General Relativity Concepts for the term, written April 26, 2011
This is due at the final exam on May 9, 2011

1. The overlap function, the Jacobian of the overlap function, the tensor transformation laws ... these are all math. Explain what these have to do with physics. Put another way, how is the idea of different observers related to the math we are doing?

2. What are all the different things the metric can do? Write them all out: equations, short explanations, pictures. For instance: raise and lower indices $v_a = g_{ab}v^b$. But lots of stuff – at least six to eight items here.

3. Write out the formulas for the connection, Γ^a_{bc}, Riemann Tensor $R^a{}_{bcd}$, the geodesic equation, the equation for parallel transport and any others you can think of. What is the physical picture involved in these? In many cases, these complicated mathematic quantities arose when we asked a physical question. What was the physical question that led to each object? Explain with words and pictures.

4. Here is a recasting of the arc of the early portion of the class. Annotate it by adding comments, material, equations, pictures, etc.

   Why is gravity geometry, and the geometry the metric?

   - On any surface, we can employ Fermat's Principle of Least Time (or the Calculus of Variations) to say that geodesics extremize

   $$\mathcal{L} = \int_a^b \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\, d\tau$$

   - Mathematical arguments say that you can always make the metric flat at any given region (flat metric and zero first derivatives

   $$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} g_{\mu\nu,\rho\sigma}\Delta x^\rho \Delta x^\sigma$$

   - The SEP says that the free fall observer is in a frame consistent with special relativity ($g_{\mu\nu} = \eta_{\mu\nu}$)
   - The SEP says $g_{\mu\nu} = e^\alpha{}_\mu e^\beta{}_\nu \eta_{\alpha\beta}$ – gravity's "accelerations" enter into "distance" rule by SEP and basic math idea
   - So gravity is geometry and geometry is the metric ...
   - Since the first derivatives of the metric don't matter, we need a second order differential equation for the metric

- We want a generally covariant differential equation for the metric, so we need to use some tensors that involve second derivatives of the metric
- Ocham's razor gets us to the Einstein Field Equation.

5. Describe the Schwarzschild metric, answering most if not all of these questions. What does it mean to be asymptotically flat? What happens at $r = 2M$? What happens to light rays emitted from near $r = 2M$ as they travel out? What happens to an observer who falls past the event horizon?

6. Describe the Robertson-Walker metrics, the big bang, the scale factor, the matter density, the possible shapes of the spatial part of the universe, etc.