1912 Old Q.M. = Bohr Atom → Balmer Lines, → r_n, E_n

Quantization from $\vec{L} = \vec{r} \times \vec{p}$ $= rmv = n\hbar$

1925 Heisenburg (HayFever)

Jordon & Born

Dirac

1926

Heis, Jordon, Born & Dirac = Matrix Q.M.

Very math.... no real attempt for physical picture

Heisenburg ≠ no excuse

1924 deBroglie, $\lambda = \frac{h}{p}$

*Schrö wave was to Q.M. as Feynman diagrams were to Q.E.D

1926 Schrö wave equation after suggestion by deBye

Big push to give physical picture

1926: Born thru

Ψ = prob amplitude

this wave Equat

Einstein & Schrö like craze, Ultimately, all these attempts fail!

M.Q.M. = 1926 = W. Q.M

Schrodinger shows dissarent reps

Vector Space (V.S.) = Function Space

Hilbert ↑

where H-Space = F.S. w/ all props of V.S.

1929 - Dirac: Schrö wave + Special Rel

1932 - Anderson = antiparticles

Balloon, Emulsion

Nobels

1932: Heisenburg

1933: Dirac & Schrö

1954: Born!

1963 - Feynman, Tokomak, Schwinger

$\Psi = e^{-}$ field + S.R.

$F^{\mu\nu}$ = E & B field

= same Footing QED

— 1900: $\vec{E}$ & $\vec{B}$ fields = waves

— 1900 –1924
1.) photo-e
2.) Compton scattering
3.) S.R.
} $\vec{E}$ & $\vec{B}$ = particles

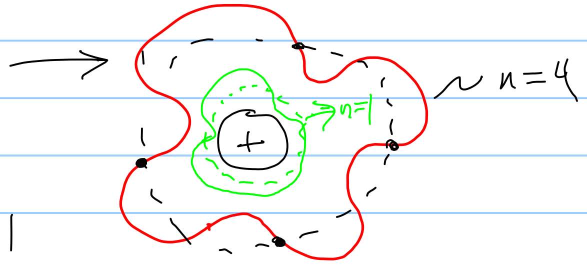
1912: e^-s = particles
But

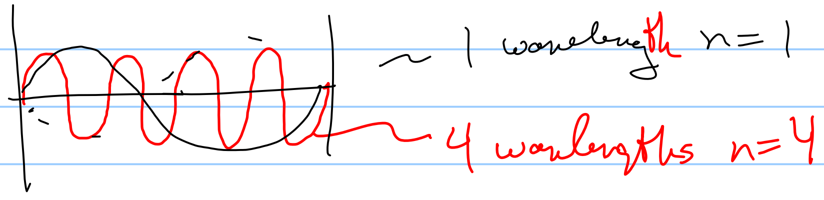
Bohr atom: e^-'s are in Quantized angular momentum states that do not radiate : $\vec{L} = n\hbar$
n=1,2,3...

1924: de Broglie $\vec{E}$ & $\vec{B}$ = particles & waves
now e^-s = particles & ?

↓ Bohr atom

Looks like Condition for Standing waves!





$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\ \mathrm{J\cdot sec}}{m_e v_e}$$

$\{$ Sure enough $L = rmv = r\frac{h}{\lambda} = n\frac{h}{2\pi}$; $r = \frac{n\lambda}{2\pi}$ $\}$ works!

so $\underbrace{\textcircled{\gamma} = \text{particles \& waves}}$

Look @ wave
Equation to Specify
E&M waves

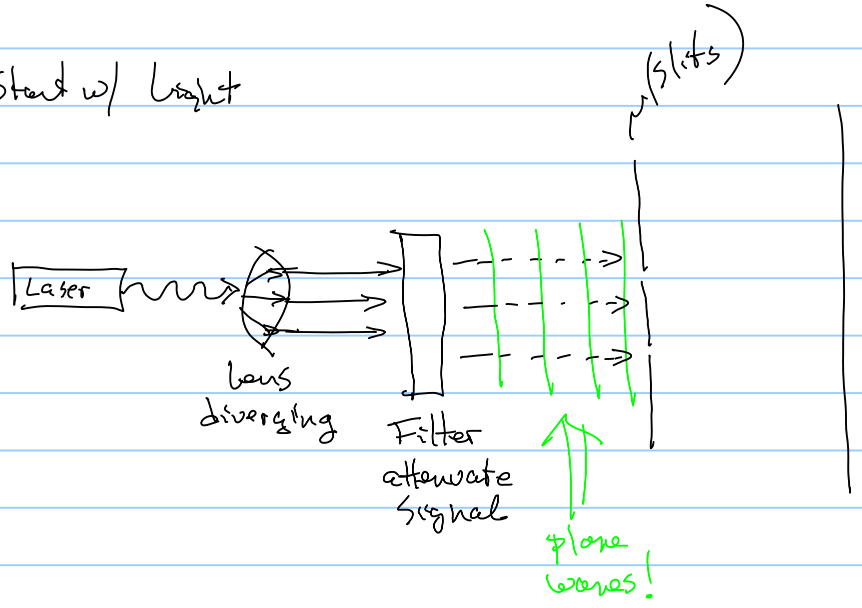
E&M wave equation w/
E&M "wave funct" soln.

now $\underbrace{e^{-}\text{s} = \text{particles + waves}}$

Look for
e^{-} wave equation
&
e^{-}, particle, wave function

So, look for experiments:

1, Start w/ Light



Side: Light ⇒ Max's Equations in Free Space ($\rho_{free} = 0$, charges/vol; $\vec{J}_f = 0$, I/Area)
completely describes all E&M phenomena, ie Light

1) $\vec{\nabla}\cdot\vec{E} = 0 + 0$

2) $\vec{\nabla}\cdot\vec{B} = 0 + 0$

3) $\vec{\nabla}\times\vec{E} = 0 + -\frac{d\vec{B}}{dt}$

4) $\vec{\nabla}\times\vec{B} = 0 + \mu_0\epsilon_0\frac{dE}{dt}$

only dynamic Sources!

