E.F. Deveney / BSC Physics. PH402 Quantum 1 — Schrö, K.G., Dirac } W. Equat's.

Note Title 3/10/2006

A better Theory must include S.R. & be Lorentz invariant. Thus $E_{TOT} = \gamma m_0 c^2 = \sqrt{p^2c^2 + m_0^2c^4}$

$\Rightarrow$ start of Quantum Field Theory.

idea: Schrö $\left[\text{particle Like}\right]\Psi = \left[\text{wave-like}\right]\Psi$

use Energy

$E_{wave} = \hbar\omega$

$\Psi \propto e^{i(kx-\omega t)}$

* not relativistic

$$\hat{H}_{classical}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Note: No spin here! Spin is missing unless add it in by HAND! S.R. Requires it!

$$\left(\frac{\hat{p}^2}{2m} + V(\vec{r})\right)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

99% right for atomic e^-'s

H(1s) $\sim 10^6$ m/s $= \frac{1}{100}$ speed of light

Better Lorentz invariance of

1.) Kinematics & Maxwell's Equat $\Rightarrow [ct, \vec{x}]$ space time mix 4-vect

2.) Dynamics $\Rightarrow \vec{p} = \gamma_u m\vec{u}$

& $E_{TOT} = \gamma m_0 c^2$

or

$$E_{TOT}^2 = p^2c^2 + m_0^2c^4$$

energy-momentum 4-vect $\Rightarrow \left[\frac{E_T}{c}, \vec{p}\right]$

$$\left[\underset{E_T}{\text{particle}}\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$\left[\hat{E}_{TOT}\ \text{wave}\right]$ doesn't change!

use $E_T^2 = p^2c^2 + m_0^2c^4$

Klein-Gordon pg 313

idea — why not? who says E_T ... a stone all making up this hybrid problem!

$$\underbrace{\left[E_T^2\right]}_{\text{particle relativistic}}\Psi = \underbrace{\left[E_T^2\right]}_{\text{wave relativistic}}\Psi$$

= H.W. 3.4 pg 59

note $E_{T\,\text{wave}}^2 = \hbar^2\omega^2$

$$c^2(-i\hbar)^2\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial x} + m_0^2c^4\Psi = -\hbar^2\frac{\partial^2\Psi}{\partial t^2}$$

$$-\hbar^2c^2\frac{\partial^2\psi}{\partial x^2} + m_0^2c^4\Psi = -\hbar^2\frac{\partial^2\Psi}{\partial t^2}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{m_0^2c^2}{\hbar^2}\Psi = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}$$

= K.G. Equat

Note: $m_0 = 0$ = mass less

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \text{wave equat}$$

for massless traveling wave @ c

ie Light = photon

= Boson Spin = 0

No spin here either!

Dirac pg 317

Awesomely Done! Great Free particle soln!

2^{nd} order diffy Q in time

$\Rightarrow$'s 2 arbitrary const.

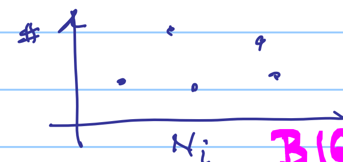
So $\Psi = A\psi_1 + B\psi_2$

Can't reconcile probability

ie $N(\psi_1 + i\psi_2)$

use this to Normalize ie get probability

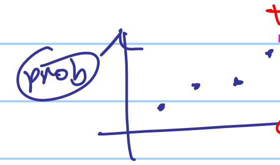
ie



N_{TOT}

BIG!

Another way to say this!



Dirac wanted a linear $\hat{O}$ so that $\hat{O}$'s keept the Ψ's in the same Hilbert Space thus pressurving probability!

$$\sum_i \frac{N_i}{N_{TOT}} = 1$$

So Dirac wanted to keep 1^{st} order in time.

