

Schurmer C.3
Schröd Schr
only to
understand
5/2006
what it
all means!

Our goal here is to understand Schröd equation & its solutions and implications.

Not how to solve for now.

- Keys:
- 1) Normalization → Hilbert → function spaces
→ connect to matrix
 - 2) Probabilistic interp of Schröd equation
 (Max Born 1926)
 - 3) Probability
 - 4) Observables
 - 5) ensembles of a Quantum state
 - 6) expectation values!

How by example: 1-D particle in Box (well) only deep.

“IDEAL”

Caution: Not a “realizable” real world problem

Why? Easy, Tremendous Insight

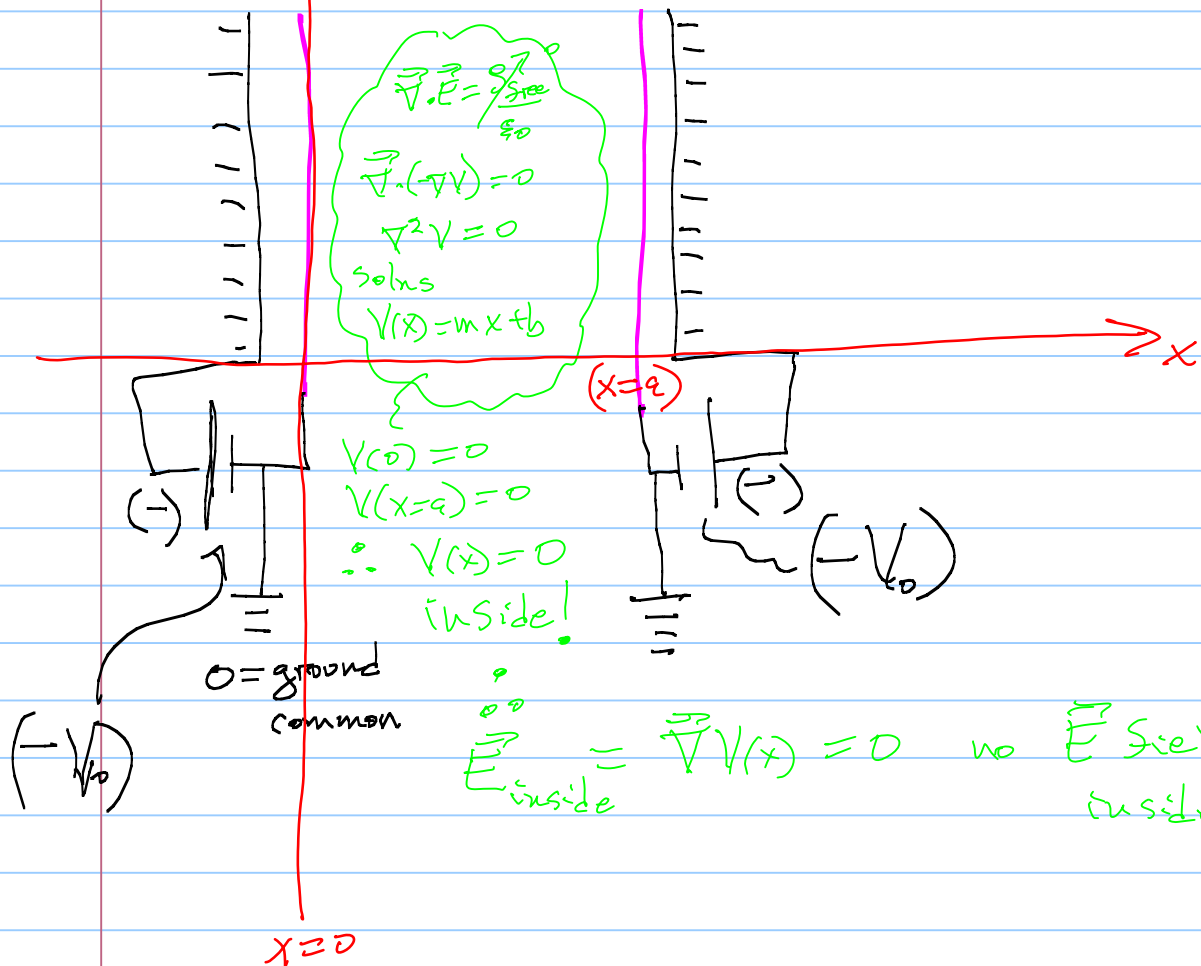
