E.F. Deveney / BSC Physics PH402 Quantum I — Scherrer Chp 3 The Schrö Equat

Note Title — 2/1/2006

1912 Old Q.M. = Bohr Atom → Balmer Lines, r_n, E_n

Quantization from $\vec{L} = \vec{r} \times \vec{p} = rmv = n\hbar$

1925 Heisenburg (Hay Fever)

Jordon & Born — Dirac

1926

Heis, Jordan, Born & Dirac = Matrix Q.M.

Very math --- no real attempt for physical picture

Heisenburg ≠ no excuse

1924 deBroglie, $\lambda = \frac{h}{p}$

*Schrö wave was to Q.M. as Feynman diagrams were to Q.E.D

1926 Schrö wave equation after suggestion by deBye

Big push to give physical picture

1926: Born thru Ψ = prob amplitude

this wave Equat

Einstein & Schrö Like crazy, Ultimately, all these attempts fail!

M.Q.M. = 1926 Schrodinger shows dissonant reps = W. Q.M

Vector Space (V.S.) = Function Space

Hilbert

where H-Space = F.S. w/ all props of V.S.

1929 - Dirac: Schrö wave + Special Rel

1932 - Anderson, Balloon, Emdsen = antiparticles

Nobels

1932: Heisenburg

1933: Dirac & Schrö

1954: Born!

1963 - Feynman, Tokomak, Schwinger

$\Psi = e^-$ field + S.R.

$F^{\mu\nu} = E$ & B field

$\Big\}$ = same Footing QED

— 1900: $\vec{E}$ & $\vec{B}$ fields = waves

— 1900 – 1924

1.) photo-e

2.) Compton scattering

3.) S.R.

$\Bigg\}$ $\vec{E}$ & $\vec{B}$ = particles

1912: e^-s = particles

But

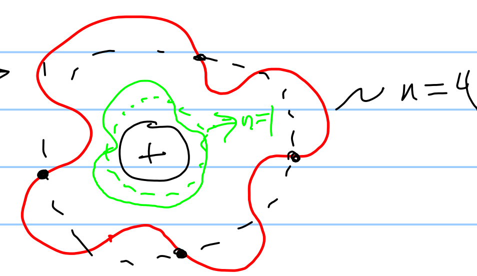
Bohr atom: e^-'s are in Quantized angular momentum states that do not radiate : $\vec{L} = n\hbar$ $n = 1, 2, 3 \ldots$

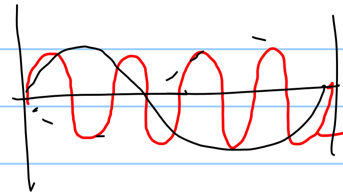
1924: de Broglie $\vec{E}$ & $\vec{B}$ = particles & waves

now e^-s = particles & ?

Bohr atom

Looks like condition for standing waves!





~ 1 wavelength $n=1$

4 wavelengths $n=4$

$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\ \text{J}\cdot\text{sec}}{m_e v_e}$$

$\Big\{$ sure enough $L = rmv = r\frac{h}{\lambda} = n\frac{h}{2\pi}$; $r = \frac{n\lambda}{2\pi}$ $\Big\}$ works!

so γ = particles & waves   now e^-s = particles + waves

Look @ wave Equation to specify E&M waves

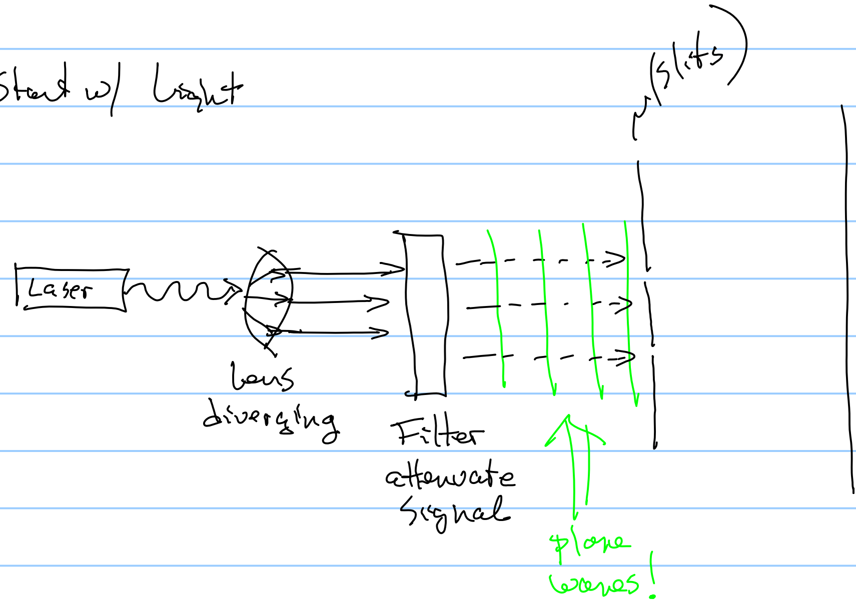
E&M wave equation w/ E&M "wave funct" soln.