Options to solve (1-4)

1) $\vec{E}$ or $\vec{B}$ one @ a time

$\vec{E}$: $\vec{\nabla}\times(\vec{\nabla}\times\vec{E})$ ...

use $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

get $\frac{d^2E}{dx^2} = \mu_0\epsilon_0\frac{d^2E}{dt^2}$

same for $\vec{B}$.

Homework on classical wave equat on Rope!
solns = $f(x \pm vt)$
= traveling wave @ veloc v
←
→

Key: 2nd order wave equat yields 2 linearly indep solns

2.) Assume a Soln $\vec{F} = E + iB$

note: anticipate in 2-Linear indep solns!

Both Techniques yield MOST GENERAL SOLN!

$$\vec{E} = E_0 e^{i(kx \pm \omega t)} \hat{y}$$

$$\vec{B} = \frac{E_0}{c} e^{i(kx \pm \omega t)} \hat{z}$$

⇑ note, not easiest, simplest But most General why →

What's so special about most general soln?

Well the $\sum_{k=-\infty}^{k=+\infty} A_n e^{i(kx-\omega t)}$ = complete basis set

$* \; k = -$

$A_n e^{i(-kx-\omega t)} = A_n e^{-i(kx+\omega t)}$

= waves ←

So, Every soln can be built from these general solns.

So, if we understand these soln's we understand properties of all solns!

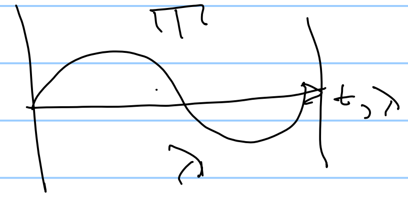
What is

$$\left.\begin{aligned} \vec{E} &= E_0 e^{i(kx-\omega t)} \hat{y} \\ \vec{B} &= \frac{E_0}{c} e^{i(kx-\omega t)} \hat{z} \end{aligned}\right\}$$

$e^{ik(x-\frac{\omega}{k}t)}$

periodic: $\cos k(x-\frac{\omega}{k}t) + i \sin k(x-\frac{\omega}{k}t)$

so periodic with $\frac{\omega}{k}$



So $v = \frac{d}{t} = \frac{\lambda}{T} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{2\pi}{T}\frac{\lambda}{2\pi}$

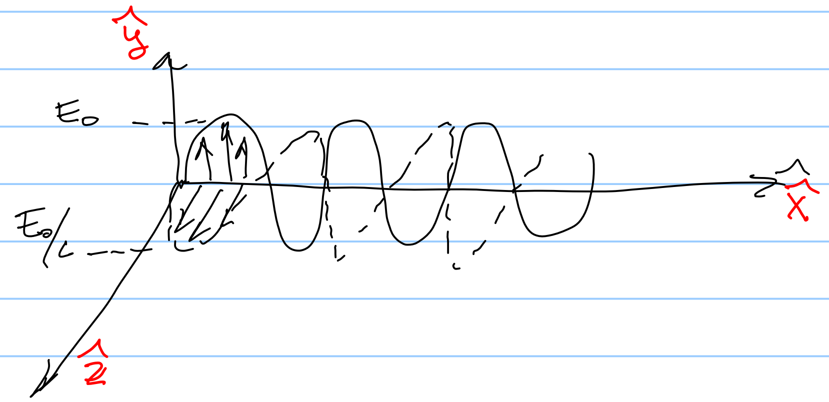
$v = \frac{\omega}{k}$

So

$e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)} = e^{ik(x-vt)}$

These are just TRAVELING WAVES!

Conclusion: soln to max's Equat in free space
= traveling waves
(self-propagating)

