MATH 142
SAMPLE FINAL EXAM

Please put your name on your blue book, show your work and give justifications for your answers. You may use a calculator, the table of integrals you are given, and both sides of an 8 1/2 by 11 sheet of notes on the test; you may not use your cell phone. Try not to spend too much time on any one problem. If you get stuck on a problem, leave a partial answer and move on to the next problem. There are exam questions on both sides of this page.

(1) (10 pts) The graph of a function $f(x)$ is shown above. Write an equation for its antiderivative $F(x)$.

(2) (5 pts) True or False: The fundamental theorem of calculus says that if $f(x) = F'(x)$, then $\int_a^b F(x)dx = f(b) - f(a)$.

(3) (10 pts) Find an antiderivative of the function $f(x) = \ln(x)$.

(4) (15 pts) Is the area of the region to the right of the line $x = 1$ and between the graph of $\frac{2}{x^3}$ and the $x$-axis finite or infinite? Show your work.
(5) (15 pts) The Lorentz curve for the distribution of salaries of lawyers in a certain country is described by the function
\[ f(x) = \frac{9}{11} x^4 + \frac{2}{11} x. \]

a) What percent of the total salaries paid to lawyers in that country is paid to the “poorest” 50% of lawyers?

b) What is the coefficient of inequality for this curve?

(6) (15 pts) Suppose that for some random variable \( x \), \( 1 \leq x \leq \infty \), the function \( f(x) = \frac{3}{x^4} \) is a probability density function.

a) What is the probability that \( x > 10 \)?

b) What is the expected value of \( x \)?

(7) (20 pts) Consider the function \( f(x, y) = x^2 - y^2 \).

a) What is \( f(2, 1) \)?

b) What is \( \frac{\partial f}{\partial x} = f_x(x, y) \)?

c) What are the critical points of \( f(x, y) \)?

d) What is \( f_{xx}(x, y) \)?

e) For each critical point, state whether it is a relative maximum, a relative minimum, or neither.

(8) (10 pts) Find the volume bounded by the planes \( x = 0 \), \( x = 2 \), \( y = 0 \), \( y = 5 \), \( z = 0 \) and the surface \( f(x, y) = x^2 + y^2 \).