Our goal here is to understand Schrödinger's equation and its solutions and implications.

Not how to solve for now.

Keys:
1. Normalization
2. Probabilistic outcome of Schrödinger equation (Max Born 1926)
3. Probability
4. Observables
5. Ensembles of a quantum state
6. Expectation values!

How by example: 1-D particle in box (well) only deep.

"Ideal"

Caution: Not a "realizable"

Real world problem

Why? Easy, Tremendous insight

BIG ⇒ Further: All problems are very tough, but almost always reduce down to there "ideal" case's to get 1st order solutions!
1-D, 0-well.

\[ V(x) = \frac{q}{\varepsilon_0} \]

\[ \nabla^2 V = 0 \]

\[ V(0) = 0 \]

\[ V(x=\ell) = 0 \]

\[ V(x) = m \cdot x + b \]

\[ \nabla^2 V = 0 \text{ inside} \]

\[ E^\text{inside} = \nabla V(x) = 0 \text{ no } E \text{ field inside.} \]

Let capacitor get very thin then here

\[ V(x=0) = 0 \]

\[ V(x=\ell) = V_0 \]

\[ x=0 \]

\[ x=\ell \]
Now bit of confusion

\[-V_0 = \text{voltage} = \frac{\text{force}}{\text{charge}}\]

So

\[E_{\text{pot}} = V(x) = e(-V_0) = (-e)(-V_0)\]

potential energy

\[E_p = eV_0 = (+)\]

So

\[E_{\text{tot}} = E_K + E_p\]

(Energy units)

\[E_{\text{tot}}(x)\]

\[\text{Region I}\]

\[E(x)\]

\[E_{\text{tot}}\]

\[\text{Region II}\]

\[V = \text{zero inside}\]

\[x=0\]

\[x=a\]

This is called a potential \(\text{WELL!}\)
Consider $E_{\text{tot}} = 1, 2, 3$

Inside $0 < x < a$

$E_{\text{tot}} = E_k + E_{\phi} = E_k$

Classically (well)

Outside $x \geq a$

$E_{\text{tot}} = E_k + E_p$

or $E_k = E_{\text{tot}} - E_p$

Problem Classically

For $E_1 < E_2 = E_T$

Thus $\Rightarrow$

$E_k = (-)$

because $E_T < E_p$

These regions are considered Classically Forbidden!

Classically NEVER can have particle there!

Regions $\Rightarrow$

II $\Rightarrow$ III

In Quantum mechanics there is no reason why not to consider solving Schrödinger in those regions.

Why? as you will see, the nature of the solutions to Schrödinger will need to be finite, smooth and continuous. So you will need to solve Schrödinger in all 3 regions except one (--- CASE ---)
Now, is ideal case (called 1-D well)
Let
\[ V_0 \to \infty \text{ Voltage} \]
Then
\[ V_{\text{potential energy}} \to \infty \text{ Joules} \]

Then this \( \infty \) well \( \to \) 's classical
and Schrödinger wave function particles can't get on there.

Thus, this \( \infty \) AND 1-D well is what we will solve in
Region I
This is "IDEAL"
Region I, \( V(x) = 0 \)

1-D Schrödinger wave Eq.

\[
\begin{align*}
H_{1D} \Psi(x,t) &= \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + i\hbar \frac{\partial \Psi(x,t)}{\partial t} \\
\hat{H}_{1D} &= \hat{H}_0 + \hat{V}_1 \\
\hat{H}_0 &= \hat{E}_x + \hat{E}_p \\
&= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)_{\text{inside}} \\
&= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}.
\end{align*}
\]

Solve for \( \Psi(x,t) \)

at \( \infty \) 1-D well

\[
\Psi(x,t) = \begin{cases} 
0 & x < 0 \\
A \sin \left( \frac{n\pi x}{a} \right) e^{\frac{i\hbar x^2}{2ma^2}} & 0 \leq x \leq a 
\end{cases}
\]