

A better Theory must include S.R. & be Lorentz invariant. Thus $E_{tot} = \gamma m_0 c^2 = \sqrt{p^2 c^2 + m_0^2 c^4}$
 \Rightarrow start of Quantum Field Theory.

idea: Sclerö $\left[\begin{matrix} \text{particle} \\ \text{like} \end{matrix} \right] \Psi = \left[\begin{matrix} \text{wave-like} \end{matrix} \right] \Psi$
 use Energy

* not relativistic

$$\hat{H}_{classical} \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$E_{wave} = \hbar \omega$$

$$\Psi_{cl} = e^{i(kx - \omega t)}$$

Note:

→ No spin here!

Spin is missing unless add it in by HAND!

S.R. Requires it!

$$\left(\frac{\hat{p}^2}{2m} + V(\vec{r}) \right) \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

99% right for atomic e's

$$H(1s) \sim 10^6 \text{ eV}$$

$$= \frac{1}{100} \text{ speed of light}$$

↓ Better

Lorentz invariance of

1) Kinematics

↳ Maxwell's Equat

$$\Rightarrow [ct, \vec{x}]$$

space-time 4-vec
mix

2) Dynamics

$$\Rightarrow \vec{p} = \gamma m \vec{u}$$

$$\& E_{tot} = \gamma m_0 c^2$$

or

$$E_{tot}^2 = p^2 c^2 + m_0^2 c^4$$

energy- 4-vec
momentum

$$\Rightarrow \left[\frac{E_{tot}}{c}, \vec{p} \right]$$

[particle] $\Psi = \begin{bmatrix} E_T \\ p \end{bmatrix}$ $\frac{\partial \Psi}{\partial t}$

doesn't change!

use $E_T^2 = p^2 c^2 + m_0^2 c^4$

Dirac-Gordan pg 313

idea

who says E_T ...
 why not? ...
 asterisk walking up
 this hybrid
 problem!

$\begin{bmatrix} E_T^2 \\ p \end{bmatrix} \Psi = \begin{bmatrix} E_T^2 \\ p \end{bmatrix} \Psi$

particle relativistic = h.w. 3.4 pg 59

nobe $E_T^2 = \hbar \omega^2$

$c^2 (-i\hbar)^2 \frac{\partial^2 \Psi}{\partial x^2} + m_0^2 c^4 \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial t^2}$

$-\hbar^2 c^2 \frac{\partial^2 \Psi}{\partial x^2} + m_0^2 c^4 \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial t^2}$

$\frac{\partial^2 \Psi}{\partial x^2} + \frac{m_0^2 c^2}{\hbar^2} \Psi = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}$

= Klein Equation

Note: $m_0 = 0$ = massless

$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}$ = wave equation

massless
 for traveling wave @ c

ie light = photon

= Boson Spin = 0

No spin here either!

Dirac pg 317

Awesome by
 Done!
 Great Steve
 particle
 soln!

2nd order diffy Ψ in time

\Rightarrow 2 arbitrary const.

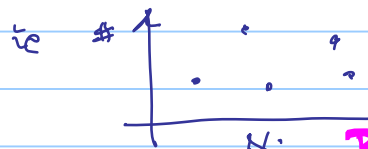
So $\Psi = A \Psi_1 + B \Psi_2$

Can't reconcile probability

ie $N(\Psi_1 + i\Psi_2)$

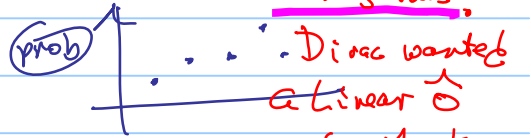
use this to

Normalize ie get probability



BIG!

Another way to say this!



Dirac wanted a linear \hat{O}

So that $\sum_i \frac{h_i}{N_{tot}} = 1$ \hat{O} 's kept the Ψ 's in the same Hilbert Space thus preserving

So Dirac wanted to keep 1st order in time.

