Ex 2 cont^{d}. $u = x^2+y^2$ does not satisfy Laplace's eq. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ bec. net flow thru any closed surface is not 0:

$$\frac{\partial u}{\partial x} = 2x \Rightarrow \frac{\partial^2 u}{\partial x^2} = 2.$$

$$\frac{\partial u}{\partial y} = 2y \rightarrow \frac{\partial^2 u}{\partial y^2} = 2.$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4.$$

This is Poisson's eq. $-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = \frac{f}{c}$, $\frac{m}{\ell^3 t}$, $\frac{m}{\ell^3}$, $\frac{\ell^2}{t\ell^2}$

where $\frac{f}{c} = -4$.

amt leaving per unit vol per unit time $f = -4c$

$\Leftrightarrow$ amt entering per unit vol per unit time $-f = 4c$.

Source produces $4c$ units of mass per unit vol per unit time

Ex 2 contd $u = x^2 + y^2$. source, 2.

If we take $\mathcal{C}$ as a circle of radius R, then Div Thm says:

$\underline{w} = c\underline{V}$

$\frac{m}{l^3}\,\frac{l}{t}$

$$\iint_S \underbrace{\operatorname{div} \underline{w}}_{\frac{m}{l^3 t}}\, \underbrace{dx\,dy}_{l^2} = \int_{\mathcal{C}} \underline{w} \cdot \underline{n}\, ds$$

$$= \iint_S 4c\, dx\,dy$$

$$= 4c \iint_S dx\,dy$$

$$= 4c(\text{area}(S))$$

$$= \underbrace{4c}_{\frac{m}{l^3 t}}\,\pi \underbrace{R^2}_{l^2}$$

So

$$\int_{\mathcal{C}} \underline{w} \cdot \underline{n}\, ds = 4c\pi R^2 .$$

If a source produces $4c$ units of mass per unit volume per unit time inside a circle of area πR^2, then $4c\pi R^2$ units of mass flow across (out of) the circle per unit length per unit time.
(the fluid produced has to go somewhere!)