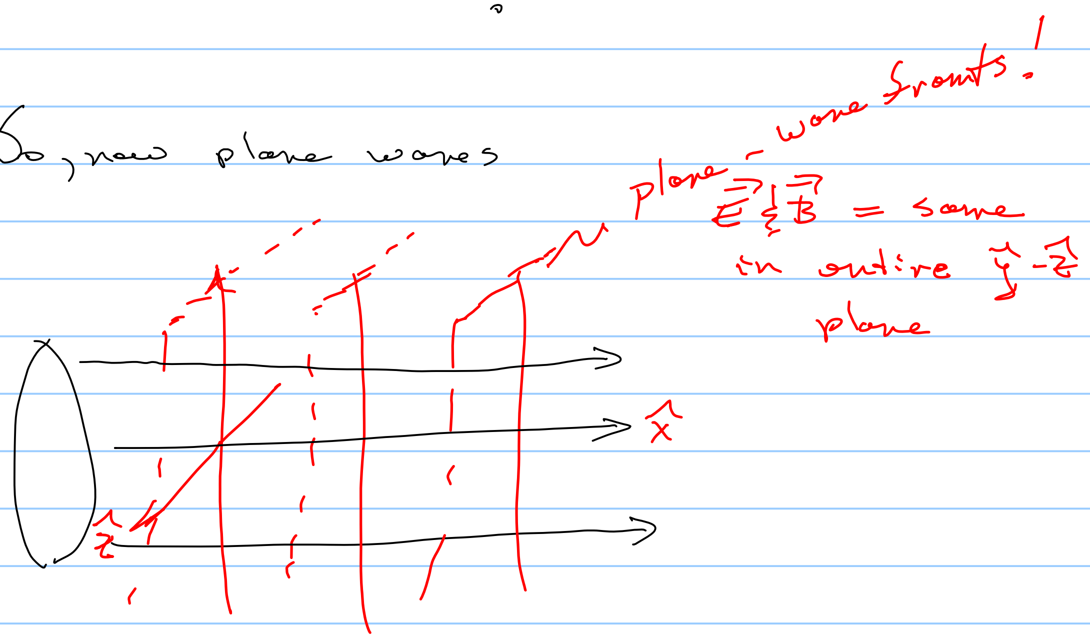


They are Plane-waves! ie $\vec{E} = E_0 e^{i(kx-\omega t)}$

$\vec{E}$ = Same vector in entire y-z plane!

O.K. not so physical
but idea is this is great bases and all solns can be built from it!

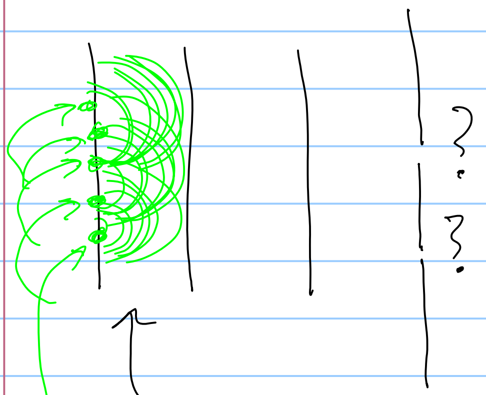
So, now plane waves



plane-wave fronts!

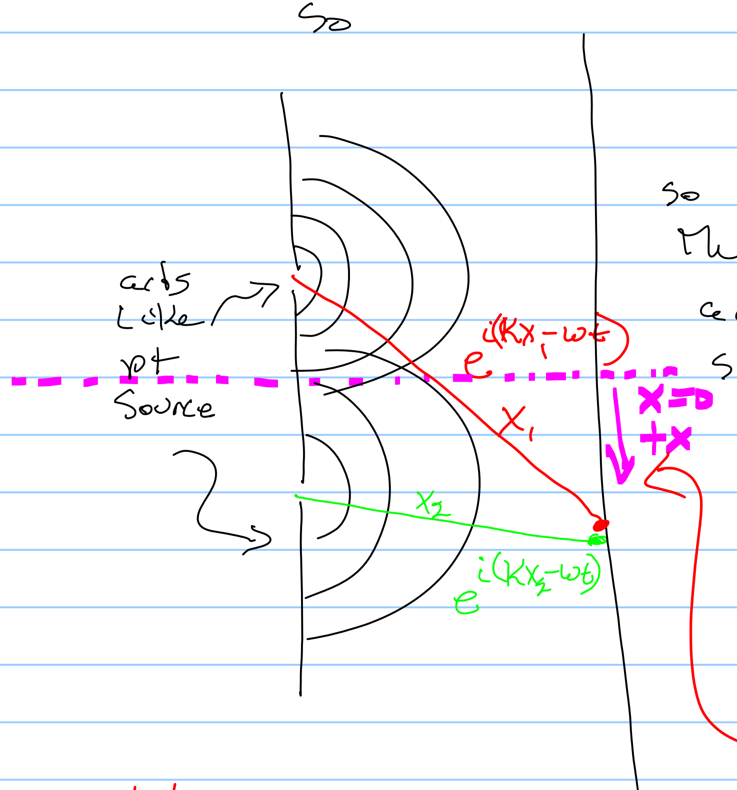
$\vec{E}$ & $\vec{B}$ = same in entire $\hat{y}$-$\hat{z}$ plane

So now look @ plane waves on a slit...



well Huygen's Princ: Plane waves = ∞ Sum of pt-like sources

So



so in every direction the plane wave acts like a source that spreads out like

$$e^{i(kx-\omega t)}$$

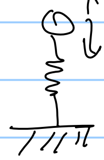
so

Note; $k=\frac{2\pi}{\lambda}$ & $\omega=\frac{2\pi}{T}$ do not change for each source.

So, if you want to find out what happens @x
you've got to add up the waves @ pt x

Now to measure waves, you measure intensity

$$I = \frac{\text{Energy}}{(m^2)(\text{sec})} \quad \text{where} \quad \underset{\uparrow}{\text{Energy}} \propto |\text{Amp}|^2$$

ie  $= \frac{1}{2}kx_0^2$

x_0 = amp

So, need the amplitude of
the $\vec{E}$ & $\vec{B}$ field @ pt x

This must be the total field @ $\vec{x}$
* will only look @ $\vec{E}$ but of course really need
$\vec{B}$ field too.

