Lecture 19: Hawking Radiation and Other Issues

Note Title 4/11/2011

One can rigorously define a theory of Black Hole Thermodynamics - see Wald, section 12.5

	Thermo	BH Thermo
Zero	T = constant in equilibrium	κ constant on horizon
1st	$dE = T\,dS$ + Work	$dM = \frac{1}{8\pi}\kappa\, dA$ + Work
2nd	$\delta S \geq 0$ in any process	$\delta A \geq 0$ in any process
3rd	Impossible to acheive $T=0$ in a physical Process	Impossible to acheive $\kappa = 0$ in a physical Process

κ - Surface gravity, A - event horizon area

1974 Hawking Radiation - Blackbody w/ $T = \frac{\hbar \kappa}{2\pi}$

↳ This is Quantum Field Theory on a curved manifold. Our goal is to understand some of the schematics.

First picture is the formation of the black hole.

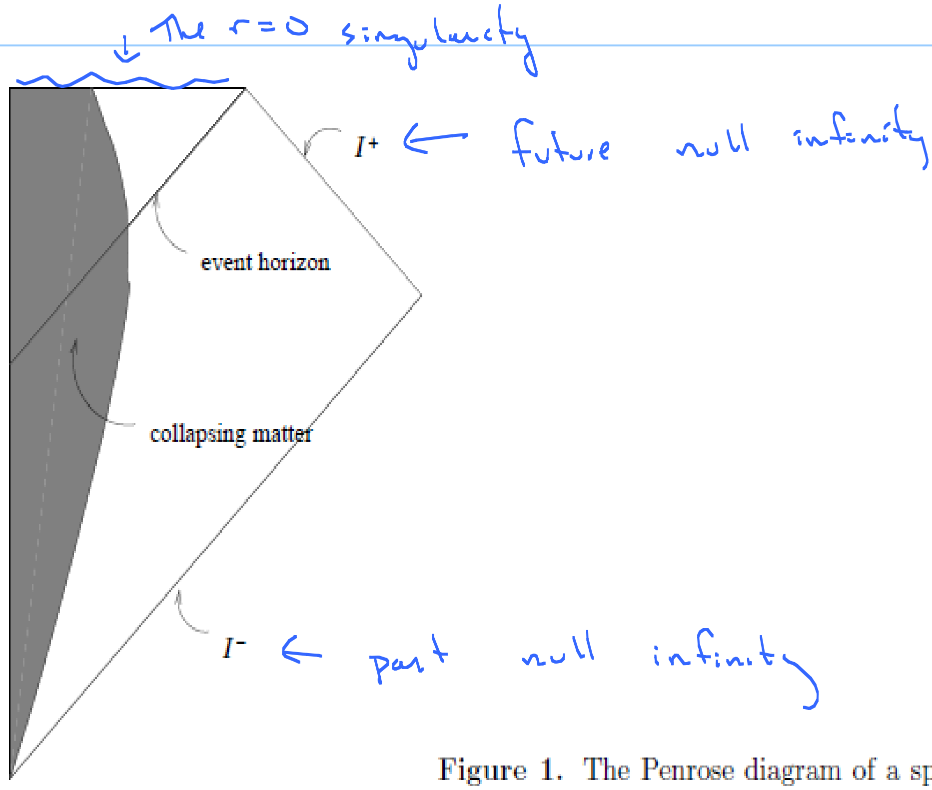


Figure 1. The Penrose diagram of a spherically symmetric object collapsing to form a black hole. The future and past null infinities $\mathcal{I}^{\pm}$ are shown, as well as the event horizon. The black hole is the region at and above the event horizon.

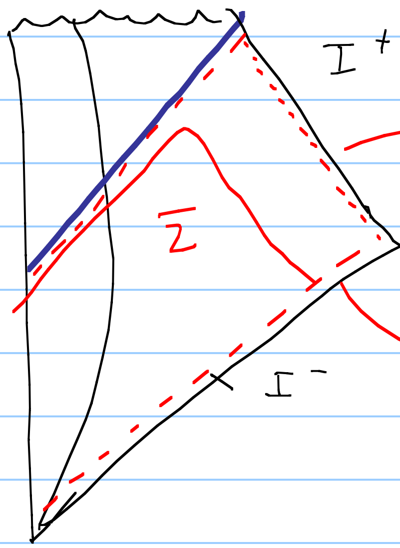
Collapsing matter causes the event horizon to form. Once it is present, we can start to do QFT to create particles (or fields).

