Class Problems and Homework

Physics 403: Mathematical Methods

January 25 2008

Class Problems: Instructor and group work.

1. Compute the gradients of the following functions:

   (a) $\varphi = x^2 + y^2 + z^2$,

   (b) $\gamma = z - \sin(x^2 + y^2)$, and

   (c) $\tau = z - 2x + 3y$.

2. Let $\vec{F} = 2xz^2\hat{x} + \hat{y} + y^3zx\hat{z}$, $\vec{G} = x^2\hat{x} + y^2\hat{y} + z^2\hat{z}$ and $f = x^2y$. Compute the following quantities:

   (a) $\vec{\nabla} f$

   (b) $(\vec{F} \cdot \vec{G})\vec{G}$

   (c) $\vec{\nabla} \times \vec{F}$

   (d) $\vec{F} \cdot (\vec{\nabla} f)$

   (e) $\vec{F} \times \vec{\nabla} f$

   (f) $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F}$

3. Compute both the divergence and curls of the following vector fields:

   (a) $\vec{A} = x\,\hat{x} + y\,\hat{y}$,

   (b) $\vec{B} = y\,\hat{x} - x\,\hat{y}$, and

   (c) $\vec{C} = xy\,\hat{x} - 2x^2yz\,\hat{y} + z\,\hat{z}$.

Homework: *Complete this assignment before coming to class on Monday, Jan. 23.*

1. *REVIEW PROBLEMS: You should be able to do all of these book problems. Look them over and do them if you are unsure.* Chapter 5: 50-54, 58, 63

2. *Good book problems:* Chapter 5: 59, 70, 77, 81

3. *Challenging!* Draw pictorial representations of the surfaces defined by $\varphi = 4$, $\gamma = 0$ and $\tau = -1$ in problem 1. On your picture, draw the gradients.

4. *Challenging and Important!* Draw pictorial representations of the vector fields $\vec{A}$ and $\vec{B}$ from problem 3. From the pictures, explain why the divergence and curls have the values they do.