So @ X, find $E_{TOT}(x) = \underset{\substack{\uparrow \\ \text{source 1}}}{E_1(x)} + \underset{\substack{\uparrow \\ \text{source 2}}}{E_2(x)}$

$$\text{Then Intensity} \propto |E_{TOT}|^2$$

$$E_T(x) = E_1(x) + E_2(x) = E_1 e^{i(kx_1-\omega t)} + E_2 e^{i(kx_2-\omega t)}$$

$$= e^{-i\omega t}\left[E_0 e^{ikx_1} + E_0 e^{ikx_2}\right]$$

since from same plane wave

Now

$I \propto |E_T(x)|^2$ * remember it is complex

$$I(x) \propto E_T^* E_T = \left(E_0 e^{i\omega t}\left[e^{-ikx_1} + e^{-ikx_2}\right]\right)\left(E_0 e^{-i\omega t}\left[e^{ikx_1} + e^{ikx_2}\right]\right)$$

$$= e^{i\omega t} e^{-i\omega t} E_0^2 \left[e^{-ikx_1} e^{ikx_2} + e^{-ikx_2} e^{ikx_2}\right.$$

$$\left. e^{ik(x_2-x_1)} + e^{-ik(x_2-x_1)}\right]$$

$$e^{ix} = \cos x + i\sin x$$

$$e^{-ix} = \cos x - i\sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$I(x) \propto E_0^2\left[2 + 2\cos k(x_2-x_1)\right] = E_0^2\left[1 + \cos k(x_2-x_1)\right]$$

is $\frac{\cancel{2\pi}}{\lambda}\Delta x = \cancel{2\pi}$

or $\Delta x = \lambda$

or

2λ

4λ

=

constructive interference

or

$$\propto \left[\underset{\substack{\uparrow \\ \text{is just} \\ \text{wave 1}}}{E_1^2} + \underset{\substack{\uparrow \\ \text{is just} \\ \text{wave 2}}}{E_2^2} + \underbrace{2E_1E_2\cos k(x_2-x_1)}_{\text{interference term}}\right]$$

ξ

$$I(x) \propto E_0^2 \left[2 + 2\cos k(x_2-x_1) \right] \Rightarrow$$

$$E_0^2 \left[1 + \cos k(x_2-x_1) \right] =$$

Note: is $\overset{k(x_2-x_1)}{\frac{\cancel{2\pi}}{\lambda}(\Delta x)} = n\,\cancel{2\pi}$

Then get constructive interference

$$\Delta\lambda = n\lambda$$

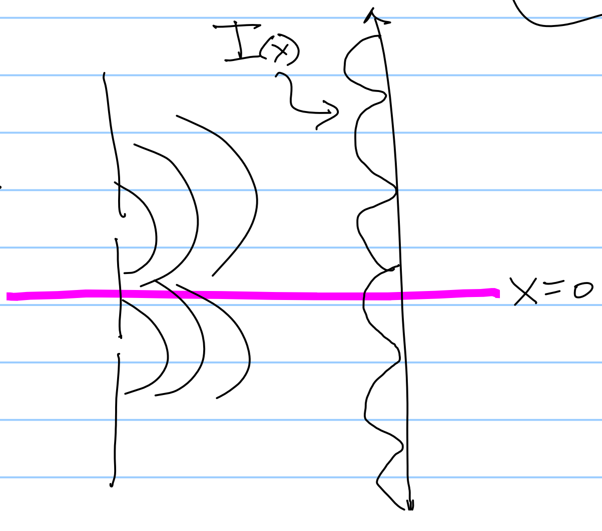
$$n = 0, 1, 2, 3 \ldots$$

& Complete destructive interference for

$$\frac{\cancel{2\pi}}{\lambda}(\Delta x) = \frac{n}{2}\,\cancel{2\pi}$$

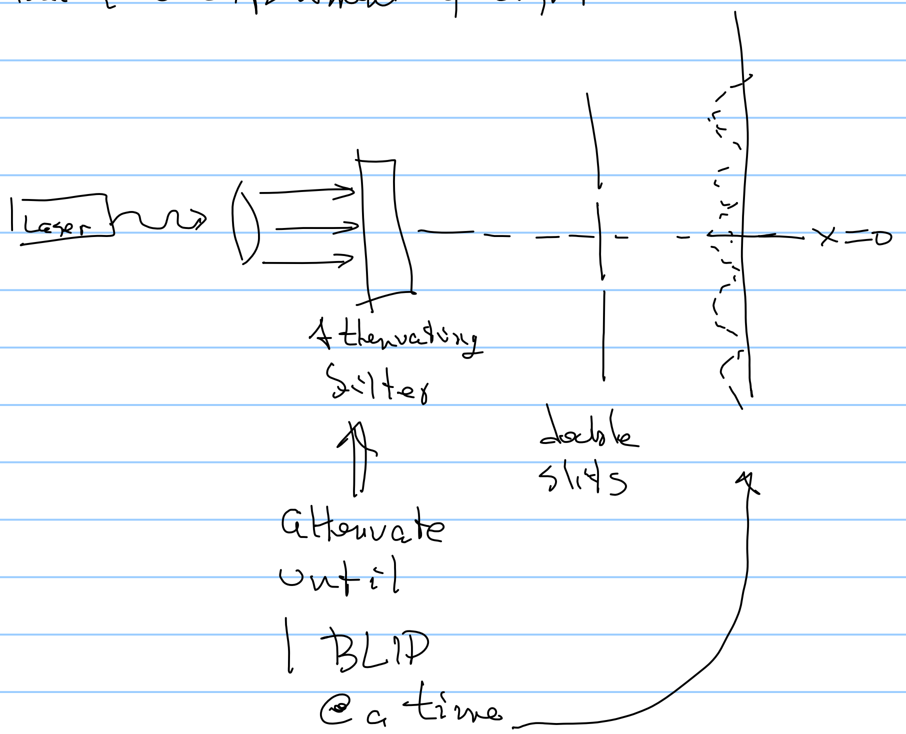
$$\boxed{\Delta x = \frac{n\lambda}{2}}$$

=



& ~~This~~ This is exactly what you get experimentally!

