E, T. Deveney / BSC Physics: PH402 Lecture

Note Title

Shiever
Schrö - generalized

Schrö time dept not free 3-D wave equat

so we've got

Free particle non relativistic Schrö

E_K only

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$-i\hbar\frac{\partial\psi}{\partial x} = -i^2\hbar k = \frac{h}{\lambda}$

λ = connection

deBroglie all particles have λ

$$\hat{E}_K = \frac{p^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

So --- Schrö

$$\hat{H}_{particle}\Psi(x,t) = \hat{H}_{wave}\Psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t}$$

Free-part non relativistic time dep Schrö

Let's do more ---

Look $\vec{p}_{classical} \neq m\vec{v}_x$

$$\vec{p}_{classical} = mv_x\hat{i} + mv_y\hat{j} + mv_z\hat{k}$$

maybe $\hat{p}_{classical} = -i\hbar\frac{\partial}{\partial x}(\hat{x}) + -i\hbar\frac{\partial}{\partial y}(\hat{y}) + -i\hbar\frac{\partial}{\partial z}(\hat{z})$

$= i\hbar\hat{\nabla}$

cause $\hat{\nabla} \equiv \hat{del} \equiv \frac{\partial}{\partial x}(\hat{x}) + \frac{\partial}{\partial y}(\hat{y}) + \frac{\partial}{\partial z}(\hat{z})$

But the problem as always is how to extract this info from a wave funct

$$\Psi \propto e^{i(kx-\omega t)} = 1\text{-D}$$

$$p = \frac{h}{\lambda} \quad \text{in 1-D}$$

from $\lambda_{\text{deBroglie}} = \dfrac{h}{p_x}$

maybe in general

$$\lambda = \frac{h}{\vec{p}}$$

$$\vec{p} = \frac{h}{\lambda} = \hbar\vec{k} = \hbar\left(k_x\hat{x} + k_y\hat{y} + k_z\hat{z}\right)$$

so

$$\Psi_{3\text{-D}} \propto e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$k_x x + k_y y + k_z z$$

$$\Psi_{3D} \propto e^{i(k_x x + k_y y + k_z z - \omega t)}$$

Now $\hat{p}_{3\text{-D}}\Psi_{3\text{-D}} \overset{?}{=} p_{3\text{-D}}\Psi$ Eigen function problem!

$$(-i\hbar\vec{\nabla})\Psi_{3\text{-D}} \overset{?}{=} p_{3\text{-D}}\Psi \quad \text{where } \hat{\vec{p}} \overset{?}{=} -i\hbar\vec{\nabla}$$

Now $\hat{p}_{3-D}\,\Psi_{3-D} \overset{?}{=} p_{3-D}\,\Psi$ Eigen function problem!

$$-i\vec{\nabla}\,\Psi_{3-D} \overset{?}{=} p_{3-D}\,\Psi_{3-D}$$

$$-i\hbar\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) e^{i(k_x x + k_y y + k_z z - \omega t)} = P_{3-D}\,\Psi_{3-D}$$

$$\left(\hbar k_x \hat{x} + \hbar k_y \hat{y} + \hbar k_z \hat{z}\right)\Psi_{3-D} = P_{3-D}\,\Psi_{3-D}$$

$$\hbar\vec{k}\,\Psi_{3-D} = P_{3-D}\,\Psi_{3D}$$

yeah! $\vec{P}_{3-D} = \hbar\vec{k}$ = eigenvalue

perfect!

Now 3-D Schrö (free particle)

E_K really $= \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$

$$= \frac{1}{2}m\,\vec{p}\cdot\vec{p}$$

so now use our new 3-D $\vec{p}$ $\hat{o}$

$$\hat{E}_K = \frac{1}{2}m\left(-i\hbar\vec{\nabla}\right)\cdot\left(-i\hbar\vec{\nabla}\right)$$

$$= -\frac{\hbar^2}{2m}\nabla^2 \qquad \left(\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

Now E_{TOT} 3-D free particle

$$\hat{H}_{classical} = E_K + \cancel{E_P} = \frac{\hat{p}^2}{2m} = \frac{-\hbar^2}{2m}\nabla^2$$

So 3-D Schrö free particle is

$$\boxed{\frac{-\hbar^2}{2m}\nabla^2 \Psi(x,t) = i\hbar\frac{\partial \Psi}{\partial t}}$$

yeah!

We built this to <u>WORK</u>
" Derived "
We designed it to <u>WORK</u>
It all works

$\Bigg\}$ For Free particle.

because

$$\Psi(x,t) = \Psi_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

Was our soln we found, argued & made to work!