BIG ⇒

Further: All problems are very tough, almost always reduce down to these

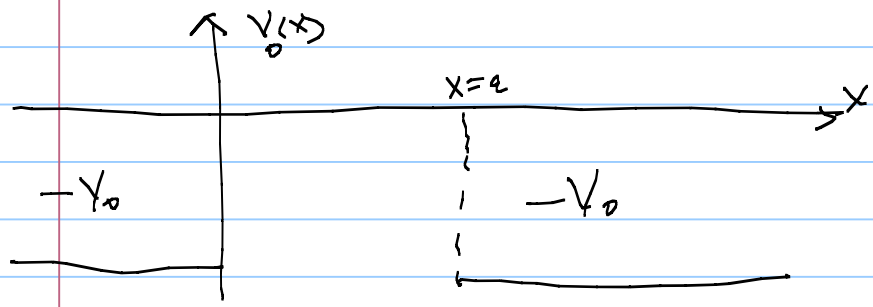
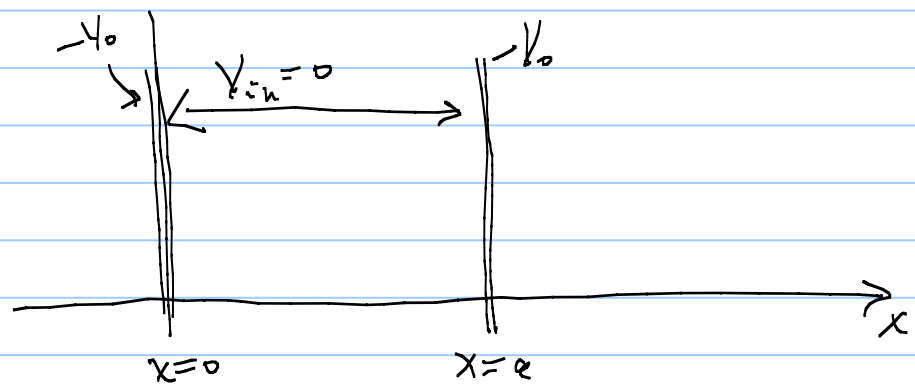
1st order “ideal” cases to get solutions!

1-D, ∞ -well.

essentially, capacitors



Let capacitor get ∞ by thin then here



now bit of confusion

$$-V_0 = \text{voltage} = \frac{\text{Joule}}{\text{charge}}$$

So

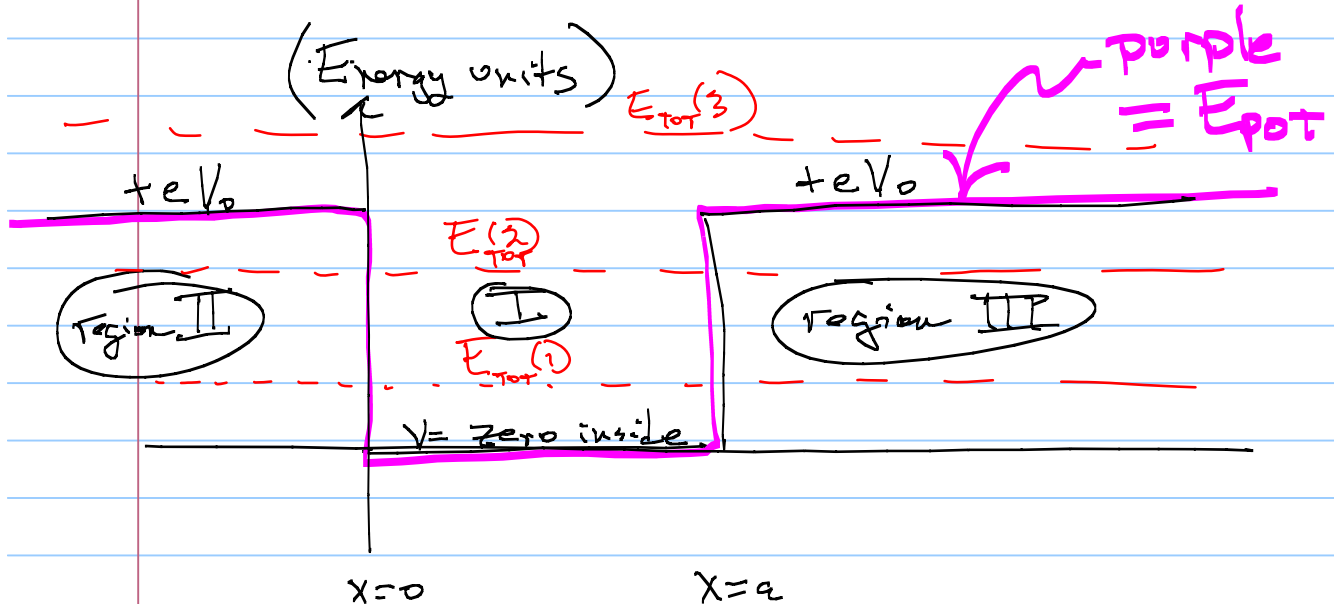
$$E_{\text{pot}} = V(x) = q(-V_0) = (-e)(-V_0)$$

↑
potential energy

$$E_p = eV_0 = (+)$$

So

$$E_{\text{TOT}} = E_k + E_p$$



This is called a potential
WELL!