Look for e^- wave equation &

e^-, particle, wave function

So, look for experiments:

1. Start w/ Light



Side: Light ⇒ Max's Equations in Free Space ($\rho_{free} = 0$, charges/vol; $\vec{J}_f = 0$, I/Area)

completely describes all E&M phenomena, i.e. Light

1.) $\vec{\nabla}\cdot\vec{E} = 0 + 0$

2.) $\vec{\nabla}\cdot\vec{B} = 0 + 0$

3.) $\vec{\nabla}\times\vec{E} = 0 + -\frac{d\vec{B}}{dt}$

4.) $\vec{\nabla}\times\vec{B} = 0 + \mu_0\epsilon_0\frac{d\vec{E}}{dt}$

only dynamic sources!

Options to solve (1–4)

1.) $\vec{E}$ or $\vec{B}$ one @ a time

$\vec{E}$: $\vec{\nabla}\times(\vec{\nabla}\times\vec{E})$ ...

use $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

get $\frac{d^2\vec{E}}{dx^2} = \mu_0\epsilon_0\frac{d^2\vec{E}}{dt^2}$

same for $\vec{B}$.

Homework on classical wave equat on Rope! solns $= f(x \pm vt)$ = traveling wave @ veloc v ⟵ ⟶

Key: 2nd order wave equat yields 2 linearly indep solns

2.) Assume a soln $\vec{F} = E + iB$

note: anticipatin 2-linear indep solns!

Both Techniques yield MOST GENERAL SOLN!

$\vec{E} = E_0\, e^{i(kx \pm \omega t)}$ $\hat{y}$

$\vec{B} = \frac{E_0}{c}\, e^{i(kx \pm \omega t)}$ $\hat{z}$

⇑ note, not easiest, simplest But most General

why ⟶

What's so special about most general soln?

Well the $\sum_{k=-\infty}^{k=+\infty} A_n e^{i(kx-\omega t)}$ = complete basis set

$* \; k = -$

$A_n e^{i(-kx-\omega t)} = A_n e^{-i(kx+\omega t)}$

= waves ←

So, Every soln can be built from these general solns.

So, if we understand these soln's we understand properties of all solns!

What is

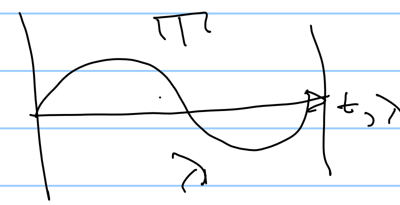
$$\vec{E} = E_0 e^{i(kx-\omega t)} \hat{y}$$

$$\vec{B} = \frac{E_0}{c} e^{i(kx-\omega t)} \hat{z}$$

$$e^{ik(x-\frac{\omega}{k}t)}$$

periodic: $\cos k(x-\frac{\omega}{k}t) + i \sin k(x-\frac{\omega}{k}t)$

so periodic with $\frac{\omega}{k}$



So $v = \frac{d}{t} = \frac{\lambda}{T} = \frac{2\pi/T}{2\pi/\lambda} = \frac{2\pi}{T}\frac{\lambda}{2\pi}$

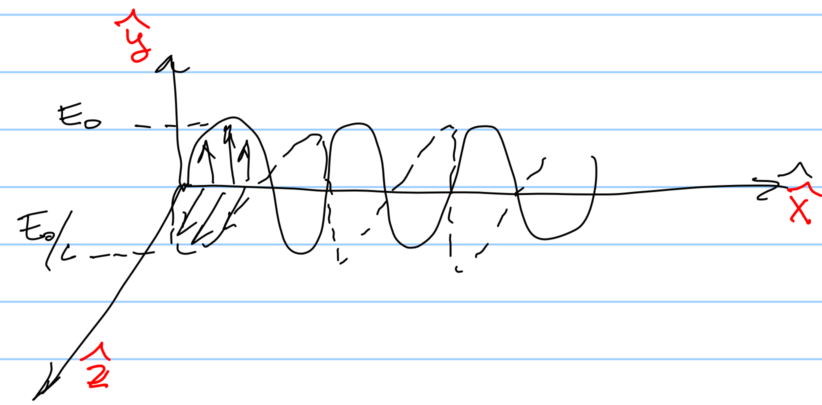
$$v = \frac{\omega}{k}$$

So

$$e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)} = e^{ik(x-vt)}$$

These are just TRAVELING WAVES!

Conclusion: soln to max's Equat in free space
= traveling waves
(self-propagating)