So Dirac wanted

$$\hat{H}_{\text{Relativistic}} \Psi = E_{\text{rel}} \Psi \quad \Leftarrow$$

doesn't change — apparent

$E_{Tot} = \hbar\omega$ = relativistic

so is $p = \frac{h}{\lambda} = \hbar k$ = relativistic

$$\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad \Leftarrow$$

note E_{rel} not E_{rel}^2

Keeps 1st order in time

Now $\hat{H}_{rel} = \hat{H}_{Dirac}$

well $E_T^2 = p^2c^2 + m_0^2c^4$ $\quad p = \gamma m \vec{u}$

& we know

$E_T \to m_0 c^2$ is $\vec{u} = 0$

& $E_T \to pc = \frac{h}{\cancel{\lambda}} \cancel{\lambda} \nu = h\nu$

is massless

So

Force $\quad E_T = pc + mc^2 = \text{ISH}$

But clearly $E_T = \sqrt{p^2c^2 + m_0^2c^4} \neq$

So what would you have to do is tried

$$\sqrt{p^2c^2 + m_0^2c^4} = c\vec{\alpha}\cdot\vec{p} + \beta m_0 c^2$$

in other words, what would $\vec{\alpha}$ & β need to be?

Note: Foreshadowing

$\vec{\alpha}$ = vector-matrix

Note this is all great Honor's proj stuff

So Dirac (1929) set out to solve

Spock story ⇕ Dirac about looking out the window

$$\hat{H}_0 \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\left[c\,\underset{\sim}{\vec{\alpha}}\cdot\vec{p} + \underset{\sim}{\beta}\, m_0 c^2\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

* = pg 317 Scherrer

* & W.B. Rolnick "Fund Particles & Their Interactions" pg 52-60

where

$$\left[c\,\underset{\sim}{\vec{\alpha}}\cdot\vec{p} + \underset{\sim}{\beta}\, m_0 c^2\right]^2 = E_T^2 = p^2c^2 + m_0^2 c^4$$

this is How Rolnick does it. See my notes From Rolnick on this!

The <u>ONLY</u> way this can be accomplished is IS

$$\underset{\sim}{\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \underset{\sim}{\mathbb{1}} & 0 \\ 0 & \underset{\sim}{\mathbb{1}} \end{pmatrix}$$

$$\&\ \underset{\sim}{\vec{\alpha}} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}}_{\alpha_x},\ \underbrace{\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}}_{\alpha_y},\ \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}}_{\alpha_z}$$

$$\underset{\sim}{\alpha_i} = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$$ where σ_i = Pauli SPIN MATRICES!

ie chpt 8 Scherrer.

spin ... yikes?

So $\hat{H}_D \Psi = i\hbar \frac{\partial \Psi}{\partial t}$ $\hat{H}_D$ = Linear $\hat{o}$ for E_{TOT} relativistic

$\Rightarrow$'s

$$\left[c \underbrace{\vec{\alpha}} \cdot \vec{p} + \underbrace{\beta} m_0 c^2 \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Relativistic REQUIRES introduction of SPIN!

was it there for Schrö

$$\left[\frac{p^2}{2m} + V(r) \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Nor for K.G.

Relativity $\Rightarrow$ invariance of
1) Kinematics
2) Max Equat
3) Dynamics

Requires Stuff called SPIN ... it is a relativistic Effect

Thought?

Since invariance of $\left\{ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \right\}$ $\Rightarrow$'s $[ct, \vec{x}]$ & $[\frac{E_0}{c}, \vec{p}]$

& it now $\Rightarrow$'s Existance of Spin!

Maybe need $\left[\frac{E_0}{c}, \text{spin}, \vec{p} \right]$ = 5 vector?

& dif frames see dif proj's of $E, S, \vec{p}$?

$$\left[c\underbrace{\vec{\alpha}}\cdot\vec{p} + \underbrace{\beta}\, m_0 c^2\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

relativistic Schrö' = Dirac Equat = requires spin!

any other surprises?

pg 320

Easy Soln: $p = 0$ (ie particle @ rest)

$$\left[c\vec{\alpha}\cdot\cancelto{0}{\vec{p}} + \beta\, m_0 c^2\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} m_0 c^2\, \Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

can't be 1 thing ... make 4

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} m_0 c^2 \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} = i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

or

$$\begin{pmatrix} m_0 c^2 \psi_1 \\ m_0 c^2 \psi_2 \\ -m_0 c^2 \psi_3 \\ -m_0 c^2 \psi_4 \end{pmatrix} = \begin{pmatrix} i\hbar\, \partial\psi_1/\partial t \\ i\hbar\, \partial\psi_2/\partial t \\ i\hbar\, \partial\psi_3/\partial t \\ i\hbar\, \partial\psi_4/\partial t \end{pmatrix}$$