Back to experiment of Light



Light apparently, (ie Plank Compton Einstein) comes in 1 lump @ a time = photon! γ

Can see, Each lump does not go to same place!

So At Best we can only try to predict where each γ will come.

Clearly prob = Big where I builds up & prob = small when I is small

$\therefore$ Can say; prob of where γ ends up $\propto I$ or $\propto |E_{TOT}^2/dx$

Wo - Alt: photons = particles
can only do probabilities
of where it will end up

&

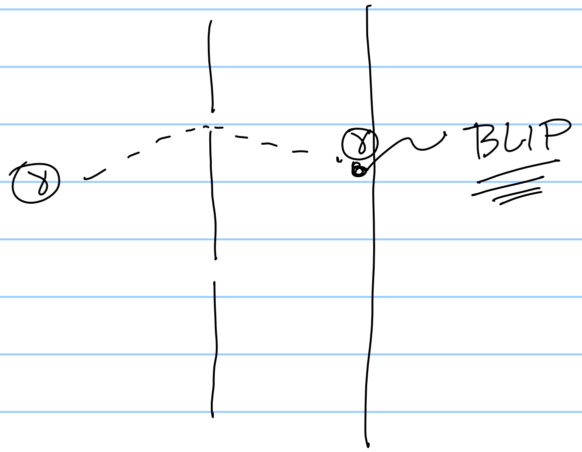
prob $\propto I$ or $\propto \left| E^2 + B^2 \right| dx$

$\uparrow$ don't forget really need this!

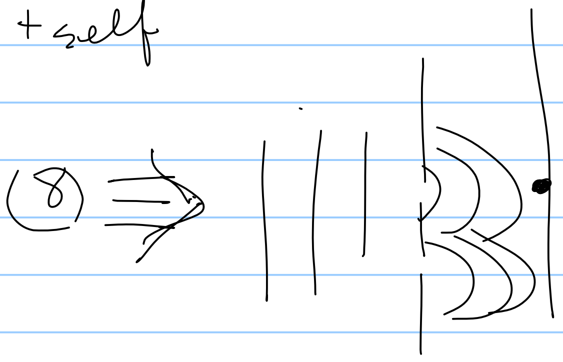
will later use

$\left| E^2 + B^2 \right|$ = energy or prob density!

Note also: this 1 photon @ a time experiment $\Rightarrow$'s γ interferes w/ itself



But only way for the "Individual" photon Intensity, or probability, to build up over time is is indeed each γ interferes w/ itself



even though it seems like it went thru just one slit!

READ Feynman Lecture NOW

(insert it)

Lect # 1 Volume III = Quantum Behavior.

especially

1-6 Watching e-s

1-8 The Uncertainty Princ.

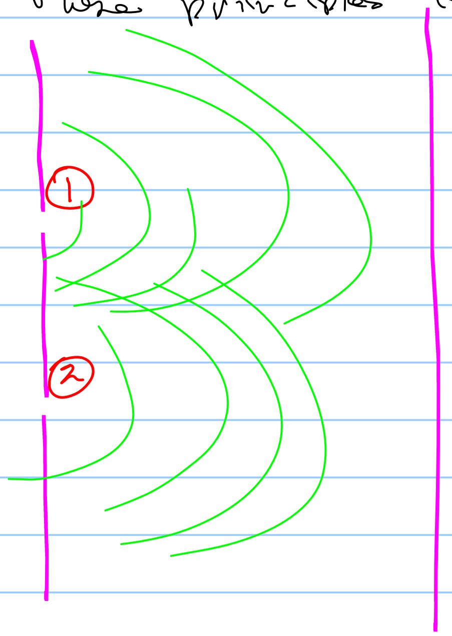
We must keep these principles in mind.