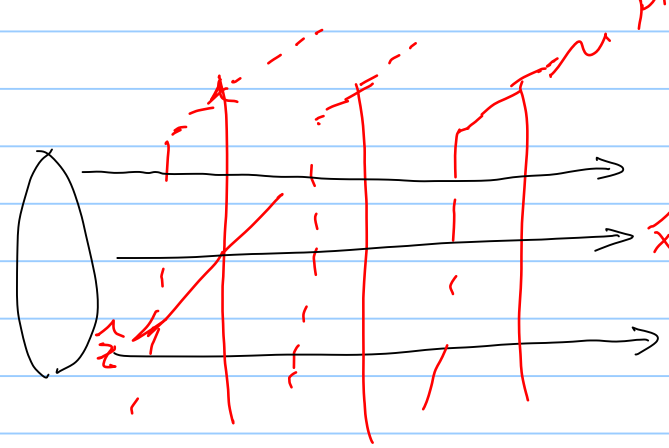


They are Plane-waves! ie $\vec{E} = E_0 e^{i(kx-\omega t)}$

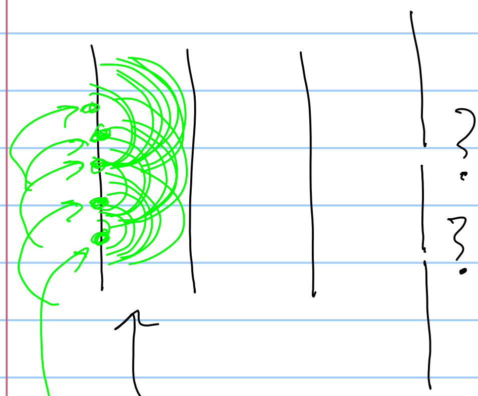
$\vec{E}$ = same vector in entire y-z plane!

O.K. not so physical
but idea is this is great bases and all solns can be built from it!

So, now plane waves

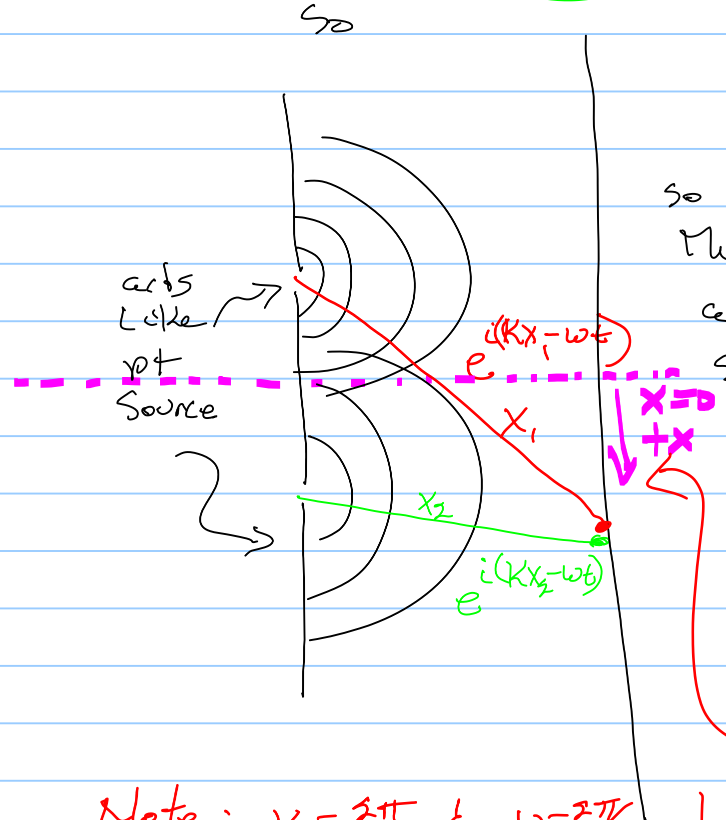


So now look @ plane waves on a slit...



well Huygen's Princ: Plane waves = ∞ sum of pt-like sources

So



so in every direction the plane wave acts like a source that spreads out like

$e^{i(kx-\omega t)}$

so

Note; $k=\frac{2\pi}{\lambda}$ & $\omega=\frac{2\pi}{T}$ do not change for each source.

So, if you want to find out what happens @ x
you've got to add up the waves @ pt x

Now to measure waves, you measure
intensity

$$I = \frac{\text{Energy}}{(m^2)(\text{sec})} \quad \text{where} \quad \text{Energy} \propto |\text{Amp}|^2$$

ie  $= \frac{1}{2} k x_0^2$

$x_0 = \text{amp}$

So, need the amplitude of
the $\vec{E}$ & $\vec{B}$ field @ pt x

This must be the total field @ $\vec{x}$
* will only look @ $\vec{E}$ but of course really need
$\vec{B}$ field too.

So @ x, find $E_{TOT}(x) = \underset{\uparrow \atop \text{source 1}}{E_1(x)} + \underset{\uparrow \atop \text{source 2}}{E_2(x)}$

Then Intensity $\propto |E_{TOT}|^2$

$$E_T(x) = E_1(x) + E_2(x) = E_1 e^{i(kx_1 - \omega t)} + E_2 e^{i(kx_2 - \omega t)}$$

$$= e^{-i\omega t}\left[E_0 e^{ikx_1} + E_0 e^{ikx_2}\right]$$

since from same plane wave

Now

$I \propto |E_T(x)|^2$ * remember it is complex

$$I(x) \propto E_T^* E_T = \left(E_0 e^{i\omega t}\left[e^{-ikx_1} + e^{-ikx_2}\right]\right)\left(E_0 e^{-i\omega t}\left[e^{ikx_1} + e^{ikx_2}\right]\right)$$

$$= e^{i\omega t} e^{-i\omega t} E_0^2 \left[e^{-ikx_1} e^{ikx_2} + e^{-ikx_2} e^{ikx_2} \ldots \right.$$

$$\left. e^{ik(x_2 - x_1)} + e^{-ik(x_2 - x_1)}\right]$$

$$e^{ix} = \cos x + i\sin x$$

$$e^{-ix} = \cos x - i\sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$I(x) \propto E_0^2\left[2 + 2\cos k(x_2 - x_1)\right] = E_0^2\left[1 + \cos k(x_2 - x_1)\right]$$

is $\frac{2\pi}{\lambda}\Delta x = 2\pi$

or $\Delta x = \lambda$

or

2λ

4λ

=

constructive interference

or

$$\propto \left[\underset{\text{is just wave 1}}{E_1^2} + \underset{\text{is just wave 2}}{E_2^2} + \underbrace{2E_1E_2\cos k(x_2 - x_1)}_{\text{interference term}}\right]$$

