E. F. Deveney / BSC Physics: PH402 Quantum.

Note Title 2/25/2006

Schaumer C3
Schrö Sch only to understand what it all means!

Our goal here is to understand Schrö equation & its solutions and implications.

Not how to solve for now.

Keys: 1) Normalization → Hilbert → function spaces
→ connect to matrix
2) Probabilistic interp of Schrö equation (Max Born 1926)
3) Probability
4) Observables
5) ensembles of a Quantum state
6) expectation values!

How by example: 1-D particle in Box (well) ∞-ly deep.

"IDEAL"

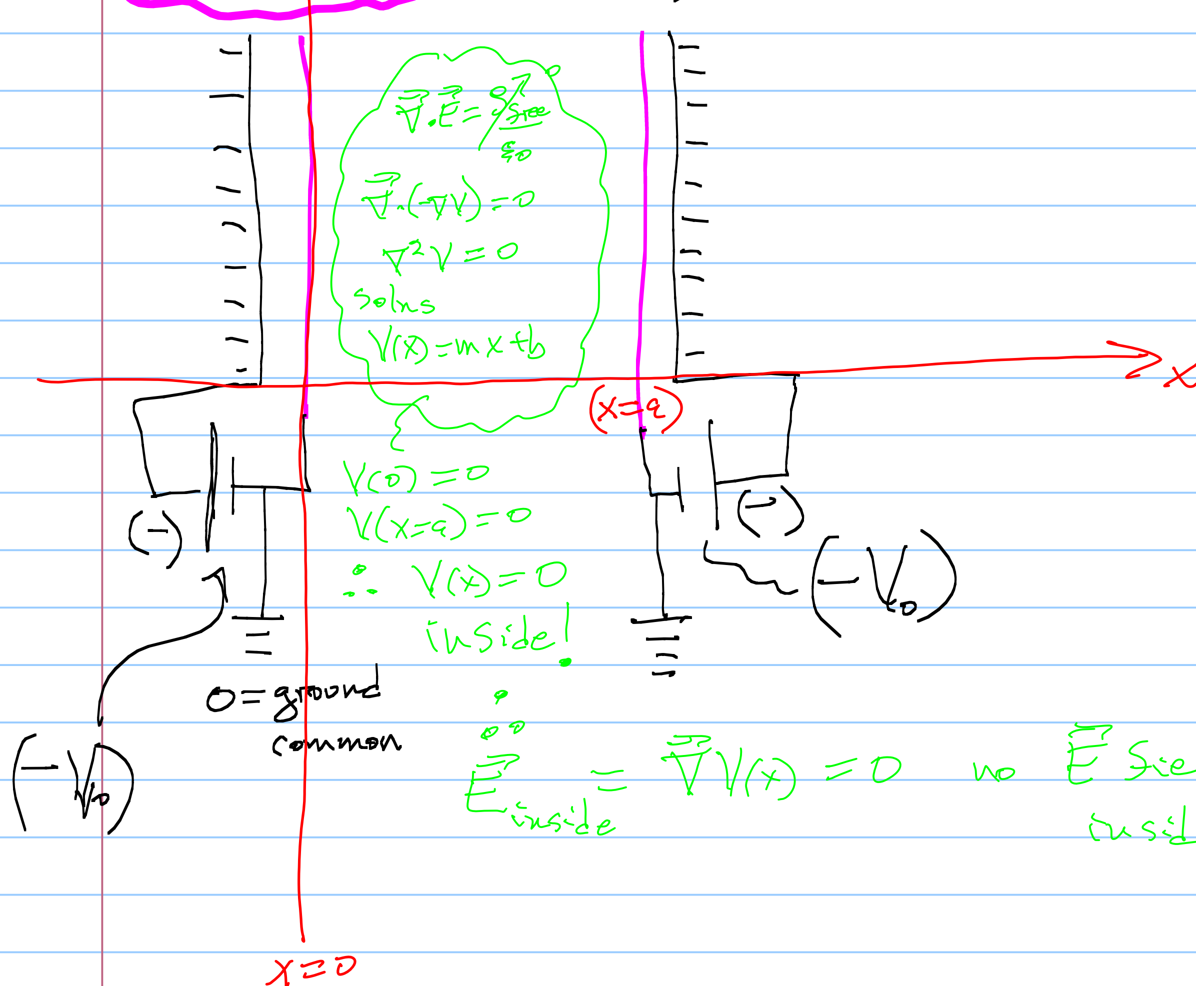
Caution: Not a "realizable" real world problem

Why? Easy, Tremendous Insight

BIG ⟹ Further: All problems are very tough, almost always reduce down to these "ideal" case's to get 1st order Soln's!

