E. F. Deveney / BSC Physics: PH402 Quantum.

Note Title 2/25/2006

HERE

Schroë C3
Schrö Soln
only to
understand
what it
all means!

Our goal here is to
understand Schrö equation &
its solutions and implications.

Not how to Solve for now.

PART 1 THIS Lect

Keys:
1) Normalization → Hilbert → function spaces; → connect to matrix
2) Probablistic interp of Schrö equation (Max Born 1926)

Next Lect.

3.) Probability
4.) Observables
5.) ensembles of a Quantum state
6.) expectation values!

How by example: 1-D particle in Box (well) ∞-ly deep.

"IDEAL"

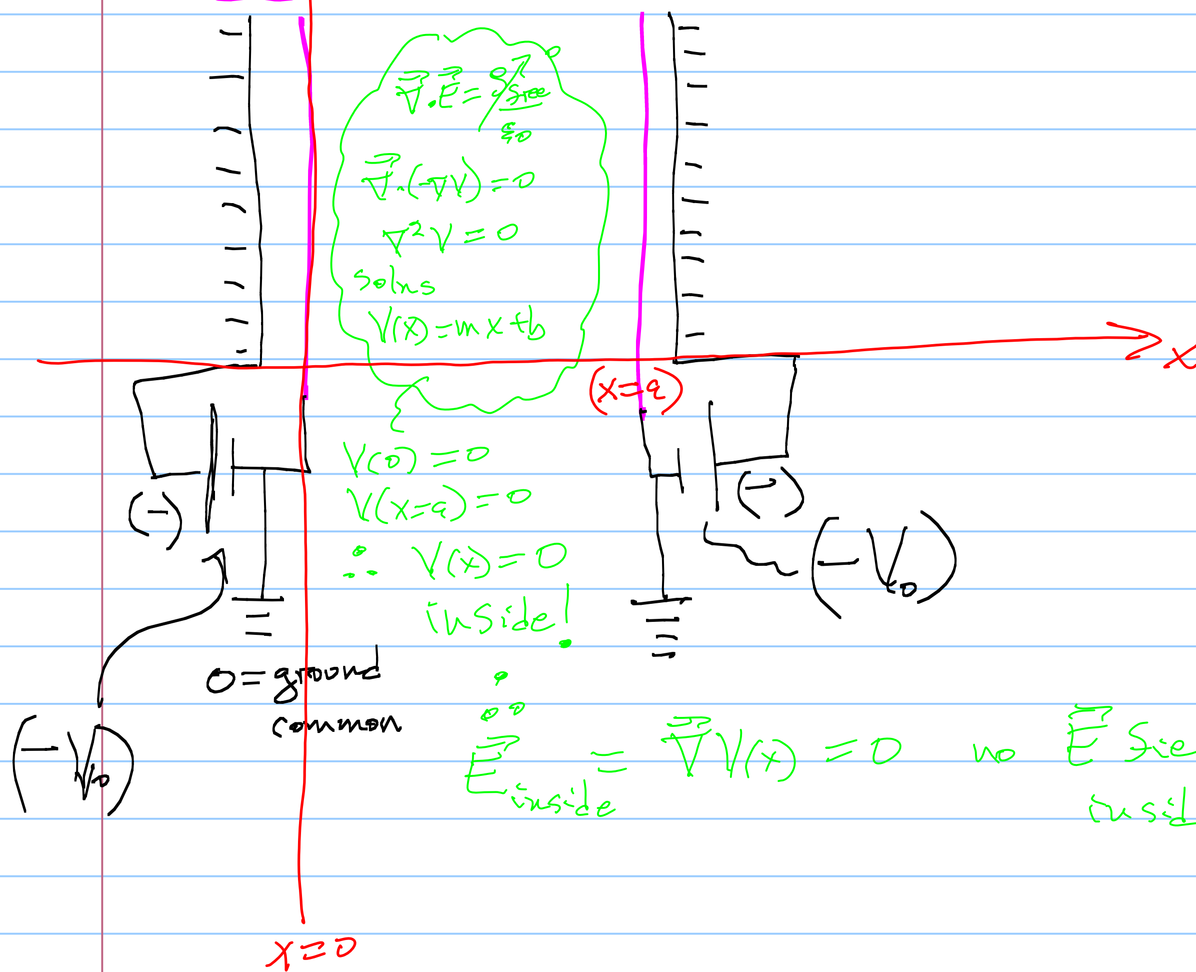
Caution: Not a "realizable" real world problem

Why? Easy, Tremendous Insight

BIG ⟹ Further: All problems are very tough, almost always reduce down to these "ideal" Case's to get 1st order Soln's!