&

$$I(x) \propto E_0^2 \left[2 + 2 \cos k(x_2 - x_1) \right] \Rightarrow$$

$$E_0^2 \left[1 + \cos k(x_2 - x_1) \right] =$$

Note: $k(x_2 - x_1)$ is $\frac{\cancel{2\pi}}{\lambda}(\Delta x) = n\,\cancel{2\pi}$

Then get constructive interference

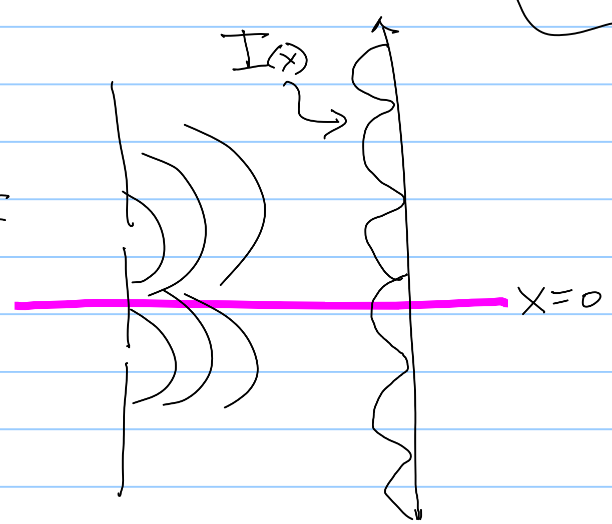
$$\Delta\lambda = n\lambda$$

$$n = 0, 1, 2, 3 \ldots$$

& Complete destructive interference for

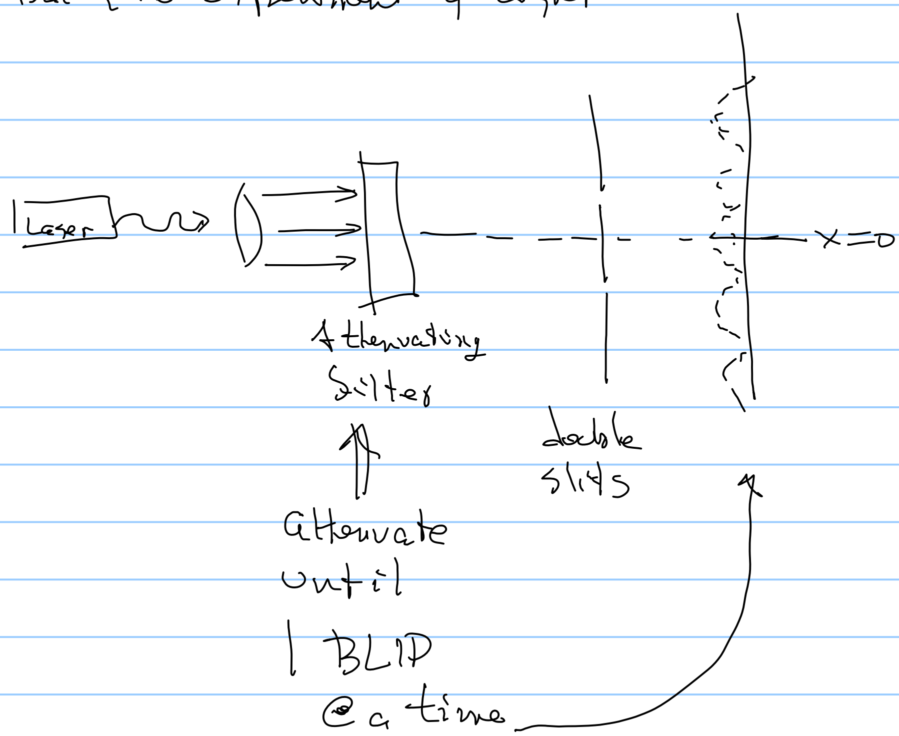
$$\frac{\cancel{2\pi}}{\lambda}(\Delta x) = \frac{n}{2}\,\cancel{2\pi}$$

$$\boxed{\Delta x = \frac{n\lambda}{2}}$$

= 

& This is exactly what you get experimentally!

Back to experiment w/ Light



Light apparently, (ie Plank, Compton, Einstein) comes in 1 Lump @ a time = photon! γ

Can see, Each Lump does not go to same place!

So At Best we can only try to predict where each γ will come.