So Dirac wanted

$$\hat{H}_{\text{relativistic}} \Psi = E_{\text{rel}} \Psi \quad \Leftarrow$$

↓
doesn't change apparent

$$E_{\text{rel}} = \hbar \omega = \text{relativistic } \hbar \omega$$

$$\text{so is } p = \frac{h}{\lambda} = \hbar k = \text{relativistic } \hbar k$$

$$\hat{H} = i\hbar \frac{\partial \Psi}{\partial t} \quad \Leftarrow$$

note E_{rel}
not E_{rel}^2

keeps 1st order
in time

New $\hat{H}_{\text{rel}} = \hat{H}_{\text{Dirac}}$

$$\text{well } E_T^2 = p^2 c^2 + m_0^2 c^4 \quad p = \gamma m \vec{u}$$

we know

$$E_T \rightarrow m_0 c^2 \quad \text{is } \vec{u} = 0$$

$$\frac{1}{c} E_T \rightarrow pc = \frac{h}{\lambda} \lambda = h \lambda$$

is massless

So

FORCE

$$E_T = pc + mc^2 = \underline{\underline{ISH}}$$

$$\text{But clearly } E_T = \sqrt{p^2 c^2 + m_0^2 c^4} \neq$$

So what would you have to try

Note:
Forslanking

\vec{A}
= vector-matrix

$$\sqrt{p^2 c^2 + m_0^2 c^4} = \underbrace{c}_{\vec{A}} \cdot \underbrace{\vec{p}}_{\vec{B}} + \underbrace{m_0 c^2}_{\vec{C}}$$

in other words, what would \vec{A} & \vec{B} need to be?

Note this is all great Honor's pro's stuff

Spock story
 ↓
 Dirac about looking out the window

So Dirac (1929) set out to solve

$$\hat{H}_D \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

* = pg 317 Schenker

* E. W. B. Rolnick "Fund Particles & Their Interactions" pg 52-60

where

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right]^2 = E_T^2 = p^2 c^2 + m_0^2 c^4$$

this is how Rolnick does it. See my notes from Rolnick on this!

The ONLY way this can be accomplished

is $\mathbb{I} \Rightarrow$

$$\beta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \uparrow & 0 \\ 0 & \downarrow \end{pmatrix}$$

$$\alpha_i = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$\alpha_x \qquad \alpha_y \qquad \alpha_z$

$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$ where $\sigma_i =$ Pauli SPIN MATRICES!
 ie. Dirac's Schenker.

SP in ... gives?

$$\sum_{\vec{s}} \hat{H}_0 \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$\hat{H}_0 =$ Linear $\hat{\sigma}$
for Error
relativistic

\Rightarrow

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

↑ Relativistic REQUIRES introduction
of SPIN!

was it there for Schröd

$$\left[\frac{p^2}{2m} + V(r) \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Non for KG.

Relativity \Rightarrow invariance of
1) Kinematics
2) Max Speed
3) Dynamics

Requires stuff called
SPIN ... it is a relativistic
Effect

* Thought?

Since invariance of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow$ $[\vec{r}, \vec{x}] \} \{ [\frac{E_0}{2}, \vec{p}]$

$\frac{1}{2}$ it now \Rightarrow Existence of SPIN!

Maybe need $[\frac{E_0}{2}, \text{Spin}, \vec{p}] = 5$ vector?

$\frac{1}{2}$ diff frames see diff proj's of E, S, \vec{p} ?

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right] \Psi = i \hbar \frac{\partial \Psi}{\partial t}$$

relativistic
Schrödinger
= Dirac Equation
= requires spin!

any other surprises?

pg 320
Easy Soln: $p=0$ (ie particle @ rest)

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right] \Psi = i \hbar \frac{\partial \Psi}{\partial t}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} m_0 c^2 \Psi = i \hbar \frac{\partial \Psi}{\partial t}$$

can't be 1 thing - ... make 4

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} m_0 c^2 \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = i \hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

$$\text{or } \begin{pmatrix} m_0 c^2 \psi_1 \\ m_0 c^2 \psi_2 \\ -m_0 c^2 \psi_3 \\ -m_0 c^2 \psi_4 \end{pmatrix} = \begin{pmatrix} i \hbar \frac{\partial \psi_1}{\partial t} \\ i \hbar \frac{\partial \psi_2}{\partial t} \\ i \hbar \frac{\partial \psi_3}{\partial t} \\ i \hbar \frac{\partial \psi_4}{\partial t} \end{pmatrix}$$