1-D, ∞-well.

essentially capacitors



$\vec{\nabla}\cdot\vec{E} = \cancel{\rho_{free}/\varepsilon_0}$

$\vec{\nabla}\cdot(-\vec{\nabla}V) = 0$

$\nabla^2 V = 0$

solns

$V(x) = mx + b$

$V(0) = 0$

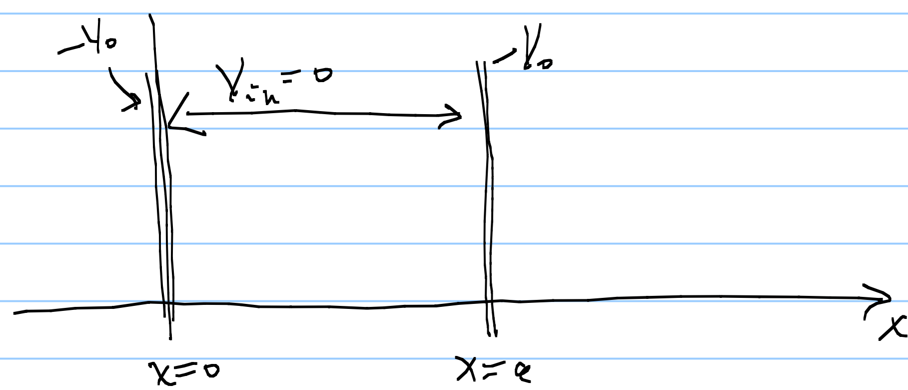
$V(x=a) = 0$

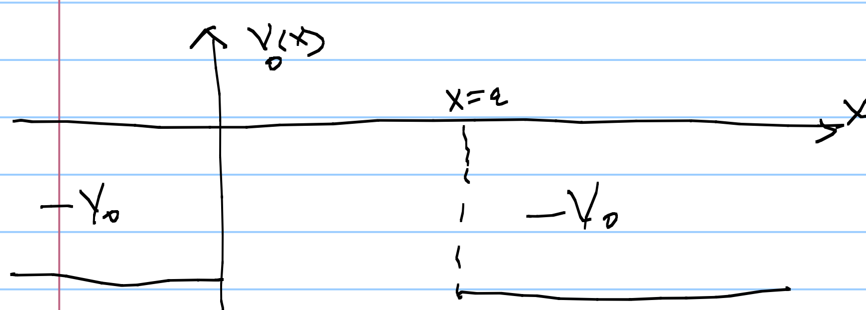
$\therefore\ V(x) = 0$ inside!

$\therefore$

$\vec{E}_{inside} = \vec{\nabla}V(x) = 0$ no $\vec{E}$ field inside.

Let capacitor get ∞ly thin then have





now bit of confusion

$$-V_0 = \text{voltage} = \frac{\text{Joule}}{\text{charge}}$$

so

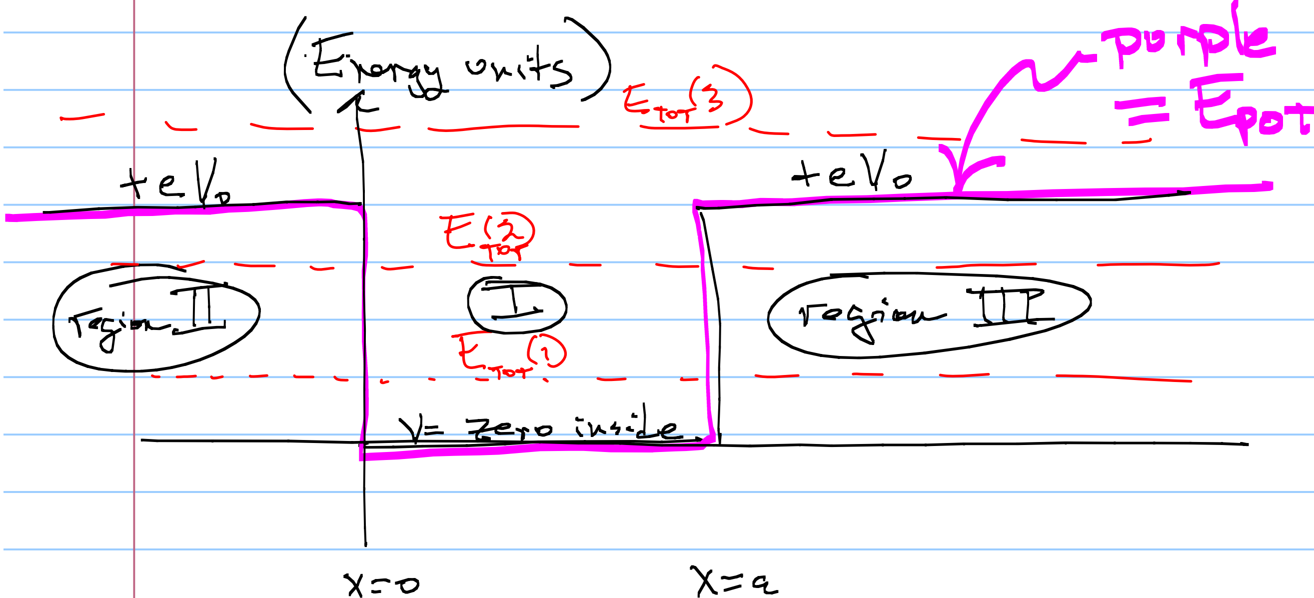
$$E_{pot} = V(x) = q(-V_0) = (-e)(-V_0)$$

