilon_0} \int_{Vol} \frac{\hat{n} \cdot \vec{P}(r')}{r^2} d\tau'$$

This is the answer, but some work makes it better.

Old HW problem $\nabla\left(\frac{1}{r}\right) = -\frac{\hat{n}}{r^2}$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\text{w/ } \nabla = (\partial_x, \partial_y, \partial_z)$$

For $\nabla' = (\partial_{x'}, \partial_{y'}, \partial_{z'})$, get a "-" difference

$$\nabla'\left(\frac{1}{r}\right) = \frac{\hat{n}}{r^2} \leftarrow \text{replace in } V \text{ above}$$

$$V = \frac{1}{4\pi\epsilon_0} \int_{V_{ol}} \vec{P} \cdot \nabla' \left(\frac{1}{r} \right) d\tau'$$

This form

Now integrate by parts:

$$\nabla \cdot (f \vec{A}) = f (\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla f$$

$$V = \frac{1}{4\pi\epsilon_0} \int \nabla' \cdot \left(\vec{P} \left(\frac{1}{r} \right) \right) d\tau'$$

$$- \frac{1}{4\pi\epsilon_0} \int \frac{1}{r} (\nabla' \cdot \vec{P}) d\tau'$$

Apply
Divergence
Theorem

Also define

$$\sigma_b = \vec{P} \cdot \hat{n}$$

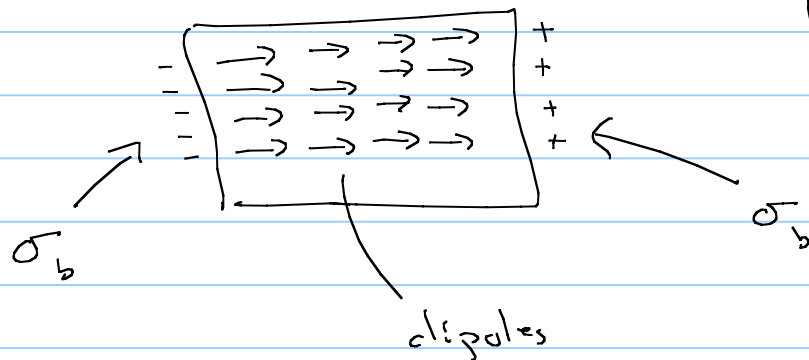
$$\rho_b = -\nabla \cdot \vec{P}$$

} bound
charge
due to
Polarization

$$V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b}{r} da' + \frac{1}{4\pi\epsilon_0} \int_{V_{ol}} \frac{\rho_b}{r} d\tau'$$

So the potential is due to the bound charge located on the surface (σ_b) or bound charge density in the volume.

Section 4.2.2 explains bound charge nicely.



Basically if there are dipoles in the material, that will leave surface charge density on the ends.

To get volume charge density, we need a non-uniform arrangement of dipoles through the bulk.

Ch. 4 HW Part 1 : 1, 4, 9, 10