$\mathcal{I}^+$ is future null infinity. Any massless scalar field that does not fall into the BH will escape to $\mathcal{I}^+$

A massless, scalar field ϕ satisfies $\nabla^a \nabla_a \phi = 0$

↳ Do a "backwards scattering" of

$$\phi_{\omega \ell m} \sim \underbrace{e^{-i\omega t}}_{\uparrow \text{ outgoing wave at } \mathcal{I}^+} Y_{\ell m}(\theta, \phi)$$



Σ^+ – Cauchy surface at future null infinity

↓ propagate back ϕ

Σ^- – Cauchy surface at past null infinity

Using time $v = t + r_*$,

$$\phi(v) \sim \phi_0 \, e^{\frac{i\omega}{\kappa}(\ln(-\alpha v))}$$

⋮ hard work

$$\langle N \rangle \sim \frac{e^{-\frac{2\pi\omega}{\kappa}}}{1 - e^{-2\pi\omega/\kappa}}$$ ← expected # of particles created through scattering

or $kT_{bH} = \dfrac{\hbar \kappa}{2\pi c}$ ← Black body emitter temp.

If BH is radiating energy, even at very low level, it should dissipate...

$$L = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2} \propto A T_H^4$$ — Luminosity carrying away energy

$$\frac{dM}{dt} = -\left(\frac{\alpha c^4 \hbar}{960 G^2}\right) n_{eff} M^{-2}$$

n_{eff} = effective # of particles being radiated

$$t_{life} \sim \left(\frac{M}{M_\odot}\right)^3 10^{65} \text{ year}$$ — lifetime before all the mass radiates away

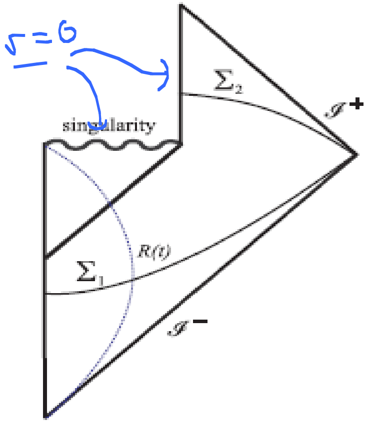
t_{life} is huge, but for low-mass BH formed at big bang: if t_{life} = Age of universe can get limit on size ~ 10^{15} grams

This is size of BH radiated away today - possibly detectable.

Information Loss Paradox: "information" is tied up with idea of entropy

When the black hole evaporates, any information in it is lost.

We can refer to Figure 2 for a view of what he had in mind. In this case we say that unitary evolution has broken down and there is loss of information because there are two surfaces Σ_1 and Σ_2 such that there is no reversible map from $\mathcal{H}_1$ to $\mathcal{H}_2$. No matter what the initial state was, the outcome is always a thermal state and it is not possible to find out from the final state what the initial state was.



From Smolin's Article

Figure 2: Hawking's evaporation scenario. The dotted (blue) line indicates the surface of the collapsing matter with radius $R(t)$. The solid (black) line is the event horizon, the thin (black) lines labeled with $\Sigma_{1/2}$ are two complete spacelike hypersurfaces.

Quantum states...

pure states $|\psi\rangle = |0\rangle$ or $|\psi\rangle = |1\rangle$

another pure state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$

(for instance, a Stern-Gerlach)

entangled state ... 2 particles c_1 + b_1

$$|\psi\rangle = \frac{1}{\sqrt{2}} |0\rangle_{c_1} |0\rangle_{b_1} + \frac{1}{\sqrt{2}} |1\rangle_{c_1} |1\rangle_{b_1}$$

Mixed States: states that can not be written in the form above, must be described w/ a density matrix

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Suppose you have 2 particles entangled as above...

- 1 particle enters event horizon, while other propogates to I^+

  ↳ No problem yet... you can't access the particle behind Event Horizon, but they are QM entangled...

  (Actually that's still confusing... suppose a person inside EH conducts experiment to localize the wave fnc, then the wave fnc of particle outside collapses too...)

- Now let BH evaporate... particle inside is gone, lost, nowhere.... what happens to the QM description of outside particle?

→ It is no longer entangled w/ anything...
↳ it falls into a mixed state.
(Information is lost)

QM + GR are unitary evolution theories:

information needed to compute state at early times = information needed to compute state at late times

But we are apparently losing information — going from an entangled pure state to a mixed state...

This is central to Quantum Gravity. Not everyone agrees whether the calculation described here is correct, or how to do it or whether to do it.

There are a range of solutions — see Smolin...

Any good theory of QG should explain this.

→ Lots of papers! see reviews by Mathur, Helfer. Articles by Smolin & Hossenfelder, Hsu. Book by Carroll "Spacetime and Geometry."