Look @ Ψ_1 $\quad \frac{\partial\psi_1}{\psi_1} = \frac{mc^2}{i\hbar}t \Rightarrow \int\frac{\partial\psi_1}{\psi_1} = \int\frac{-imc^2}{\hbar}dt$

$\ln\psi_1 = \frac{-imc^2}{\hbar}t + C \Rightarrow \Psi_1 = N e^{-\frac{imc^2 t}{\hbar}}$

So $\Psi_1 = N e^{\frac{-imc^2}{\hbar}t}$

we know $\hat{H}_{wave}\Psi = -i\hbar \frac{\partial \Psi}{\partial t} = E\Psi$

$$\hat{H}_w \Psi = -i\hbar\left(\frac{-imc^2}{\hbar}\right)\Psi$$

$$H_w\Psi = +mc^2\,\Psi = E\,\Psi$$

$$E_1 = m_0 c^2$$

For Ψ_2 also get $E_2 = m_0 c^2$

Ψ_3 get $E_3 = -m_0 c^2$ $\quad \Psi_3 = N e^{\frac{+imc^2}{\hbar}t}$

Ψ_4 get $E_4 = -m_0 c^2$

So:

$$\left[c\,\underline{\alpha}\cdot\vec{p} + \underline{\beta}\, m_0 c^2\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

has 4 solutions!

2 w/ $E = m_0 c^2$ = spin ↑ & spin ↓

2 w/ $E = -m_0 c^2$

Now recall $\underline{\alpha}$ = spin matrix is $|\text{spin}| = \frac{1}{2}$

Then $S = \overset{1/2}{\uparrow}$ or $\overset{-1/2}{\downarrow}$

Thus

Then interpret neg energy solns as antiparticles!

$$m_0 c^2 = \uparrow, \downarrow \quad ; \; e^-$$

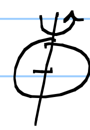
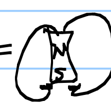
$$-m_0 c^2 = \uparrow, \downarrow \quad ; \; e^+ \Big\}$$

Consistent w/ Pauli Exclusion.... any Solns for

Fermion Spin $= n\left(\frac{1}{2}\right)$ $\quad n = 1, 2, 3 \ldots$

Say

e^-, spin $= \left(\frac{1}{2}\right)$ can have 2 states of same energy $\uparrow$ & $\downarrow$

* Note: since  = tiny $\vec{\mu}$ =  so will have

$\vec{\mu} \cdot \vec{B}_{ext}$ energy so will change energy

& thus will depend on Spin $\uparrow$ or $\downarrow$

<u>here</u> we have not included these perturbations into $\hat{H}$

so

are degenerate!

Conclude:

Non relativistic
No Spin
Schrö

$$\left[\frac{\hat{P}^2}{2m}+V\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Klien-Gordon
relativistic
Spin = 0 only

$$\frac{\partial^2\psi}{\partial x^2}-\frac{m_o^2c^4\Psi}{\hbar^2}=\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

Dirac
relativistic
Spin $n(\frac{1}{2})$, $n=1,2\ldots$

$$\left[c\vec{\alpha}\cdot\vec{p}+\beta m_o c^2\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$\Uparrow$

For 99% of things you want to do having to do with the mascropic world
people
medicine
Technolog!

ie Engineers of the Future will start here instead of $\vec{F}=m\vec{a}$ EED

$\Downarrow$

Spin = 0
Bosons = Forces
ie The photon!
γ = spin 0

But not
$W^-, W^+, Z^0 \Rightarrow$ spin 1

Graviton $\Rightarrow$ spin 2

$\downarrow$

e^-'s, ν's
& q's

FERMIONS
= particles

particle physics
Fundamental physics
RIGHT NOW!

$\rightarrow$ to do better start w/

$$\hat{H}=E_K+E_P$$

Then add in other energies, $\hat{H}'$ = perturbations
such as spin-orbit
$\vec{\mu}_s\cdot\vec{B}_{orbit}$ } that are semi-classical approximations!