Schrö time dept not free 3-D wave eqn

so we've got

Free particle non relativistic Schrö

E_K only

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$-i\hbar \frac{\partial \psi}{\partial x} = -i^2 \hbar k = \frac{h}{\lambda}$$

= connection

deBroglie all particles have λ

$$\hat{E}_K = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

So ... Schrö

$$\hat{H}_{particle}\,\Psi(x,t) = \hat{H}_{wave}\,\Psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = i\hbar\frac{\partial \Psi}{\partial t}$$

Free-part non relativistic time dep Schrö

Let's do more ...

Look $\vec{p}_{classical} \neq m\vec{v}_x$

$$\vec{p}_{classical} = m v_x \hat{i} + m v_y \hat{j} + m v_z \hat{k}$$

maybe

$$\hat{p}_{classical} = -i\hbar \frac{\partial}{\partial x}(\hat{x}) + -i\hbar\frac{\partial}{\partial y}(\hat{y}) + -i\hbar\frac{\partial}{\partial z}(\hat{z})$$

$$= i\hbar \hat{\nabla}$$

cause $\hat{\nabla} \equiv \hat{del} \equiv \frac{\partial}{\partial x}(\hat{x}) + \frac{\partial}{\partial y}(\hat{y}) + \frac{\partial}{\partial z}(\hat{z})$

But the problem as always is how to extract this info from a wave funct

$$\Psi \propto e^{i(kx-\omega t)} = \text{1-D}$$

$$p = \frac{h}{\lambda} \quad \text{in 1-D}$$

$$\text{from} \quad \lambda_{\text{deBroglie}} = \frac{h}{p_x}$$

maybe in general

$$\lambda = \frac{h}{\vec{p}}$$

$$\vec{p} = \frac{h}{\lambda} = \hbar \vec{k} = \hbar(k_x \hat{x} + k_y \hat{y} + k_z \hat{z})$$

so

$$\Psi_{3\text{-}D} \propto e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$k_x x + k_y y + k_z z$$

$$\Psi_{3D} \propto e^{i(k_x x + k_y y + k_z z - \omega t)}$$

Now $\hat{p}_{3\text{-}D} \Psi_{3\text{-}D} \stackrel{?}{=} p_{3\text{-}D} \Psi$ Eigen function problem!

$(-i\hbar \vec{\nabla})\Psi_{3\text{-}D} \stackrel{?}{=} p_{3\text{-}D} \Psi$ where $\hat{\vec{p}} \stackrel{?}{=} -i\hbar\vec{\nabla}$

Now $\hat{p}_{3-D}\,\Psi_{3-D} \stackrel{?}{=} p_{3-D}\,\Psi$ Eigen function problem!

$$-i\vec{\nabla}\,\Psi_{3-D} \stackrel{?}{=} p_{3-D}\,\Psi_{3-D}$$

$$-i\hbar\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) e^{i(k_x x + k_y y + k_z z - \omega t)} = p_{3-D}\,\Psi_{3-D}$$

$$\left(\hbar k_x \hat{x} + \hbar k_y \hat{y} + \hbar k_z \hat{z}\right)\Psi_{3-D} = p_{3-D}\,\Psi_{3-D}$$

$$\hbar\vec{k}\,\Psi_{3-D} = p_{3-D}\,\Psi_{3D}$$

yeah! $\vec{p}_{3-D} = \hbar\vec{k}$ = eigenvalue

perfect!

Now 3-D Schrö (free particle)

$$E_K \text{ really} = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}mv_z^2$$

$$= \tfrac{1}{2}m\,\vec{p}\cdot\vec{p}$$

so now use our new 3-D $\vec{p}$ $\hat{o}$

$$\hat{E}_K = \tfrac{1}{2}m\left(-i\hbar\vec{\nabla}\right)\cdot\left(-i\hbar\vec{\nabla}\right)$$

$$= \frac{-\hbar^2}{2m}\nabla^2 \qquad \left(\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

Now E_{TOT} 3-D free particle

$$\hat{H}_{classical} = E_K + \cancel{E_P} = \frac{\hat{p}^2}{2m} = \frac{-\hbar^2}{2m}\nabla^2$$

So 3-D Schrö free particle is

$$\boxed{\frac{-\hbar^2}{2m}\nabla^2\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}}$$

yeah!