potential energy

$$E_p = eV_0 = (+)$$

so

$$E_{TOT} = E_K + E_p$$



This is called a potential

WELL!

BIG

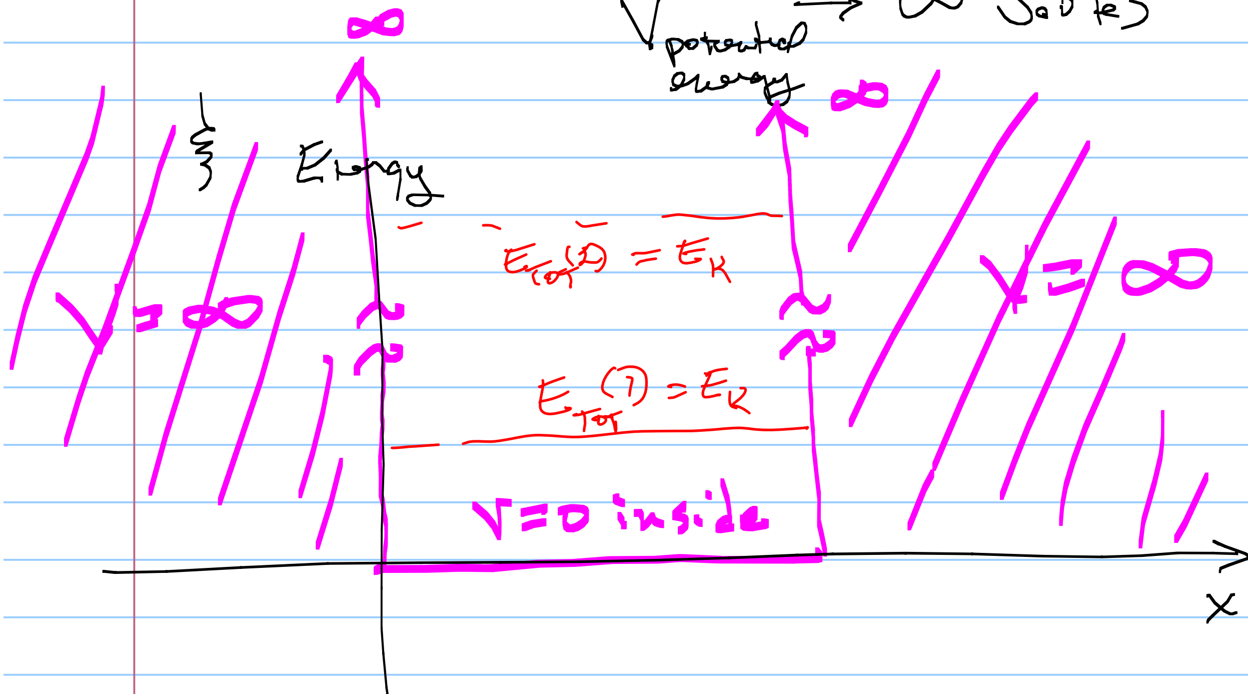
Consider E_{TOT} 1, 2, 3

inside $= (0 < x < a)$	outside $x < 0$, $x > a$
$E_{TOT} = E_K + \cancel{E_P}^{0} = E_K$	$E_{TOT} = E_K + E_P$ or $E_K = E_{TOT} - E_P$
Classically cool!	problem classically For E_1 & $E_2 = E_T$, Thus $\Rightarrow$'s $E_K = (-)$ because $E_T < E_P$
Regions II & III Classically NEVER can have particle there! $\Rightarrow$	These regions are considered Classically Forbidden!

