Electric & Magnetic Field

Wave equation

Then prob = $|E_1 + E_2|^2$

Lagrange's & Schrödinger → build with...!

Some thing...

Debye Field

2D $e^-$ wave function

3D $e^-$ wave equation!

+ Debye get $\frac{\hbar}{e} = \frac{\hbar}{F_\text{Deb}}$ to solve

Bohr atom... stable state

Schrödinger equation

lead, Schrödinger will talk about

wave-like behavior

$\Psi^2$ Debye must... there is the wave equal

so we start by saying, assuming 3D wave function = prob

$\Psi_1 + i\Psi_2$

are not vectors

$\Psi$ is linearly indep.

Super position

$\Psi^2$ is close to $\Psi_1$ & $\Psi_2$ vectors, direction

3D vector direction = $\Psi_1 + i\Psi_2$
Here's where we are:

1. $E\times B$ Sells $i$ $\text{e}^T$

2. $E\times B$ wave function $\psi = E_x e^{i(kx - \omega t)} + E_y e^{-i(kx + \omega t)}$

*note 2 linearly indp solns

3. $E\times B$ wave Equation: Max's

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\omega^2}{c^2} \psi$$

4. Yields probabilities in limit of single photon

$$\frac{\partial^2 |\psi(x)|^2}{\partial x^2} = \text{prob density}$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

Now let's yield:

1. $E\text{e}^T$

2. $\text{e}^T$ wavefunction $(\psi = \psi_1 + i\psi_2)$

3. $\text{e}^T$ wave equation $= \text{Schrod}$

4. $\text{e}^T$ prob density $|\psi|^2 = \text{prob}$
Electromagnetic Field

so \( E_M : \mathbf{F} = E_0 e^{-\frac{(x^2-x^2)}{2}} \times \mathbf{v} \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \)

\( \omega = 2\pi f = \frac{2\pi}{\lambda} \)

Wave Equation

\[
\frac{\partial^2 E}{\partial x^2} = \frac{1}{\mu_0 e_0} \frac{\partial^2 E}{\partial t^2} \quad \text{or} \quad \nu = \frac{1}{\sqrt{\frac{\mu_0 e_0}}}
\]

Speed of light: \( c = \frac{1}{\sqrt{\mu_0 e_0}} \)

So, let's get physical of the \( E_x \) wave equation.

We know the \( E_x B \) field carries energy & momentum & it is a traveling wave @ \( c \)

\[
E_x = \frac{1}{2} E^2 \quad \text{or} \quad E = \frac{1}{2} c \frac{dE}{dt} = \nu \frac{dE}{dt}
\]

\[
\mathbf{B} = \frac{1}{c} \frac{dE}{dt}
\]

\[
\nu = \frac{E_0}{2} - \frac{1}{2} m c^2
\]

\[
\mathbf{p} = \frac{E_x}{c} \quad \text{or} \quad \frac{E}{m} = \frac{1}{c} \mathbf{v} = \frac{\lambda}{\nu} = \frac{\lambda c}{\nu} = \frac{\lambda c}{\nu}
\]

\[
\mathbf{p} = \frac{\lambda c}{\nu} = \nu \mathbf{c}
\]

So Maxwell's equations yield

\[
\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \left( \frac{1}{\mu_0 e_0} \frac{\partial^2 E}{\partial t^2} \right)
\]

\[
\frac{\partial^2 E}{\partial x^2} \quad \text{or} \quad \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}
\]

\[
\frac{\partial^2 E}{\partial x^2} \quad \text{or} \quad \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}
\]

\[
\nu = \frac{\lambda c}{\nu} = \frac{\lambda c}{\nu} = \frac{\lambda c}{\nu} = \frac{\lambda c}{\nu}
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\[
\mathbf{v} = \nu \mathbf{c} = \frac{\lambda c}{\nu}
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\[
\mathbf{v} = \nu \mathbf{c} = \frac{\lambda c}{\nu}
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\[
\mathbf{v} = \nu \mathbf{c} = \frac{\lambda c}{\nu}
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\[
\mathbf{v} = \nu \mathbf{c} = \frac{\lambda c}{\nu}
\]

\[
\mathbf{v} = \nu \mathbf{c} = \frac{\lambda c}{\nu}
\]
Thus Maxwell's equations, the \( E \times B \) wave equation is a sort of eigenvalue problem that looks for a solution, the \( E \times B \) wavefunction:

\[
D_\phi \vec{E} = \frac{1}{c} \nabla \times \vec{E} \quad \Rightarrow \quad \nabla \overrightarrow{D} \equiv \frac{1}{\omega} \overrightarrow{B}
\]

That contains:

- Energy \( \propto \omega \)
- \( \frac{1}{2} \) momentum \( \propto k \)

\( \frac{1}{2} \) is a massless wave that travels \( \propto c \).

