E.F. Dorney / BSC Physics: PH402 : Special Relativity & General Relativity Intro & Guage fields!

Note Title

S.R. to preserve

1.) Kinematics (ie constant accel)

2.) Maxwell's Equations

in inertial frames $\Rightarrow \vec{x} \rightarrow \underbrace{(ct, \vec{x})}_{\text{4-vector space-time}}$

& Lorentz transformation

3.) Dynamics (ie cons of $\vec{p}$)

$\Rightarrow$;

$$\left.\begin{array}{l} \vec{p} \rightarrow \gamma m \vec{v} \\ \text{\& } E_{Rest} = m_0 c^2 \end{array}\right\} \vec{p} \rightarrow \underbrace{\left(\frac{E_0}{c}, \vec{p}\right)}_{\text{4-vector energy-momentum}}$$

$$E_{TOT} = \gamma m_0 c^2 \; ; \; E_{TO}^2 = p^2 c^2 + m_0^2 c^4$$

But when

$V = c$, $m_0 = 0$

γ = undefined

use

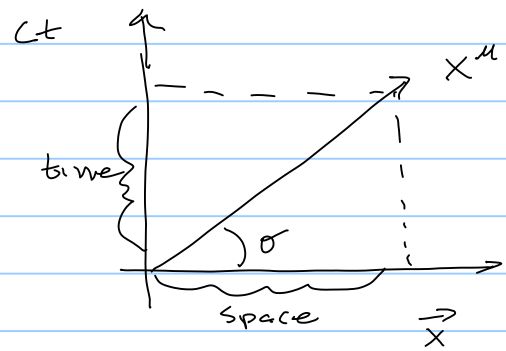
$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\therefore E = \frac{h}{\lambda} c$$

* ie massless field particles carry energy w/o mass

S.R. "mixes" things ....

basic idea:



Space-time

x^μ = 'contravariant' 4-vector

$= (ct, \vec{x})$

σ depends on frame

so of x^μ, some see space & others see time $\Big\}$ relative

Same w/ $p^\mu = \left(\frac{E_0}{c}, \vec{p}\right)$

Now clearly something is constant to all frames, that's $\sim$ to vector magnitude

here called the invariant

In 3-D $\qquad |\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}} \quad$ = magnitude

$$\underbrace{|\vec{A}|^2 = \vec{A}\cdot\vec{A}}_{\text{normal dot prod}} \quad = \text{magnitude}^2$$

w/ 4-vectors invent new 4-vector dot prod

$$|I|^2 = \sum_{\mu=0,1,2,3} x_\mu x^\mu = x_\mu x^\mu$$

Einstein Sum Convention,

where

x_μ = covariant 4-vector

and you 'make' X_μ according to

$$X_\mu = \sum_{(\nu=0,1,2,3)} g_{\mu\nu} x^\nu$$

$\Uparrow$

metric, defines the geometry of space

Then

$$|x^\mu|^2 = x_\mu x^\mu = c^2t^2 - x^2 - y^2 - z^2 = \text{constant for all observers}$$

$$\xi \quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This is the goal of G.R.

From Hartle: AJP 74 (1) 14-21 (2006)

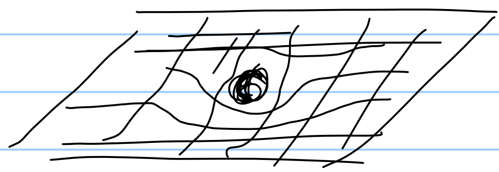
$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

$$\underbrace{R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(x) R}_{\text{curvature of space}} = \underbrace{\frac{8\pi G}{c^4} T_{\mu\nu}}_{\text{Stress Energy-momentum Tensor}} = \text{Einstein's Equation}$$

goal is solve

= 10 nonlinear, partial diff-Q's.

ex: symmetric, non radiating, spherical mass m



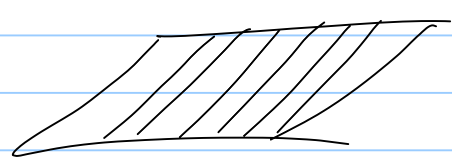
$$x^\mu = (ct, \underbrace{r, \theta, \phi}_{\text{spherical symmetry}})$$

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\,dt)^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2$$
$$+ r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

or

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{2GM}{c^2 r} & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

Note if $M = 0$; you should have flat space



& then $ds^2 = -(c\,dt)^2 + \underbrace{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi}_{\text{just Flat (3-D) spherical coords}}$

dVol

From S.R., also see how <u>invariance</u> can lead to introducing fields.

