Physics 429: General Relativity
Homework Assignment # 8

In this assignment, we will consider metric tensors. You should work together on this assignment, but each individual should turn in their own work.

1. For $r > 2m$, the Schwarzschild solution has a metric tensor field given by

$$g_{\mu\nu} = \begin{pmatrix} (1-2m/r) & 0 & 0 & 0 \\ 0 & -(1-2m/r)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix},$$

where the coordinates are labelled according to $t \equiv x^0$, $r \equiv x^1$, $\theta \equiv x^2$, and $\phi \equiv x^3$. (Here, units are chosen such that $G = c = 1$.) Find the lengths of the following vectors and the angles between them:

(a) $\lambda^\mu = \delta^\mu_0$;
(b) $\mu^\mu = \delta^\mu_1$;
(c) $\nu^\mu = \delta^\mu_0 + (1-2m/r)\delta^\mu_1$.

2. Classify each vector above as timelike, spacelike, or null.

3. Write the Minkowski metric in spherical coordinates (t, r, θ, ϕ). What are the components of $\eta_{\mu\nu}$ in spherical coordinates?

Consider the following coordinate transformation on Minkowski spacetime:

$$u = t + r \qquad\qquad v = t - r$$

$$\theta = \theta \qquad\qquad \phi = \phi$$

where the coordinates are labelled according to $u \equiv x^0$, $v \equiv x^1$, $\theta =\equiv x^2$ and $\phi \equiv x^3$.

4. Write the Minkowski metric in (u, v, θ, ϕ) coordinates. What are the components of $\eta_{\mu\nu}$ in these coordinates?

5. For the coordinates (u, v, θ, ϕ), find the lengths of

   (a) $\alpha^\mu = \delta^\mu_0$, and
   (b) $\beta^\mu = \delta^\mu_1$.

6. What is the inner product between α^μ and β^μ?