Clearly prob = Big where I builds up & prob = small when I is small

$\therefore$ Can say: prob of where γ ends up $\propto I$ or $\propto |E_{TOT}|^2/dx$

W2 - Alt: photons = particles
can only do probabilities
of where it will end up

&

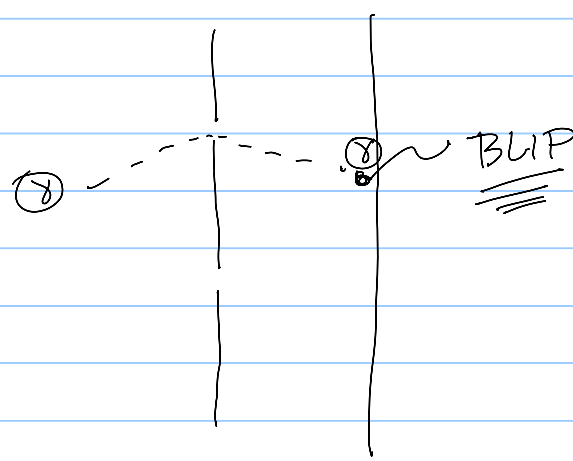
prob $\propto I$ or $\propto \left| E^2 + B^2 \right| dx$

don't forget really need this!

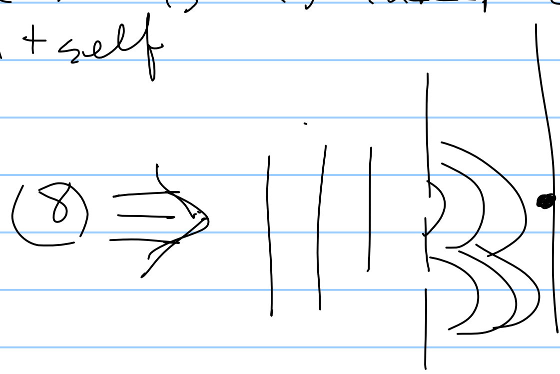
will later use

$\left| E^2 + B^2 \right|$ = energy or prob density!

Note also: this 1 photon @ a time experiment $\Rightarrow$'s γ interferes w/ itself



But only way for the "Individual" photon Intensity, or probability, to build up over time is is indeed each γ interferes w/ itself



even though it seems like it went thru just one slit!

READ Feynman Lecture NOW

(insert it)

Lect # 1 Volume III = Quantum Behavior.

especially

1-6 Watching e-s

1-8 the Uncertainty Princ.

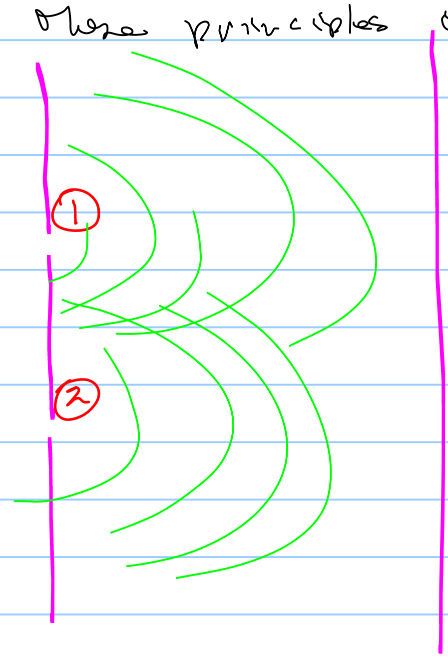
We must keep these principles in mind.