1-D, ∞-well.

essentially capacitors



$\vec{\nabla}\cdot\vec{E} = \cancel{\rho_{free}}/\varepsilon_0$

$\vec{\nabla}\cdot(-\nabla V) = 0$

$\nabla^2 V = 0$

solns

$V(x) = mx + b$

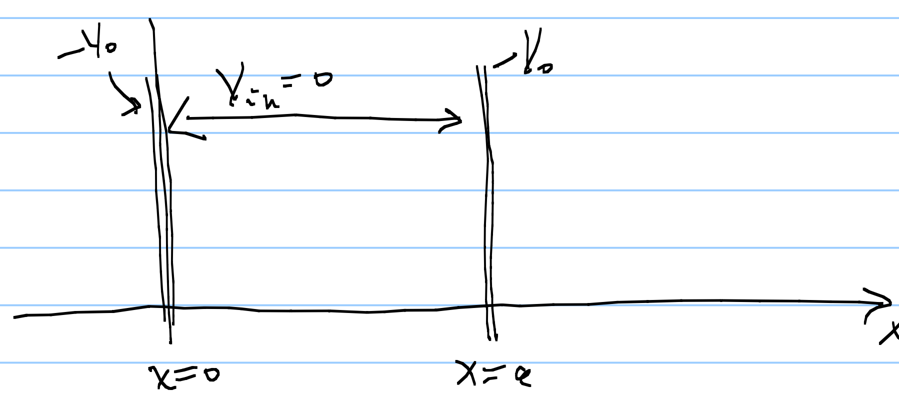
$V(0) = 0$

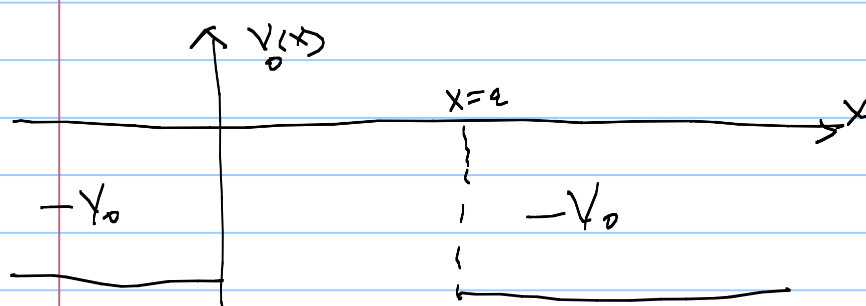
$V(x=a) = 0$

$\therefore V(x) = 0$ inside!

$\therefore \vec{E}_{inside} = \vec{\nabla}V(x) = 0$ no $\vec{E}$ field inside.

Let capacitor get ∞ly thin then have





now bit of confusion

$$-V_0 = \text{voltage} = \frac{\text{Joule}}{\text{charge}}$$

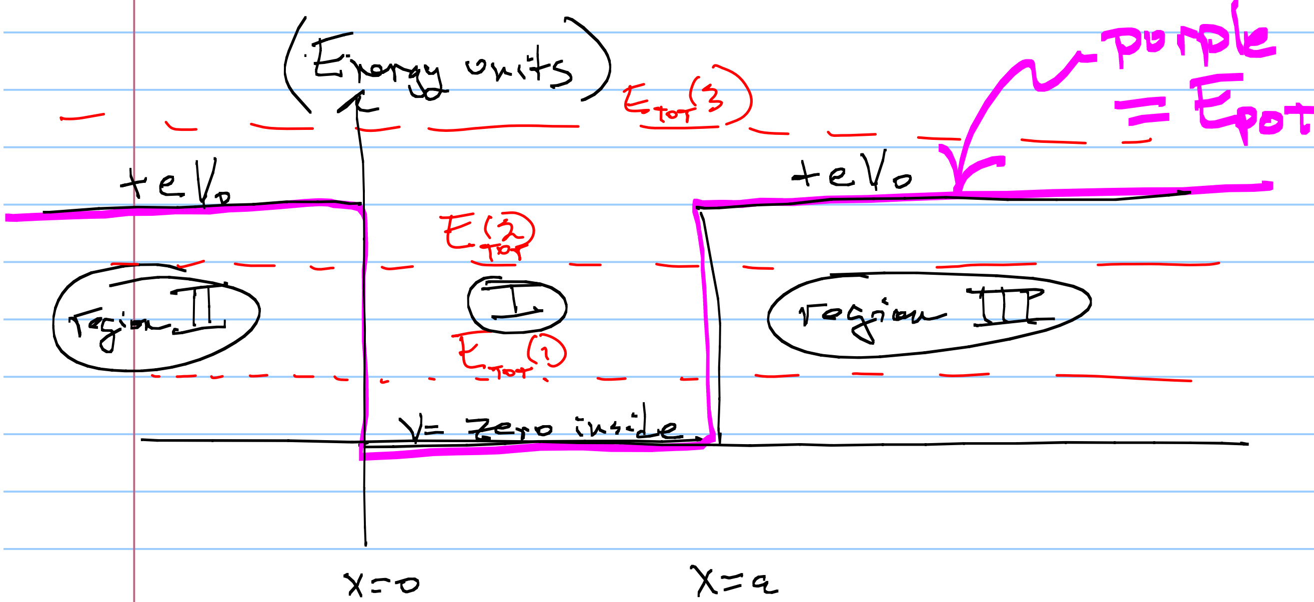
so

$$E_{pot} = V(x) = q(-V_0) = (-e)(-V_0)$$

potential energy

$$E_p = eV_0 = (+)$$

so $E_{TOT} = E_K + E_p$



This is called a potential WELL!