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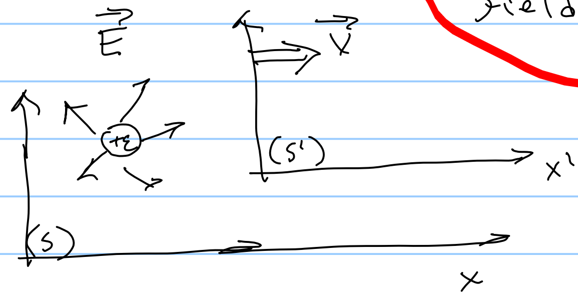


in (S) $\vec{E}$ only

in (S') $\vec{E}$ & $\vec{B}$

Since niether S or S' is preserved, must be that $\vec{E}$ & $\vec{B}$ are part of Bigger field that depending on Frame, leads one to conclude $\vec{E}$, $\vec{E}$ & $\vec{B}$ and different projections of Both!

$$\text{Field Tensor} = \begin{Bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & B_x & 0 \end{Bmatrix} \equiv \underbrace{F^{\mu\nu}}_{\substack{\text{most} \\ \text{Basic} \\ \text{descrip'l of} \\ \vec{E}\ \&\ \vec{B}}}$$

So we have $\underbrace{x^\mu, p^\mu}_{\substack{\text{Tensor} \\ \text{rank 1}}}$ & $\underbrace{F^{\mu\nu}}_{\substack{\text{Tensor} \\ \text{rank 2}}}$

ideas from Griffith, Intro to particles, Chpt 11 (Elementary)

So, invariance leads to introducing new fields

How can you do physics then?

Well, Euler-Lagrange equations of motion work (ie minimize action integral .... later because of invariant phase $\to e^{iS/\hbar}$ & smallness of $\hbar$

$S = \int L\,dt =$ energy-time

But in any case, you start by constructing a Lagrangian

$\mathcal{L} = T - V$ Then

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0$$

gives $\sum \vec{F} = m\vec{a}$

So when you "build" your $\mathcal{L}$ you have to ask *

1) does your $\mathcal{L}$ look the same to all observers (ie ... must have be S.R. include $\mathcal{L}$

(ie so space-time energy-momentum invariant)

2) Any other invariances?

The answer is .... yes. Global → Local
Guage Theory

invariance must be conserved everywhere but remember, 'causal' things can't happen faster than c (ie can't be instant)

So need to add Local or Guage fields to make $\mathcal{L}$ be Global → Local invariant

$$\mathcal{L} = \underbrace{T - V}_{\text{w/ S.R.}} + \underbrace{(\text{Guage fields})}_{\text{to make Global} \to \text{Local invariant.}}$$

in Terms of fields & field densities so Φ_{e^-} field

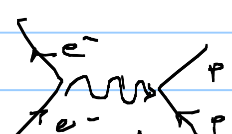
w/ S.R. to mak space-time energy-moment invariant

to make Global → Local invariant.

Then do E.L. to get Dynamics! w/ Feynman diagrams

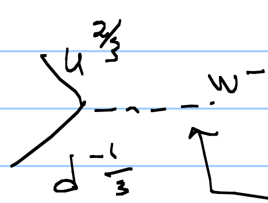
These are things we can measure!

1) collisions



where you interpret all the math in between

2) decays (lifetimes)



Field looks like $F^{\mu\nu}$

or

Weak W^-, W^+, Z^0

or gluons!

Any thing else, invariance, you need to worry about in the $\mathcal{L}$?

Yeah.... Symmetries that got Lost!
Broken Symmetries
↳ ie symmetries you don't see now but may have been there before (ie perm magnet)

you need to add these to

$$\mathcal{L} = \underbrace{(T-V)}_{S.R.} + \text{Gauge fields} + \underbrace{\text{Spont Broken Symmetry fields}}_{\text{These are Higgs fields!}}$$

Now all fields start out massless

$E_T^2 = p^2c^2 + m^2 c^4$ But still carry Energy.

When do the dynamics of $\mathcal{L}$ by E.L. The Feynman diagrams can look like give the fields mass!

Now must look again, This is $\cong$ Story! → from mem.
See Griffiths Chp 11, Intro to Elem. Particles!