(γ) = $\vec{E}$ & $\vec{M}$
plane waves

(e⁻), $\Psi_{\text{Schrö}}$

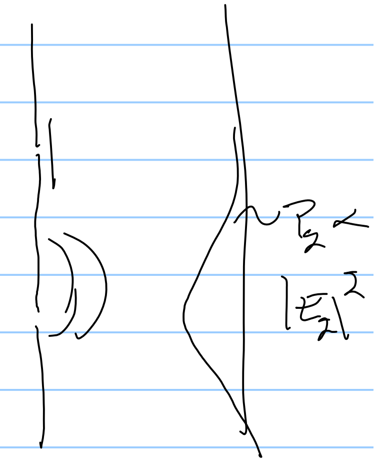
or

regular waves

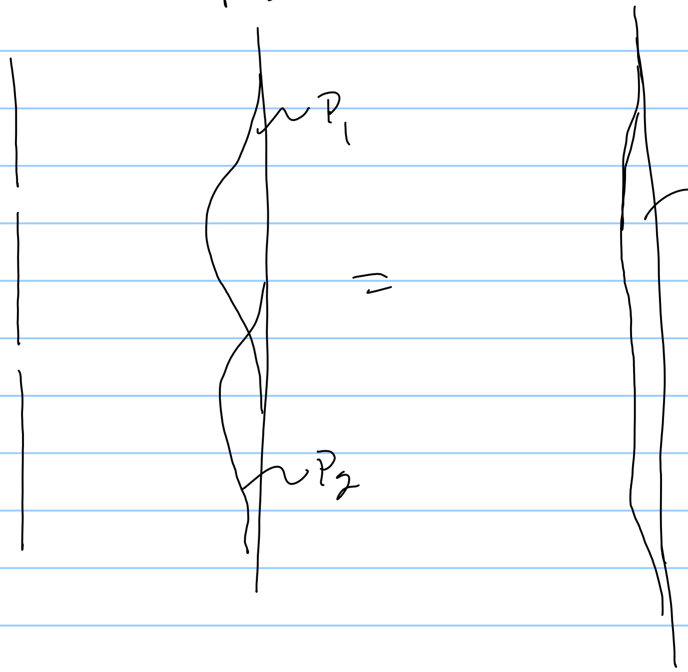


if slit 2 is blocked

if slit 1 is blocked



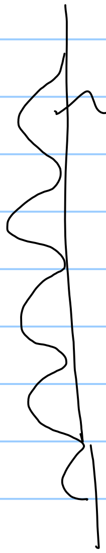
Now both open



$P_1 + P_2 = P_{TOT}$

$P_T \propto (E_1^2 + E_2^2)$

But we don't get that instead we get



$$P_{TOT} \propto (\underbrace{\vec{E}_1 + \vec{E}_2}_{\text{where (call } E_1 \,\&\, E_2 \text{ prob amplitudes}})^2 = \underbrace{E_1^2 + E_2^2}_{\text{each slit open independently}} + \underbrace{2E_1E_2\cos(k(x_2-x_1))}_{\text{interference TERM!}}$$

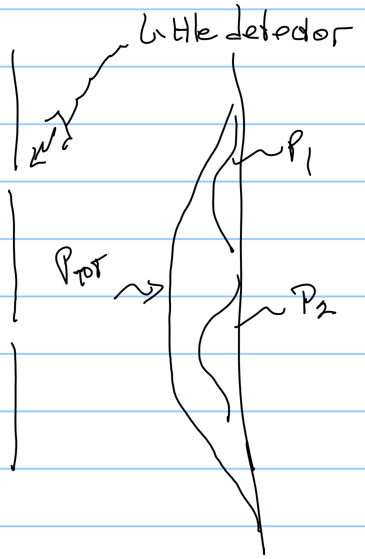
Therefore:

In the limit of 1 γ @ a time, to get the correct results you must

Construct a total probability by adding the waves from each slit @ the same time together & then squaring! =

conclude the γ wavefunction takes both paths at once and ∴ THE γ interferes w/ itself.

If you try to look @ which way the photon did go, you destroy the interference pattern!



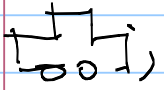
Interference is DESTROYED

P_T looks like $\propto E_1^2 + E_2^2$
Like it went thru either slit 1 or 2.

I will conclude w/ Q.M. Big Picture

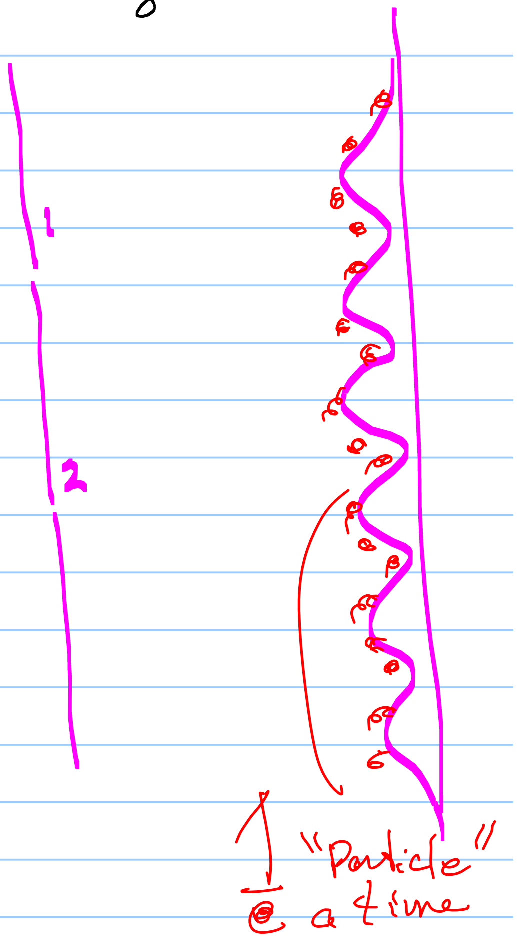
(γ); $\vec{E}$ & $\vec{M}$ fields = wave funct