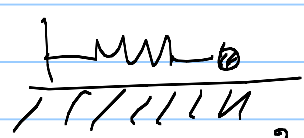
BUT

BIG

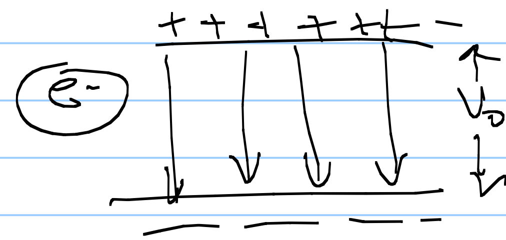
We have not derived this FOR ANY other CASE AT ALL

$\Psi = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ worked

But let's think, are all problems about free particles?



$E_p = \frac{1}{2}kx^2$



$E_p = qV_0$

No way....

So $\hat{H}\text{amiltonian}_{\text{classical}} = E_K + E_p +$ maybe other stuff.

So maybe

$$\hat{H}_{classical}\,\Psi = i\hbar\frac{\partial \Psi}{\partial t} \quad ?$$

$$\left[\frac{\hat{p}^2}{2m} + \hat{V}(x)\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

But now $\Psi = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ will might not work

BUT we will

Postulate, that indeed if we

Solve for $\Psi \equiv$ call it the particle wavefunction for

$$\left[\frac{\hat{p}^2}{2m} + \hat{V}(\vec{r})\right]\Psi = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

That indeed that Ψ, although no longer $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ exactly

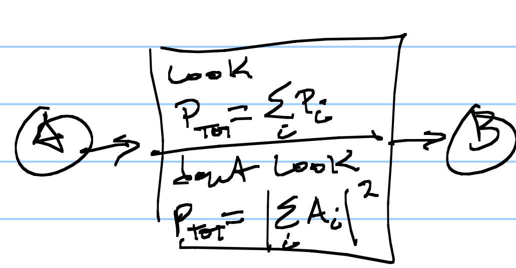
Will still work!

This is Schrö equation & it is a postulate

1) Never has not worked

2) Can be $\boxed{\text{derived}}$* also into from Feynmans

$$e^{i\frac{S}{\hbar}}\Psi$$



Then now: Schrö 3-D (~~not~~ ~~free~~ but any particle)

Wave Equation!

remember still kind of Hybrid eigen funct problem

$$\hat{H}_{\substack{TOT\\classical}}\ \Psi_{(x,t)} = i\hbar \frac{\partial \Psi_{(x,t)}}{\partial t}$$

$$\hat{H}_{\substack{classical\\particle}}\ \Psi = \hat{H}_{wave}\ \Psi$$

or

$$E_{\substack{TOT\\particle}}\ \Psi = E_{\substack{TOT\\wave}}\ \Psi$$

* In general.... not a bad way to approach any new problem you might want to solve

$$\left[\begin{matrix}\text{particle-}\\\text{like property}\end{matrix}\right] \Psi = \left[\begin{matrix}\text{wave like}\\\text{property}\end{matrix}\right] \Psi$$

expressed as $\hat{O}$

For $\hat{H}_{\substack{tot\\classical}} = E_K + E_p$

$$\hat{H}_{C_T} = \frac{-\hbar^2}{2m}\nabla^2 + V(x,y,z)$$

a potential energy can ALWAYS be found for any conservative Force!

{ All Force ultimately are Conservative ie conserve energy

So $$\hat{H}_{C_T}\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\Psi(x,y,z,t)}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right) + V(x,y,z,t)\,\Psi(x,y,z,t) = i\hbar\frac{\partial\Psi(x,y,z,t)}{\partial t}$$

= Schrö, <u>time depend</u>, <u>not free</u>

Wave Equat in 3-D.

In 1-D

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Scherrer "possible most import equation in all of 20th century physics"

Schrö, time depend, not free
Wave Equat in 3-D.

In 1-D

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

Scherrer "possible most import equation in all of 20th Century physics"

1) Medicine
Nobel Prizes
NMR
Atomic Structure

magne PET
SQUIDS
Nuc medicine → radioactive isotope tagging
Radiation oncology
e^-, γ, n beams

2.) Economy 30% of current economy = built on Q.M.
Lasers
transistors
medical

3.) Future: Clean energy
Medicine

4.) Philosophy!

All physics majors must be able to write thus @ any time

& any place

& hopefully have something to say about it!