Maxwell's Equations & its solution therefore contain all these into there is about light!  

\textcolor{red}{\text{Light = particle + wave}}

We will thus seek a similar answer &

\( \textcolor{red}{e^- = \text{particle} + \text{wave}} \)

\textcolor{red}{IDEA Here is}

Look for a sort of Hybrid Eiger problem that incorporates both wave-like & particle-like properties.

Because Energy \( \textcolor{red}{(\text{Classical})} \) but in this case is conserved, look for
Our hybrid eigen problem states "Find \( \Psi \) such that when you act on it with classical and wave-like \( \hat{O} \)'s that go out consistent results."

So the goal is to send the \( \Psi \) wavefunction = state function that can contain all the particle-like and wave-like properties.

Argue: The connection of this hybrid eigen problem should be energy.

\[
\begin{bmatrix}
\text{E} \\
\text{classical energy}
\end{bmatrix}
\Psi = \begin{bmatrix}
\text{E} \\
\text{wave-like}
\end{bmatrix}
\Psi
\]

Recognize as the Hamiltonian \( \hat{H} \) classically already.

For starters: world w/ light aether \( \hat{\Psi} \), it is

\[\hat{H} \Psi = \hat{\Psi} \Psi\]
Now light is massless & Held, but $E_{\text{tot}} = \hbar \omega = \hbar k c = \hbar \frac{\omega}{c}$

Because in 1924 de Broglie
wavelength $\lambda = \frac{\hbar}{p}$

$\hat{H}_{\text{particle}} = \frac{\hbar}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi$

$\hat{H} = E = \frac{\hbar c}{\gamma}$

$\hat{H}_{\text{particle}} = \hbar k < c$

$c = \frac{1}{\gamma}$

Because $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
So Schrödinger’s wave function is really the Schrödinger wave equation.

But now motivated to look for a wave equation that carries anything about a classical particle and mass particle. The free electron.

$$E_k = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

So start with the classical particle:

Free particle for now!

$$E_{tot} = E_k + E_p$$

No non-relativistic $$\frac{v}{c}$$.

$$H = E_k + E_p = E_k = \frac{1}{2m} p^2$$

"Massive"

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

why? well $$p \rightarrow \hbar k$$

Connection

$$\chi = \frac{\hbar}{\lambda}$$

$$p = \frac{\hbar}{\lambda}$$

so $$p = \frac{\hbar}{2\pi} \frac{2\pi}{\lambda} = k \frac{\lambda}{2\pi}$$

now we are talking about (k x - w t).
So, \( \phi = x k \)

What is \( w \) for

\[ \phi = -i \frac{\hbar}{2m} \frac{\partial}{\partial x} \]

Then, \( \phi \phi = -i x \frac{\partial}{\partial x} e^{i(kx - \omega t)} \)

So,

\[ \phi \phi = \hbar k \psi = E \text{ or quantum}
\]

\[ \phi \phi = \text{Ensemble}
\]

\[ \psi = \text{momentum}
\]

Thus,

\[ \phi = -i \frac{\hbar k}{2m} = \text{Momentum} \phi
\]

Back to math issue

\[ H_{\text{free particle, classical}} = \frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \]

Free particle Schröd

\[ \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (\psi(x,t)) = i \hbar \frac{\partial}{\partial t} (\psi(x,t)) \]

\[ H_{\text{classical}} \psi(x,t) = \psi(x,t) \]
So we have our Schrödinger's equation for a free particle:

\[ H \Psi = E \Psi \]

Classical \quad Wave

\[ \hat{H}_{\text{classical}} = \hat{H}_{\text{wave}} = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2m} \frac{\partial^2}{\partial \xi^2} = \text{Schrödinger time-dep equation} \]

Now in general:

\[ \hat{H}_{\text{classical}} + \hat{H}_{\text{wave}} = \hat{H} = \hat{H}_{\text{ion}} + \hat{H}_{\text{other}} \]