(e⁻); Ψ_{e^-} from Schrö

; Ψ_{car} from Schrö

Everything

A WAVE FUNCTION! $\vec{E}$ or Ψ



Pt A

@ Pt B

RULES

A →

If know it went thru 1 or 2

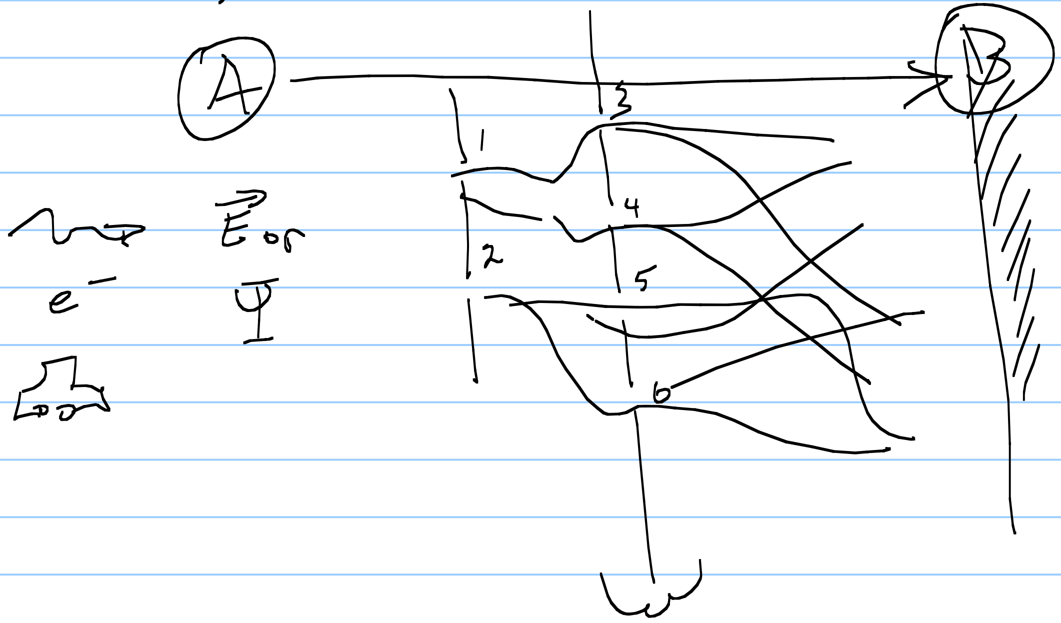
$P_T = P_1 + P_2 = E_1^2 + E_2^2 \Rightarrow$ no interference

If don't know then

$P_{TOT} = P_1 + P_2 + \text{interference} = E_1^2 + E_2^2 + 2E_1E_2\cos(k(x_2 - x_1))$

→ B

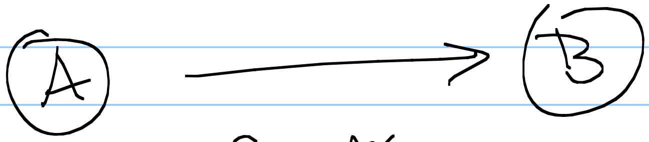
A Thirsty Feynman then Says.....
what is



Why Not.

Why not more & more until in the Limit None?

Conclude:



requires

$$P_{Tot} = \left| A_1 + A_2 + A_3 + \cdots\cdots \infty \right|^2$$

↑ prob Amplitude for path (i) ie E_1 or Ψ_1

For all paths from A-B

So that all paths from ALL A to B contribute

to the prob of getting from A to B
& All of these paths interfere
w\ each other!

YIKES!

We see that what will need is
some way to

1) Find the Quantum field
of a particle

2) Find the prob amplitudes A_i
from 1

3) Sum up (perhaps ∞ #)
of A_i

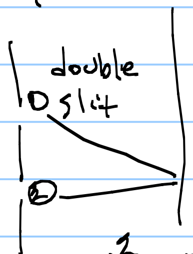
4) Square them

5) $P_{TOT} = \left| \sum_{i=1}^{\infty} A_i \right|^2$

ex. w/ γ: 1) Field is $\vec{E}$ & $\vec{B}$
from $\frac{d^2E}{dx^2} = \mu_0 \epsilon_0 \frac{d^2E}{dt^2}$

2) Solve for $\vec{E}_1 = E_1 e^{i(kx-\omega t)}$

3) Sum all paths



$= E_1 + E_2$

4) & 5) $P_{tot} = |E_1+E_2|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos[k(x_2-x_1)]$

* quite different from $P_{TOT} = \sum_i E_i^2 = E_1^2 + E_2^2$ (allows no interfere)

How can this idea of nature be correct? After all, it must reduce to classical physics!

Well, what do we know about classical physics?

It <u>all</u> come from Euler-Lagrange minimization of the action integral

$$S = \int \mathcal{L}\,dt \qquad \mathcal{L} \equiv \underset{\substack{\uparrow \\ \text{kinet}}}{T} - \underset{\substack{\uparrow \\ \text{potential}}}{V}$$

S is minimized for solutions that satisfy

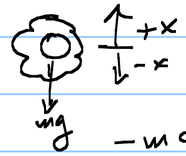
$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = 0 \quad : \text{E-L}$$

From which all of classical mechanics follows.

The E-L. equation <u>just</u> helps us figure out which path <u>minizes</u> S.