We built this to WORK
" Derived "
We designed it to WORK
It all works

For Free particle.

because

$$\Psi(x,t) = \Psi_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Was our soln we found, argued & made to work!

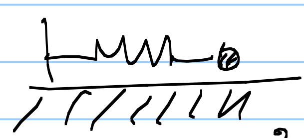
BUT

BIG

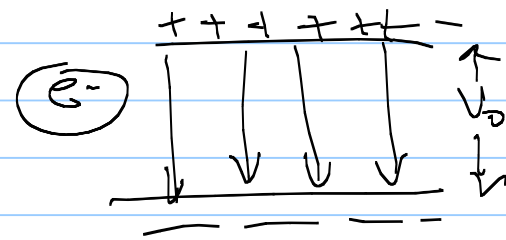
We have not derived this FOR ANY other CASE AT ALL

$\Psi = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ worked

But let's think, are all problems about free particles?



$E_p = \frac{1}{2}kx^2$



$E_p = qV_0$

No way....

So $\hat{\text{Hamiltonian}}_{\text{classical}} = E_k + E_p +$ maybe other stuff.

So maybe

$$\hat{H}_{\text{classical}}\Psi = i\hbar\frac{\partial\Psi}{\partial t} \quad ?$$

$$\left[\frac{\hat{p}^2}{2m} + \hat{V}(x)\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

But now $\Psi = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ ~~will~~ might not work

BUT we will

Postulate, that indeed if we

Solve for $\Psi \equiv$ call it the particle wavefunction for

$$\left[\frac{\hat{p}^2}{2m} + \hat{V}(\vec{r})\right]\Psi = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

That indeed that Ψ, although no longer $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ exactly

Will still work!

This is Schrö equation & it is a postulate

1) Never has not worked

2) Can be $\boxed{\text{derived}}$ * also into from Feynmans

$$e^{i\frac{S}{\hbar}}\Psi$$

$$(A) \rightarrow \boxed{\begin{array}{l} \text{Look} \\ P_{TOT} = \sum_i P_i \\ \hline \text{don't Look} \\ P_{TOT} = \left|\sum_i A_i\right|^2 \end{array}} \rightarrow (B)$$

Then now: Schrö 3-D (~~not~~ free but any particle)

Wave Equation!

remember still kind of Hybrid eigenfunct problem

$$\hat{H}_{\substack{Tot \\ classical}} \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

$$\hat{H}_{\substack{classical \\ particle}} \Psi = \hat{H}_{wave} \Psi$$

or

$$E_{\substack{Tot \\ particle}} \Psi = E_{\substack{Tot \\ wave}} \Psi$$

✱ In general.... not a bad way to approach any new problem you might want to solve

$$\begin{bmatrix} \text{particle-} \\ \text{like property} \end{bmatrix} \Psi = \begin{bmatrix} \text{wave like} \\ \text{property} \end{bmatrix} \Psi$$

expressed as $\hat{O}$

For $\hat{H}_{\substack{Tot \\ classical}} = E_K + E_p$

$$\hat{H}_{CT} = \frac{-\hbar^2}{2m}\nabla^2 + V(x,y,z)$$

a potential energy can ALWAYS be found for any conservative Force!

{ All Force ultimately are Conservative ie conserve energy

so $$\hat{H}_{CT}\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\Psi(x,y,z,t)}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right) + V(x,y,z,t)\,\Psi(x,y,z,t) = i\hbar\frac{\partial\Psi(x,y,z,t)}{\partial t}$$

= Schrö, time depend, not free

Wave Equat in 3-D.

In 1-D

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Scherrer "possible most import equation in all of 20th Century Physics"

Schrö, time depend, not free Wave Equat in 3-D.

In 1-D

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

Scherrer "possible most import equation in all of 20th Century physics"

1) Medicine

Nobel Prizes

NMR

Atomic Structure

maybe PET

SQUIDS

Nuc medicine → radioactive isotope tagging

Radiation oncology

e^-, γ, n beams

2.) Economy 30% of current economy = built on Q.M.

Lasers

transistors

medical

3.) Future: Clean energy

Medicine

4.) Philosophy!

All physics majors must be able to write this @ any time

& any place

& hopefully have something to say about it!