BIG

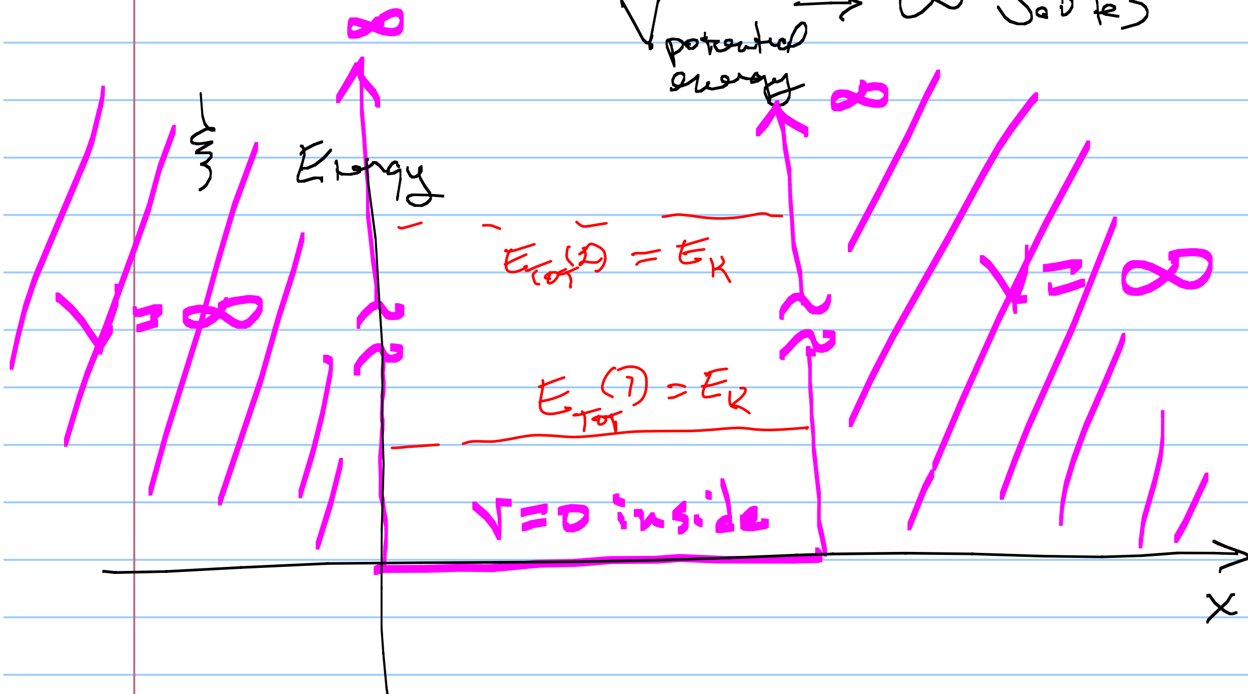
Consider E_{TOT} 1, 2, 3

inside = $(0 < x < a)$	outside $x < 0$, $x > a$
$E_{TOT} = E_K + \cancel{E_P}^{0} = E_K$	$E_{TOT} = E_K + E_P$ or $E_K = E_{TOT} - E_P$
Classically Cool!	problem classically For E_1 & $E_2 = E_T$, Thus $\Rightarrow$'s $E_K = (-)$ because $E_T < E_P$
Regions II & III Classically NEVER can have particle there! $\Rightarrow$	These regions are considered Classically Forbidden!

In Quantum mechanics There is No reason why NOT to consider Solving Schrö in these regions.

Why? as you will see, The nature of the Solutions to Schrö will need to be Finite, smooth & continuous ∴ you will need to solve Schrö in all 3 regions except in I - CASE!

Now, if Ideal case
called ∞ 1-D well,
let
$V_0 \to \infty$ Voltage
Then
$V_{\text{potential energy}} \to \infty$ Joules



Then this ∞ well →'s classical
and Schrö's also gives particles
can't get in there.

Thus, this ∞ AND 1-D well is
what we will solve in
Region I

This is "IDEAL"

Region I, $V(x) = 0$

1-D Schrö time dep Equat

$$\hat{H}_{tot}\,\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

$$\hat{H}_{tot} = \hat{E}_k + \hat{E}_p$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \underbrace{V(x = \text{inside})}_{\to 0}$$

$$= \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

so

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

Solve for $\Psi(x,t)$

particle in ∞ 1-D well

$$\Psi(x,t) = \begin{cases} 0 & x < 0 \\ & x > a \\ A\sin\left(\frac{\pi x}{a}\right) e^{\frac{i\hbar\pi^2 t}{2ma^2}} & 0 \le x \le a \end{cases}$$