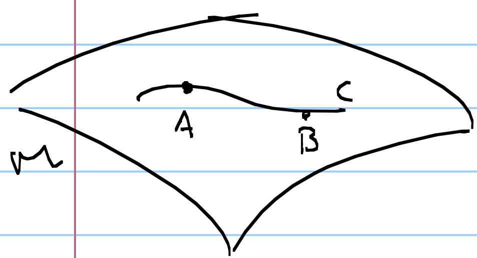
Lecture 11: What are Coordinates?

Note Title

<u>Review</u> There are two roles for $g_{\mu\nu}$



(1) $\underline{g_{\mu\nu}}$ is the metric: $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

$\Rightarrow$ $g_{\mu\nu}$ gives the length of curves:

$$L(c) = \int_A^B d\tau \, \frac{ds}{d\tau} = \int_A^B d\tau \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

(2) $\underline{g_{\mu\nu}}$ is the gravitational field. + $\ddot{x}^\mu = -\Gamma^\mu_{\nu\rho}\dot{x}^\nu \dot{x}^\rho$ is the physical equation for the gravitational force.

$\Longrightarrow$ In SR, coordinates are distances, <u>physical distances</u> in an observer's reference frame from coordinate axes.

$\rightarrow$ In GR, distance is <u>NOT</u> the coordinates! The definition of distance is given by $L(c)$, the coordinates are something complicated.

So what are the coordinates?

In GR, coordinates are literally nothing more than labels. They are not physically determined in any way as a general rule.

→ Coordinates are simply a way to label the points in the manifold. These labels can be chosen completely randomly, or associated w/ some observer.

- choose x^μ, then have $g_{\mu\nu}$

$\Updownarrow$

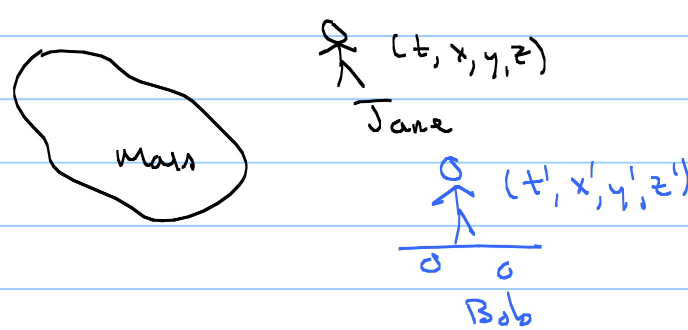
Choose y^μ or x'^μ, have $g'_{\mu\nu}$

Same physics, but w/ different descriptions.

The general form of the metric due to a static, weak field gravitational potential, ϕ is

$$ds^2 = (1+2\phi)\,dt^2 - (1-2\phi)(dx^2+dy^2+dz^2)$$

in (t,x,y,z) coordinates at rest w/ respect to the potential.



Let Bob move at β in the x-dir relative to Jane

Question: What gravitational field does Bob feel?

→ Work to 1st order in β.

- Lorentz transformation $\Rightarrow$ $t = \gamma(t' + \beta x') \simeq t' + \beta x'$

$$x = \gamma(x' + \beta t') \simeq x' + \beta t'$$

$$\underset{(\text{Bob})}{g'_{\mu\nu}} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} \underset{(\text{Jane})}{g_{\alpha\beta}}$$

$$\frac{\partial t}{\partial t'} = 1, \quad \frac{\partial t}{\partial x'} = \beta, \quad \frac{\partial x}{\partial t'} = \beta, \quad \frac{\partial x}{\partial x'} = 1$$

$$g'_{00} = \frac{\partial x^\alpha}{\partial t'}\frac{\partial x^\beta}{\partial t'} g_{\alpha\beta} = \frac{\partial t}{\partial t'}\frac{\partial t}{\partial t'} g_{00} + \frac{\partial x}{\partial t'}\frac{\partial x}{\partial t'} g_{11}$$

$$\underline{g'_{00}} = (1)(1)(1+2\phi) + \beta\cdot\beta\cdot(-1)(1-2\phi) \simeq \underline{1 + 2\phi}$$

$$\underline{g'_{11} = -(1-2\phi)} \quad \text{(similar calculation)}$$

But $$g'_{01} = \frac{\partial x^\alpha}{\partial t'}\frac{\partial x^\beta}{\partial x'} g_{\alpha\beta} = \frac{\partial t}{\partial t'}\frac{\partial t}{\partial x'} g_{00} + \frac{\partial x}{\partial t'}\frac{\partial x}{\partial x'} g_{11}$$

$$g'_{01} = \beta(1+2\phi) + \beta(-1)(1-2\phi) = 4\beta\phi \,!$$

Not zero!

Metric for Bob: $ds^2 = g'_{ab}\, dx^a dx^b$

$$ds^2 = (1+2\phi)dt'^2 - (1-2\phi)(dx'^2 + dy'^2 + dz'^2) + 8\beta\phi\, dt'dx'$$

indicates motion in x-dir w/ respect to mass.

Another way:

$$t = t' + \beta x' \qquad x = x' + \beta t' \implies dt = dt' + \beta dx' \qquad dx = dx' + \beta dt' \qquad dy = dy' \qquad dz = dz'$$

$$ds^2 = (1+2\phi)dt^2 - (1-2\phi)(dx^2 + dy^2 + dz^2)$$

$$ds^2 = (1+2\phi)(dt'^2 + 2\beta dt'dx') - (1-2\phi)(dx'^2 + 2\beta dx'dt') - (1-2\phi)(dy'^2 + dz'^2)$$

(Substituting in, working to 1st order)

$$ds^2 = (1+2\phi)dt'^2 - (1-2\phi)(dx'^2 + dy'^2 + dz'^2) + 8\beta\phi\, dt'dx'$$

(By collecting terms.)

Note how the relational view comes out here...

Jane's coordinates set her in a particular relation to the matter around her — she is at rest relative to the matter — and that shows up in the metric.

Likewise, Bob moves relative to the matter, and that shows up in his metric as a g_{01} term...