$(\gamma) = \vec{E}$ & $\vec{M}$ plane waves

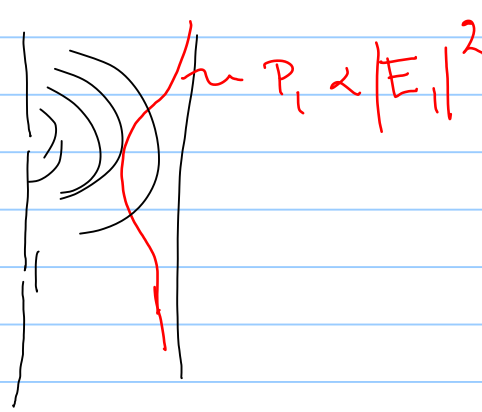
(e^-), $\Psi_{\text{Schrö}}$

or

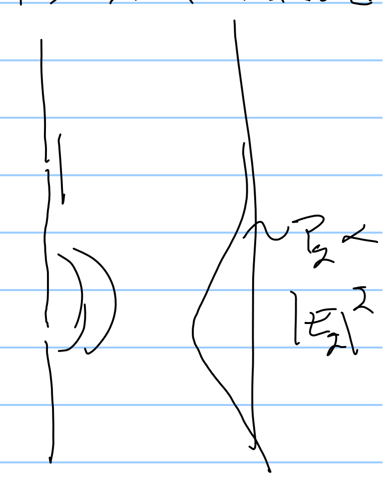
regular waves



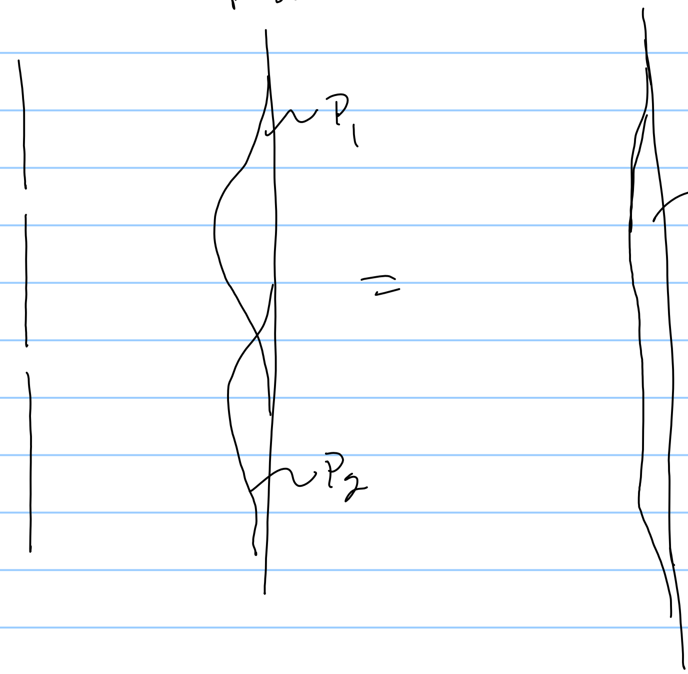
if slit 2 is blocked



if slit 1 is blocked



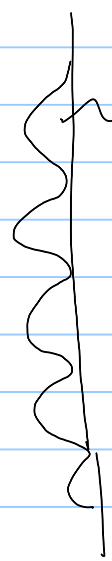
Now both open



$P_1 + P_2 = P_{TOT}$

$P_T \propto (E_1^2 + E_2^2)$

But we don't get that instead we get



$$P_{TOT} \propto \Big(\underbrace{\vec{E}_1 + \vec{E}_2}\Big)^2 = \underbrace{E_1^2 + E_2^2}_{\text{each slit open independently}} + \underbrace{2E_1E_2\cos(k(x_2-x_1))}_{\text{interference TERM!}}$$

where (call E_1 & E_2 prob amplitudes

each slit open independently

interference TERM!

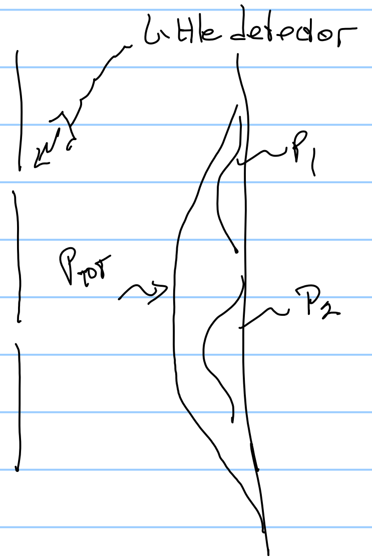
Therefore:

In the Limit of 1 ⊗ @ a time, to get the correct results you must

Construct a total probability by adding the waves from each slit @ the same time together & then Squaring! =

conclude the γ wavefunction takes both paths at once and ∴ THE γ interferes w/ itself.

If you try to look @ which way the photon did go, you destroy the interference pattern!



Interference is DESTROYED

P_T looks like $\propto E_1^2 + E_2^2$
Like it went thru either slit 1 or 2.

I will conclude w/ Q.M. Big Picture

(γ); $\vec{E}$ & $\vec{M}$ fields = Wave funct

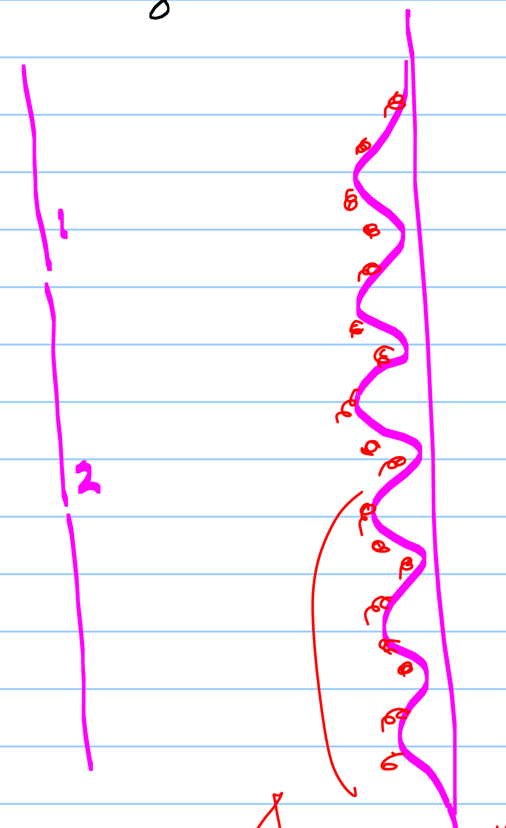
(e^-); Ψ_{e^-} from Schrö

(car); Ψ_{car} from Schrö

Everything

A WAVE FUNCTION!

