Lecture 18: Schwarzschild Spacetime & Black Holes 3

Note Title

What are black holes?

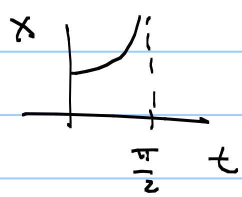
→ Event horizon at $r = 2m$ → Does anything weird happen there? ← No!

Problem is in the coordinates. – silly example.

(A) $x\ddot{x} - 2\dot{x}^2 - x^2 = 0$ w/ $x = 1$, $\dot{x} = 0$, $t = 0$

Question: Does a solution exist for all t?

↳ Sol $x = (\cos t)^{-1}$



← No! Sol. blows up at $t = \frac{\pi}{2}$.

Now let $q(t) = \frac{1}{x(t)}$ ⇒ (A) ⇒ $\ddot{q} = -q$!

↑ sol. $q = \cos t$ exists for all time.

The problem was in the coordinates.

We can find coordinates which extend Schwarzschild solution for full range in r. No troubles w/ infinities in metric or any other tensor.

For example: $v = t + r + 2m \log\left(\frac{r}{2m} - 1\right)$, r, θ, ϕ

$$ds^2 = \left(1 - \frac{2m}{r}\right) dv^2 - 2\,dv\,dr - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2$$

(Eddington – Finkelstein coordinates)

$r = 2m$ is called a coordinate singularity
$r = 0$ is the singularity.

→ at $r = 2m$, no scalar quantity blows up.

just a coordinate problem.

$r = 2m$ (event horizon) is important → It separates 2 distinct regions of spacetime, & we belive that there will be no "naked singularities" or singularities not enclosed by an event horizon.

Formation of Black Holes

take a $20 M_\odot$ star

→ a typical star forms from a cloud of H gas & contracts under gravity. As contracts $T \uparrow$ & thermonuclear fusion occurs.

Millions of years

→ Fusion of H supports the star.

→ As all H is used up, contract again until T rises to support He → C fusion.

↓

Continues on, until Fe core from fusion

very nuclearly stable, high binding energy per nucleon

→ density $\sim 10^{11} \frac{kg}{m^3}$, $T \sim 10^9$ K

→ At this point electron degeneracy pressure holds up the star → Pauli exclusion principle.

Chandrasekhar (1931) showed that if $M > 1.4 M_\odot$, the degeneracy pressure can not hold the star up, + it collapses to a black hole.

If $M < 1.4 M_\odot$, star collapses to a neutron star or white dwarf.

→ Astrophysical black holes! Supermassive black holes are believed to exist at the centers of most galaxies.

→ Most important bh Solutions
(1) Schwarzschild
(2) Kerr (or) Kerr-Newman
↑ rotating b.h.

- Since stars have non-zero angular momentum, they form a nearly exact Kerr BH.

- As time goes on, the nearly Kerr BH radiates gravitational waves, + spin down until they no longer radiate, ending up as Schwarzschild BH.