BIG

Consider

$E_{TOT} 1, 2, 3$

inside $\equiv (0 < x < a)$

$$E_{TOT} = E_K + E_P^{\text{pot}} = E_K$$

classically
(val)

outside $x < 0$
outside $x > a$

$$E_{TOT} = E_K + E_P$$

$$\text{or } E_K = E_{TOT} - E_P$$

problem
classically

$$\text{For } E_1 \text{ or } E_2 = E_T,$$

thus \Rightarrow 's

$$E_K = (-)$$

because $E_T < E_P$

Regions I & II
Classically NEVER!
can have particle there!

These regions are considered

Classically Forbidden!

In Quantum mechanics there is no reason why NOT to consider solving Schrödinger in these regions.

Why? as you will see, the nature of the solutions to

Schrödinger will need to be finite, smooth & continuous
 \therefore you will need to solve Schrödinger in all 3 regions except in

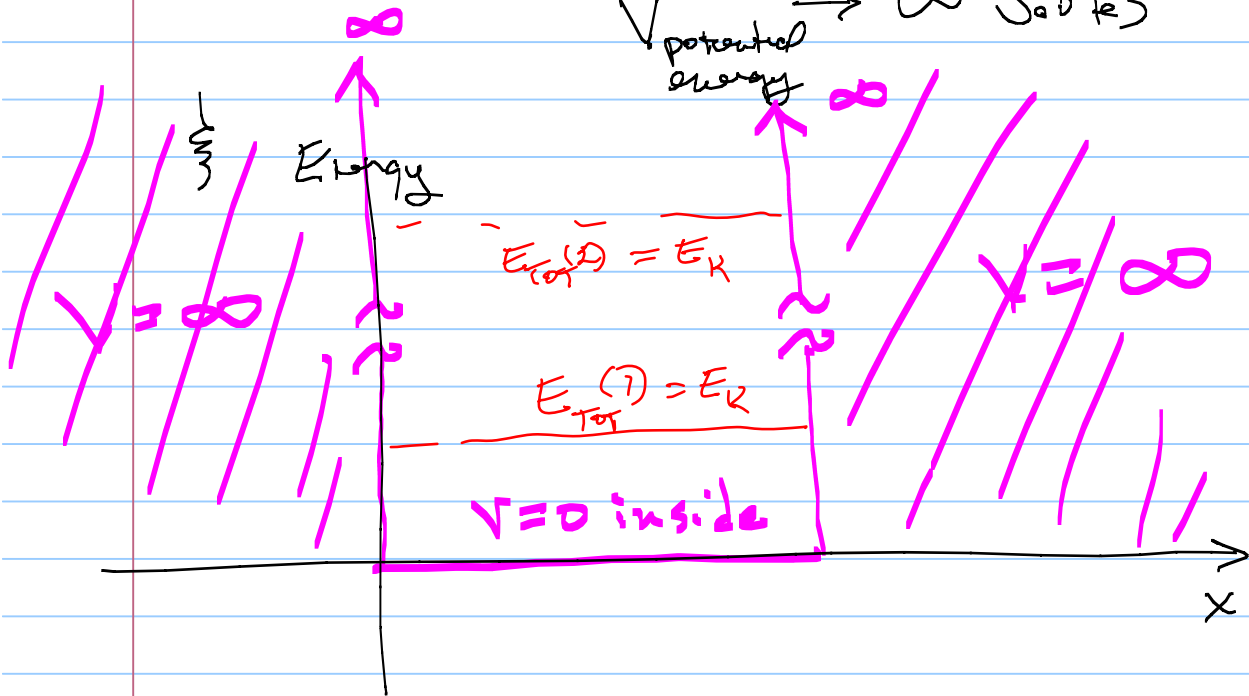
1 - CASE!

Now, is Ideal case
called ∞ 1-D well,
let

$V_0 \rightarrow \infty$ Voltage

Then

$V_{\text{potential energy}} \rightarrow \infty$ Joules



Then this ∞ well \rightarrow 's classical
and Schrod's also gives particles
can't get in there.

Thus, this ∞ AND 1-D well is
what we will solve in
Region I

This is "IDEAL"

Region I, $V(x)=0$

1-D Schrödinger time dep Eqn

$$\hat{H}_{\text{tot}} \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

$$\hat{H}_{\text{tot}} = \hat{E}_K + \hat{E}_P$$

$$= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x \neq \text{inside})$$

↘
→ 0

$$= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

> 0

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

Solve for $\Psi(x,t)$

particle
in
 ∞ 1-D
well

$$\Psi(x,t) = \begin{cases} 0 & x < 0 \\ 0 & x > a \\ A \sin\left(\frac{\pi x}{a}\right) e^{\frac{i\hbar \pi^2 t}{2ma^2}} & 0 \leq x \leq a \end{cases}$$