So we've got

<table>
<thead>
<tr>
<th>Free particle</th>
<th>non-relativistic</th>
<th>Schröd</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = -ic\frac{\partial \psi}{\partial x}$</td>
<td>$E_x = \frac{\psi^2}{2m} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$</td>
<td>$\nabla \times \text{curl} = 0$</td>
</tr>
</tbody>
</table>

So... Schröd

$\hat{H}_{\text{particle}} \psi(x,t) = \hat{H}_{\text{wave}} \psi(x,t)$

Free-particle non-relativistic wave Schröd

Let's do more...

Look... $\vec{p}_{\text{classical}} \neq m \vec{v}$

$\vec{p}_{\text{classical}} = m \vec{v}_x \hat{x} + m \vec{v}_y \hat{y} + m \vec{v}_z \hat{z}$

maybe $\vec{p}_{\text{classical}} = -i\hbar \frac{\partial \psi}{\partial x} + -i\hbar \frac{\partial \psi}{\partial y} + -i\hbar \frac{\partial \psi}{\partial z}$

$\Rightarrow \nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}$
But the problem as always is hard to extract
This is so from a wave function
\[ \psi \propto e^{-i(kx - \omega t)} = 1 - D \]

\[ \Psi = \frac{\hbar}{\mathcal{P}} \text{ in } 1-D \]

From \( \lambda_{\text{def}} = \frac{\hbar}{\mathcal{P}} \)

maybe in general

\[ \gamma = \frac{\hbar}{\mathcal{P}} \]

\[ \mathcal{P} = \frac{\hbar}{\sqrt{\gamma}} = \frac{2}{\gamma} \]

\[ \Psi = \frac{\hbar}{\mathcal{P}} = \hbar \left( k_x x + k_y y + k_z z \right) \]

\[ \Omega_0 \quad i \left( k \cdot \mathcal{P} - \omega t \right) \]

\[ \left( \begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \right) \quad \rightarrow \quad \left( \begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \right) = \hbar \left( k_x x + k_y y + k_z z \right) \]

\[ \left( \begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \right) \quad \rightarrow \quad \left( \begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \end{array} \right) = \hbar \left( k_x x + k_y y + k_z z \right) - \omega t \]

Now \( \hat{p} \Psi = \mathcal{P} \Psi \quad \text{Eigenfunction Problem!} \)

\[ \left( -i \hbar \nabla \right) \Psi = \mathcal{P} \Psi \quad \text{alone} \quad \frac{\hbar^2}{\mathcal{P}} = -i \hbar \nabla \Psi \]
Now \( \mathbf{p}_3 \cdot \mathbf{r}_3 = p_{3D} F \)

Eigen-Solution Problem!

\[-i \mathbf{k} \left( \frac{\hbar}{m} \mathbf{x} + \frac{\hbar}{2m} \mathbf{y} + \frac{\hbar}{2m} \mathbf{z} \right) e^{i(kx + ky + kz - \omega t)} = p_{3D} \mathbf{F}_{3D} \]

\[(\frac{\hbar}{m} \mathbf{k} \mathbf{x} + \frac{\hbar}{2m} \mathbf{k} \mathbf{y} + \frac{\hbar}{2m} \mathbf{k} \mathbf{z}) \mathbf{F}_{3D} = p_{3D} \mathbf{F}_{3D} \]

\[\frac{\hbar^2 \mathbf{k}}{m} \mathbf{F}_{3D} = p_{3D} \mathbf{F}_{3D} \quad \text{freq} \quad p_{3D} = \hbar \mathbf{k} = \text{eigenvalue} \]

Perfect!

Now 3-D Solutions (Sinc particle)

\[E_{12} \text{ potential} = \frac{1}{2} m V_x^2 + \frac{1}{2} m V_y^2 + \frac{1}{2} m V_z^2 \]

\[= \frac{1}{2} m \mathbf{p} \cdot \mathbf{p} \]

so now use our new 3-D \( \mathbf{p} \cdot \mathbf{p} \)

\[E_k = \frac{1}{2} m \left(-i \hbar \mathbf{k} \cdot \mathbf{\nabla} \right) \cdot \left(-i \hbar \mathbf{k} \cdot \mathbf{\nabla} \right) \]

\[= -\hbar^2 \mathbf{k}^2 \nabla^2 \]

\[
\left( \nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)
\]

\[
\left( \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 \right)
\]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)
\]
Now for 3-D free particle

\[ H_{\text{classical}} = E_k + E_p = \frac{p^2}{2m} = \frac{k^2}{2m} \]

So 3-D Schrödinger free particle is

\[ \psi(x,t) = i \hbar \frac{\partial}{\partial t} \frac{-\hbar^2}{2m} \Delta \psi(x,t) = i \hbar \frac{\partial}{\partial t} \psi(x,t) \]

Yeah!