In Quantum mechanics There is No reason why NOT to consider Solving Schrö in these regions.

Why? as you will see, The nature of the Solutions to Schrö will need to be Finite, smooth & continuous $\therefore$ you will need to solve Schrö in all 3 regions except in 1-CASE!

Now, is Ideal case
called ∞ 1-D well,
let
$V_0 \to \infty$ Voltage
Then
$V_{\text{potential energy}} \to \infty$ Joules



Then this ∞ well $\to$'s classical
and Schrö's also give's particles
can't get in there.

Thus, this ∞ AND 1-D well is
what we will solve in
Region I
This is "IDEAL"

Region I, $V(x)=0$

1-D Schrö. time dep Equat

$$\hat{H}_{tot}\,\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

$$\hat{H}_{tot} = \hat{E}_k + \hat{E}_p$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x = \text{inside}) \rightarrow 0$$

$$= \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

so

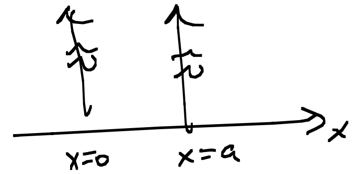
$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

Solve for $\Psi(x,t)$

particle in ∞ 1-D well

$$\Psi(x,t) = \begin{cases} 0 & x<0 \\ & x>a \\ A\sin\left(\frac{\pi x}{a}\right) e^{\frac{i\hbar\pi^2 t}{2ma^2}} & 0 \le x \le a \end{cases}$$

So we've got solns to



$$\left[\frac{p^2}{2m} + V\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$ for 1-D particle in ∞ well

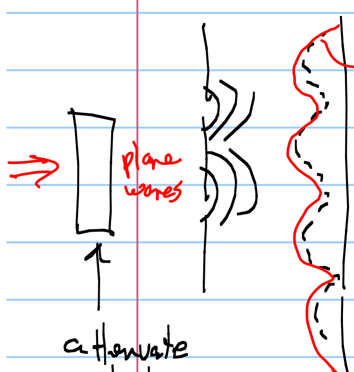
$$\Psi(x,t) = \begin{cases} A\sin\left(\frac{\pi x}{a}\right) e^{\frac{-i\hbar\pi^2}{2ma^2}t} & 0 \le x \le a \\ 0 & x<0 \\ & x>a \end{cases}$$

(we used B.C.s to get soln)

note A is arbitrary

Now what? recall our lead ----

Light = wave + particle



attenuate until 1 click @ a time

red = wave soln, wave funct, $\vec{E}$ of Max Equation = Wave equation

Wave Equat = Max
Wave Funct = $\vec{E}$
} Intensity $= |E(x)|^2$ (really $|E+B|^2$)

