MA 100 SAMPLE MIDTERM SOLUTIONS

(1) Answer the following questions for the graph shown above.

a) (5 pts) Is this the graph of a function? Why or why not?

If the graph passes the vertical line test (p. 42) then it is the graph of a function.

b) (5 pts) Estimate which points on the graph have y coordinate $-1$.

Draw the horizontal line $y = -1$ on the graph, then give the coordinates of the points at which your horizontal line crosses the graph.

(2) Let $g(x) = \frac{3x - 1}{x^2 - 4}$.

a) (5 pts) What is the domain of $g(x)$?

$g(x)$ is defined whenever $x \neq 2$ and $x \neq -2$, so the domain of $g(x)$ is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

b) (5 pts) What are the zeros of $g(x)$?

The zero of $g(x)$ is at $x = 1/3$.

c) (5 pts) What is the y-intercept of the graph of $g(x)$?

$g(0) = 1/4$, so the y-intercept of the graph is at $(0, 1/4)$.

(3) (10 pts) If $h(x) = x^2$, what is $h(x + 2)$? Sketch the graph of $h(x + 2)$.

$h(x + 2) = (x + 2)^2 = x^2 + 4x + 4$. Its graph is a parabola that opens up with its vertex at $(0, -2)$.

(4) (10 pts) Solve for $x$: $3x^{-2} + x^{-1} = 0$.

Multiply both sides of the equation by $x^2$; you’ll find that $x = -3$.

(5) (10 pts) Simplify: $\frac{x + 2}{x - 3} \cdot \frac{2}{x^2 - 4} = \frac{2}{(x - 3)(x - 2)}$

(6) (15 pts) What is the distance between the points $(1, 5)$ and $(4, 1)$?

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5 \]

(7) (15 pts) True or false: If a line has a negative slope then any line perpendicular to that line has positive slope. Justify your answer.

True. The slope of a line is the negative reciprocal of the slope of a line perpendicular to it.

(8) (15 pts) Is it possible for a function to be both even and odd? Justify your answer.

The graph of the function $f(x) = 0$ is symmetric about the $x$-axis and symmetric about the origin, so it is possible for a function to be both even and odd.