E or Ψ



"Particle" @ a time

Pt A

@ Pt B

RULES

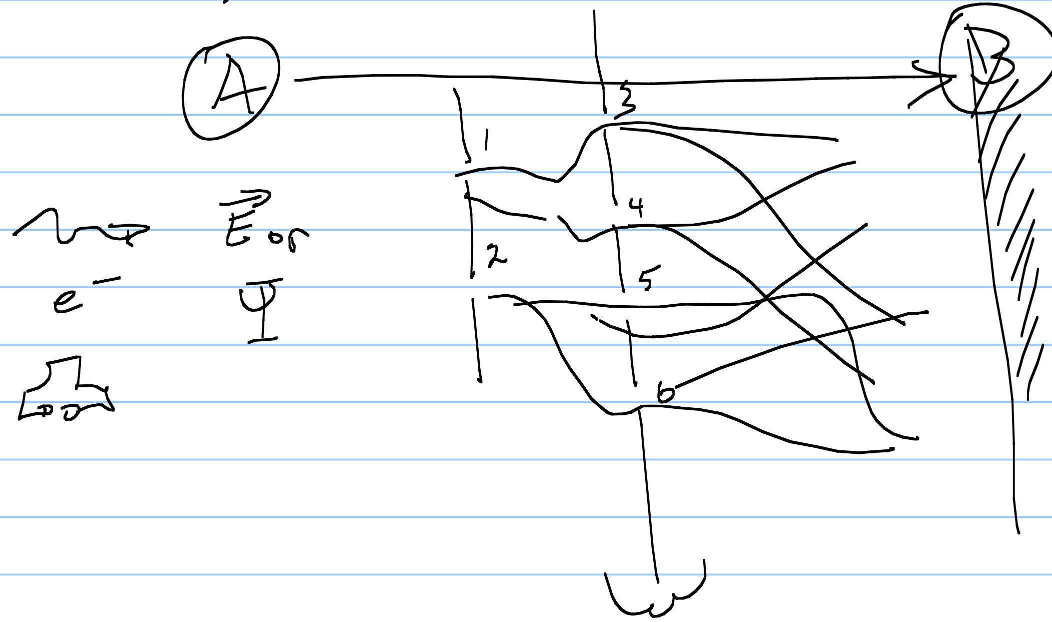
A → If know it went thru 1 or 2

$P_T = P_1 + P_2 = E_1^2 + E_2^2 \Rightarrow$ no interference → B

If don't know then

$P_{TOT} = P_1 + P_2 + \text{interference} = E_1^2 + E_2^2 + 2E_1E_2\cos(k(x_2 - x_1))$

A Trinity Feynman then Says.....
what is



Why Not.

Why not more & more until in the Limit None?

Conclude:

$$A \longrightarrow B$$

requires

$$P_{Tot} = \left| A_1 + A_2 + A_3 + \cdots \infty \right|^2$$

prob Amplitude for path (i) ie E_i or Ψ_i

For all paths from A–B

ALL

So that all paths from A to B contribute to the prob of getting from A to B

& All of these paths interfere w/ each other!

YIKES!

We see that what will need is some way to

1.) Find the Quantum field of a particle

2.) Find the prob amplitudes A_i from 1

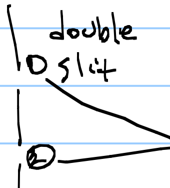
3.) Sum up (perhaps ∞ #) of A_i

4.) Square them

5.) $$P_{TOT} = \left| \sum_{i=1}^{\infty} A_i \right|^2$$

ex. w/ γ: 1.) Field is $\vec{E}$ & $\vec{B}$ from $$\frac{d^2E}{dx^2} = \mu_0\epsilon_0 \frac{d^2E}{dt^2}$$

2.) Solve for $\vec{E}_1 = E_1 e^{i(kx-\omega t)}$

3.) Sum all paths $\left|\right.$ 

$\left.\right| = E_1 + E_2$

4.) & 5.) $P_{tot} = |E_1+E_2|^2 = E_1^2 + E_2^2 + 2E_1E_2\cos[k(x_2-x_1)]$

* quite different from $P_{TOT} = \sum_i E_i^2 = E_1^2 + E_2^2$ (allows no interference)

How can this idea of nature be correct? After all, it must reduce to classical physics!

Well, what do we know about classical physics?

It all comes from Euler-Lagrange minimization of the action integral

$$S = \int \mathcal{L}\,dt \qquad \mathcal{L} \equiv \underset{\text{kinetic}}{T} - \underset{\text{potential}}{V}$$

S is minimized for solutions that satisfy

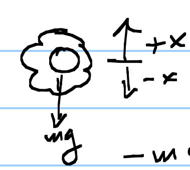
$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = 0 \quad : \text{E-L}$$

From which all of classical mechanics follows.

The E-L equation just helps us figure out which path minimizes S.

Nature works to minimize S.