We built this to work

"Derived"

We designed it to work

For free particle.

It all works

because

\[ \psi(x,t) = \psi(0) e^{i(k \cdot x - \omega t)} \]

Was our solution sound, argued

\[ \frac{\hbar}{m} \text{ made to work!} \]

BUT
We have not derived this for any other case at all. For some cases it worked.

But let's think, are all problems about free particles?

\[ H = -\frac{\hbar^2}{2m} \nabla^2 + V(x) \]

\[ E_p = \frac{1}{2} k x^2 \]

No way...

So Hamiltonian = \( E_k + E_p \) + maybe other stuff.

So maybe

\[ H_{\text{classical}} \Phi = i\hbar \frac{\partial}{\partial t} \Phi \]

\[ \left[ \frac{\hbar^2}{2m} + V(x) \right] \Phi = i\hbar \frac{\partial \Phi}{\partial t} \]

But now \( \Phi = e^{i(k \cdot r - \omega t)} \) might not work.

\[ \Phi \]

But we will...
Postulate, that indeed is we, solve for \( \psi \) call it the particle wavefunction for

\[
\left[ \frac{\hat{\mathcal{L}}}{2m} + \hat{V}(x) \right] \psi = i \hbar \frac{\partial \psi}{\partial t}
\]

That indeed that \( \psi \) although no longer exactly

We'll still work.

This is Schrödinger equation if it is a postulate.

\( \Delta \) Never has not worked

\( \Box \) Can be inferred also into

\[ e^{i S / \hbar} \]

\[ \frac{P_{\text{tot}}}{\hbar} \]

\[ P_{\text{tot}} = |E A_0|^2 \]
New now: Schröd 3-D (not but any particle)

Wave Equation!

Remember still hard of Hybrid exponential function problem

\[ \hat{H}_{\text{classical}} \Psi(x,t) = \frac{-i}{\hbar} \frac{\partial \Psi(x,t)}{\partial t} \]

\[ \hat{H}_{\text{wave}} \Psi \]

or

\[ \Psi_{\text{classical}} = \Psi_{\text{wave}} \]

\[ \forall \tilde{p} = \tilde{E} \text{ for particle wave} \]

\( \checkmark \) In general now not a bad way to approach any new problem you might want to solve

\[ \left[ \text{particle-like property} \right] \Psi = \left[ \text{wave-like property} \right] \Psi \]

expressed as \( \hbar / 2 \)

For classical \[ \hat{H}_{\text{classical}} = E + \tilde{p} \]
\[ H = \frac{-\hbar^2}{2m} \nabla^2 + V(x,y,z) \]

A potential energy can always be found for any conservative force.

1. All force ultimately one conservative force, i.e., can conserve energy.

So
\[ \hat{H} \psi(x,t) = i\hbar \frac{\partial \psi}{\partial t} \]
\[ -\frac{\hbar^2}{2m} \nabla^2 \psi + V(x) \psi = i\hbar \frac{\partial \psi}{\partial t} \]
\[ -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + V(x,y,z) \psi = i\hbar \frac{\partial \psi}{\partial t} \]

= Schroedinger time-dependent, not time-free wave equation in 3-D.

In 1-D
\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi = i\hbar \frac{\partial \psi}{\partial t} \]
Schrödinger "possible most important equation in all of 20th century physics"
Schrödinger's time dependent wave equation in 3-D.

\[ \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t) \psi(x,t) = \frac{\hbar^2 \partial^2 \psi(x,t)}{2m \partial t^2} \]

Schrödinger “possible most important equation in all of 20th century physics”

1) **Medicine**
   - Nobel Prizes
   - NMR
   - Atomic structure

   Maybe PET

   - SQUIDS
   - Nuc medicine
   - Isotope tagging
   - Radiation oncology
   - \( e^-, \gamma, n \) beams

2) **Economy** 30% of current economy
   - built on EMI.
   - Lasers
   - Transistors
   - Medical

3) **Future:** clean energy
   - Medicine

4) **Philosophy!**
All physics majors must be able to write this at any time in any place.

I hopefully have something to say about it!