Nature works to minimize S.

ex: $\mathcal{L} = \frac{1}{2}m\dot{x}^2 - mgx$



$-mg - m\ddot{x} = 0$

$-mg = m\ddot{x}$

$-mg = m\vec{a}$

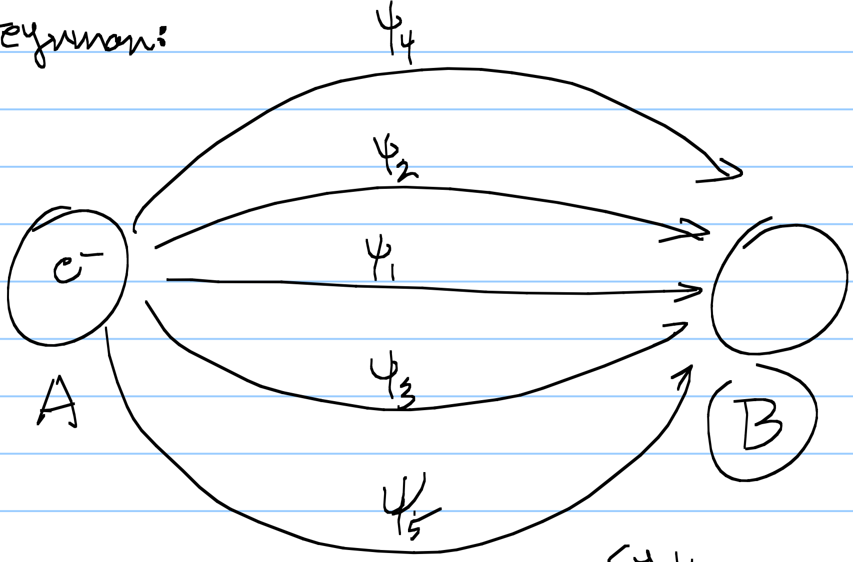
Just newton's 2^{nd} Law.

Solve for x & you're solved for the position function that minimizes the action integral.

& this = Nature.

So, Bigger question is, why does nature always use the path that minimizes S?

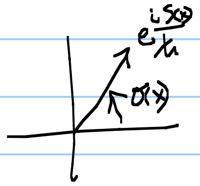
Feynman:



Let each $\psi \to e^{iS/\hbar}\psi_i = e^{i\frac{\int \mathcal{L}dt}{\hbar}}\psi_i$

$\vdots$

$e^{i\frac{\text{Energy}(X)}{\hbar}}\psi_i$



ψ_i

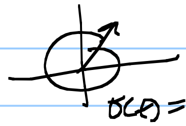
So each ψ carries a action integral clock

$\theta(t)$

$$e^{i\frac{S(x)}{\hbar}}\psi = e^{i\theta(t)}\psi$$

$$S = \text{Energy} \begin{cases} \dfrac{S_{Baseball}}{\hbar} = \dfrac{J\cdot sec}{10^{-34} J.s} = \boxed{10^{34} t} \\ \dfrac{S_{e^-}}{\hbar} = \dfrac{(10^{-19} J)\cdot sec}{10^{-34} Js} = \boxed{10^{15} t} \end{cases}$$

Look @ Baseball clock $e^{i10^{34}}\psi$

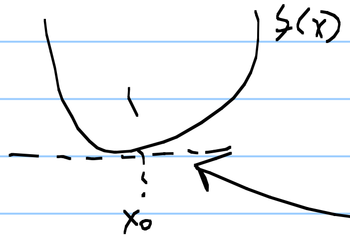


$\theta(t) = 10^{34}\ \frac{rad}{sec}$

so

since goes so quickly

all but on



$$S(x) = S(x_0) + \boxed{\frac{dS}{dx}\Big|_{x_0}}\Delta x + \frac{1}{2}\frac{d^2S}{dx^2}\Big|_{x_0} x^2$$

$$\Rightarrow$$

$$S(x) \cong S(x_0) + \underbrace{\frac{1}{2}\frac{d^2S}{dx^2}\Big|_{x_0}}_{small} x^2 + ---$$

So This means

1) you are on a path

$\vec{F} = \vec{E} + i\vec{B}$ ← complex #

keeping track of 2 linearly indep vectors

(γ)

$$\underbrace{\frac{d^2\vec{E}}{dx^2} = \mu_0\varepsilon_0 \frac{\partial^2\vec{E}}{dt^2}}_{\text{wave equation}} \; ; \; \underbrace{\begin{matrix} \vec{E}_i = E_i e^{i(kx-\omega t)} \\ B_i = \frac{E_i}{c} e^{i(kx-\omega t)} \end{matrix}}_{\text{wave functions}}$$

or prob Amplitudes

then

$$\text{prob} = \left|E_1 + E_2\right|^2$$

$$\text{prob}(x) = E_1^2 + E_2^2 + 2E_1E_2\cos(k(x_2-x_1))$$

now e^-'s

↑ same thing!

e^-

+ deBye got $\lambda_{e^-} = \frac{h}{p_e}$ to solve Bohr atom stable-state problem

Led. Schrö to talk about wave-like Behavior & DeBye to insist --- where is the wave equa?

So we start by saying, assuming, an e^- wave funct = prob Ampt.

$$\psi_{tot} = \psi_a + i\psi_B$$

} Like $\vec{F}$ except not a vector

⇒'s Linearly indep superposition state functs ≠ dif vector direction

= |dead⟩ + i|alive⟩

w/ this, we will argue for a wave equation for the e^-

Try $\psi_a = \psi_0 e^{i(kx-\omega t)}$

where for waves $k = 2\pi/\lambda$, $\omega = 2\pi/T$ we now know $\lambda = h/p$

& $E = h\nu = hc/\lambda$ & $p = E/c = \frac{h}{\lambda}$