ex: $\mathcal{L} = \frac{1}{2}m\dot{x}^2 - mgx$



$-mg - m\ddot{x} = 0$

$-mg = m\ddot{x}$

$-mg = m\vec{a}$

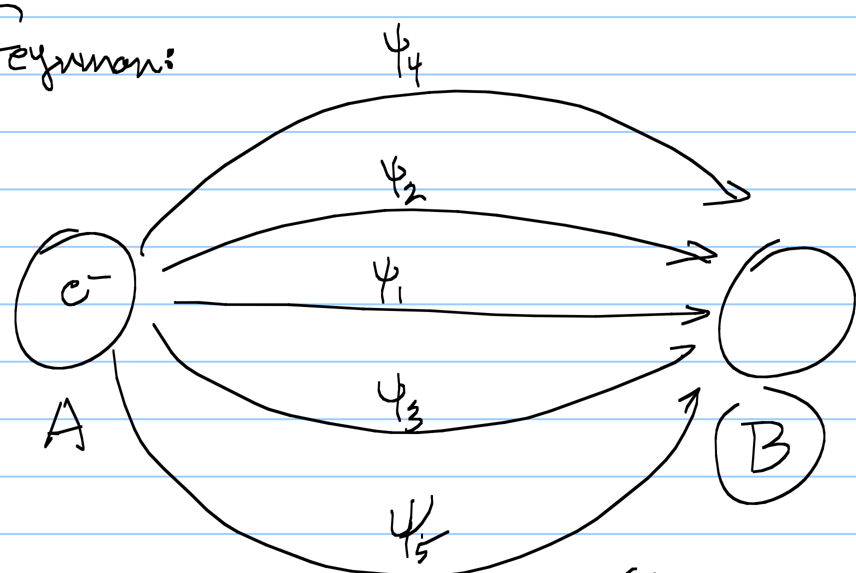
Just newton's 2nd Law.

Solve for x & you've solved for the position function that minimizes the action integral.

& this = Nature.

So, Bigger question is, why does nature always use the path that minimizes S?

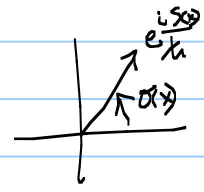
Feynman:



let each $\psi \to e^{iS/\hbar}\psi_i = e^{i\frac{\int \mathcal{L}\,dt}{\hbar}}\psi_i$

$\vdots$

$e^{i\frac{\text{Energy}(x)}{\hbar}}\psi_i$



ψ_i

So each ψ carries a action integral clock

$\theta(t)$

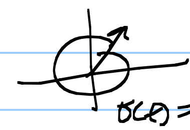
$$e^{i\frac{S(x)}{\hbar}}\psi = e^{i\theta(t)}\psi$$

$S = \text{Energy}$

$$\frac{S_{Baseball}}{\hbar} = \frac{J\cdot sec}{10^{-34}\, J\cdot s} = 10^{34}\, t$$

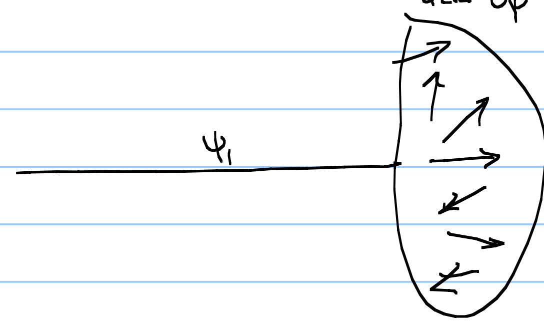
$$\frac{S_{e^-}}{\hbar} = \frac{(10^{-19}\, J)\cdot sec}{10^{-34}\, Js} = 10^{15}\, t$$

Look @ Baseball clock $e^{i10^{34}}\psi$

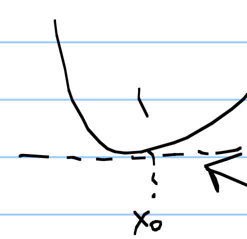


$\theta(t) = 10^{34}\,\frac{rad}{sec}$

so



since goes so quickly cell but on



$$S(x) = S(x_0) + \frac{dS}{dx}\Big|_{x_0}\Delta x + \frac{1}{2}\frac{d^2S}{dx^2}\Big|_{x_0} x^2$$

$\Rightarrow$

$$S(x) \cong S(x_0) + \underbrace{\frac{1}{2}\frac{d^2S}{dx^2}\Big|_{x_0}}_{\text{small}} x^2 + \cdots$$

So this means

1) you are on a path

'

$\vec{F} = \vec{E} + i\vec{B}$ ← complex #

keeping track of 2 linearly indep vectors

(8)  $\underbrace{\frac{d^2\vec{F}}{dx^2} = \mu_0\epsilon_0 \frac{\partial^2\vec{F}}{dt^2}}_{\text{wave equation}}$; $\underbrace{\vec{E}_i = E_i e^{i(kx-\omega t)} \quad B_i = \frac{E_i}{c} e^{i(kx-\omega t)}}_{\text{wave functions}}$

or prob Amplitudes

then

$$\text{prob} = |E_1 + E_2|^2$$

$$\text{prob}(x) = E_1^2 + E_2^2 + 2E_1E_2\cos(k(x_2 - x_1))$$

now e^-'s ↑ same thing!

e^- 

+ deBye got $\lambda_{e^-} = \frac{h}{p_e}$ to solve Bohr atom stable-state problem

Led. Schrö to talk about wave-like Behavior

& DeBye to insist --- where is the wave eqn?

So we start by saying, assuming, an e^- wave funct = prob Ampt.

$$\Psi_{tot} = \Psi_a + i\Psi_B$$

} Like $\vec{F}$

except not a vector

⇒'s Linearly indep superposition

state functs ≠ dif vector direction

$= |dead\rangle + i|alive\rangle$

w/ this, we will argue for a wave equation for the e^-

Try $\Psi_a = \Psi_0 e^{i(kx-\omega t)}$

where for waves $k = 2\pi/\lambda$, $\omega = 2\pi/T$, $\lambda = h/p$

we now know

& $E = h\nu = hc/\lambda$ & $p = E/c = \frac{h}{\lambda}$