Look @ ψ_1

$$\frac{\partial \psi_1}{\psi_1} = \frac{m_0 c^2}{i \hbar} t \Rightarrow \int \frac{\partial \psi_1}{\psi_1} = \int \frac{-i m_0 c^2}{\hbar} dt$$

$$\ln \psi_1 = \frac{-i m_0 c^2}{\hbar} t + C \Rightarrow \psi_1 = N e^{\frac{-i m_0 c^2}{\hbar} t}$$

$$\text{So } \Psi_1 = N e^{\frac{-im_0 c^2}{\hbar} t}$$

we know $\hat{H}_{\text{Dirac}} \Psi = -i\hbar \frac{\partial \Psi}{\partial t} = E \Psi$

$$\hat{H}_0 \Psi = -i\hbar \left(\frac{-im_0 c^2}{\hbar} \right) \Psi$$

$$H_0 \Psi = +m_0 c^2 \Psi = E \Psi$$

$$E = m_0 c^2$$

For Ψ_2 also get $E_2 = m_0 c^2$

Ψ_3 get $E_3 = -m_0 c^2$

Ψ_4 get $E_4 = -m_0 c^2$

$$\Psi_3 = N e^{\frac{+im_0 c^2}{\hbar} t}$$

So:

$$\left[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2 \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

has 4 solutions!

$$2 \omega / E = m_0 c^2$$

$$2 \omega / E = -m_0 c^2$$

$\uparrow = \text{spin } \uparrow$ $\downarrow = \text{spin } \downarrow$

Now recall $\alpha_3 = \text{spin matrix}$ is $|\text{spin}| = \frac{1}{2}$

then $\frac{1}{2}$ $-\frac{1}{2}$
 $S = \uparrow$ or \downarrow

thus

Then interpret neg energy solns as anti-particles!

$$\begin{aligned} m_0 c^2 &= \uparrow, \downarrow ; e^- \\ -m_0 c^2 &= \uparrow, \downarrow ; e^+ \end{aligned}$$

Consistent w/ Pauli Exclusion.... any
Solns for

$$\text{Fermion spin} = n\left(\frac{1}{2}\right) \quad n=1, 2, 3, \dots$$

Say

e^- , spin = $\left(\frac{1}{2}\right)$ can have 2
states of same energy $\uparrow \ \ \ \downarrow$

* Note: since $\vec{u} = \text{tang } \vec{u} = \vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ so will have

$\vec{u} \cdot \vec{B}_{\text{ext}}$ energy so will change energy
 $\frac{1}{2}$ thus
will depend on spin \uparrow or \downarrow

here we have not included these
perturbations into \vec{u}
so
are degenerate!

Conclude:

Non relativistic
No spin
Schro

$$\left[\frac{\hat{p}^2}{2m} + V \right] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Klein-Gordon
relativistic
Spin = 0 only

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{m_0^2 c^4 \Psi}{\hbar^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}$$

Dirac
relativistic
Spin $n(\frac{1}{2}), n=1,2, \dots$

$$[c \vec{\alpha} \cdot \vec{p} + \beta m_0 c^2] \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

For 99% of things you want to do having to do with the macroscopic world people medicine technology!

Spin = 0
Bosons = Feyn
ie The photon!
 $\otimes = \text{spin } 0$
But not
 $W^-, W^+, Z^0 \Rightarrow \text{spin } 1$
Graviton $\Rightarrow \text{spin } 2$

$e^-s, \nu s$
 $\frac{1}{2} q s$
FERMIONS
= particles
particle physics
Fundamental physics
RIGHT NOW!

ie Engineering of The Future

will start here instead of $\vec{F} = m\vec{a}$
EFTD

to do better start w/
 $\hat{H} = E_k + E_p$

Then add in other energies, $\hat{H}' \equiv$ perturbations
such as spin-orbit } that are semi-classical approximations!
 $\vec{M}_S \cdot \vec{B}$ orbit }