(probability are (γ) @ a time @ x) $\propto I(x)$

Where $I(x)$ is Big

$P(x)$ is Big

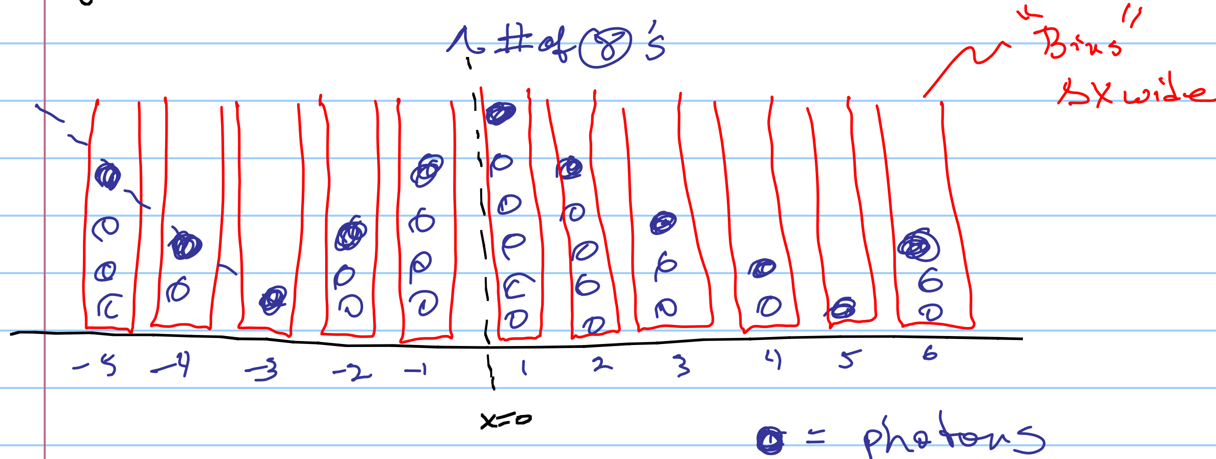
$P(x) \propto |\text{wave funct}|^2$

$P(x) \propto |E|^2 \Leftarrow$ E & M

Now $\Longrightarrow$

Probability Side Note #1/2

got $P(x) \propto |E|^2$ need more



N_i = # of photons in Bin (i)

$$N_{TOT} = \sum_{i=-\infty}^{+\infty} N_i$$

Now $P_i = \frac{N_i}{N_{TOT}}$ = prob that photon will end up in bin (i)

$$\& \sum_{i=-\infty}^{+\infty} P_i = 1 = \cdots \frac{N_1 + N_2 + N_3 \cdots \; N_1 + N_2 + N_3 + \cdots}{N_{TOT}}$$

ex: B-days Feb 14; $P_i = \frac{1}{365}$ = Finite "Bin" size
Bin = day

But what is Sat Feb 14, 9–10 AM

$$P_i = \frac{1}{365 \cdot 24}$$

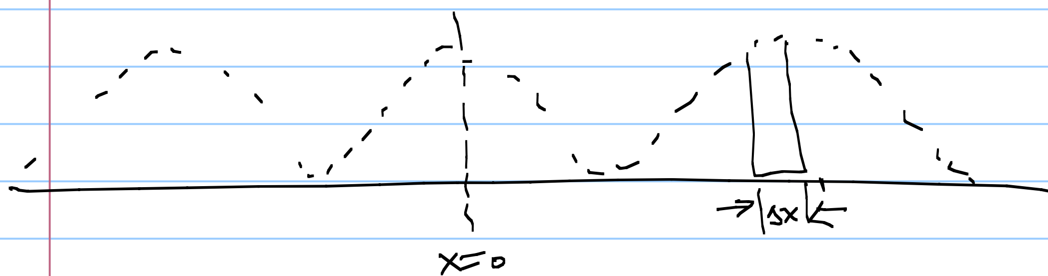
what is Feb 14, 9.25 AM 3.7652...... seconds

can eventually $P \propto dt$

since prob must be unitless, need

$P = (\text{density})\, dt$ where density $= \rho = \frac{1}{\text{time}}$

So back to our bin



clearly if we let bin $\delta x \to dx$

finite $\uparrow$ $\uparrow$ infinitesimal

Then

$P(x)$ must also be $\propto dx$

$P(x) \propto dx$ now to make unitless need density

$\therefore P(x) = (\text{density})\, dx$

$P(x) = \rho\, dx$

& ρ must have units of $\frac{1}{\text{length}}$ $dx = 1$

end of prob side note #1

So

$$P(x) = \rho(x)\, dx$$

We have argued up to now that

$$P(x) \propto |E(x)|^2$$

So How about $P(x) = |E(x)|^2 dx$

where $|E(x)|^2$ = prob density of where γ might end up $= \dfrac{\text{prob}}{\text{unit length}}$

Now just like before, if add up all prob, you get 1

So say

$$\int_{-\infty}^{+\infty}(d\,\text{prob}) = \int_{-\infty}^{+\infty}|E(x)|^2\,dx = 1$$

$$P_{TOT} = \underbrace{\int_{-\infty}^{+\infty}|E(x)|^2\,dx = 1}_{\text{means}}$$

The photon prob for ending up somewhere between $x=-\infty$ to $x=+\infty$ $= 1$

THIS IS IT

Now back to Q.M.

γ, $\vec{E}$ & $\vec{B}$

e^-, Ψ

γ = wave & particle

wave equat = Max

Got this → wave funct ⇒ $\vec{E} + i\vec{B}$

for γ's $\int_{-\infty}^{+\infty}|E|^2dx = 1$

$p(x) = |E(x)|^2\,dx$

---

e^-'s: = particle + wave

wave equat = Schrö

wave funct = Ψ

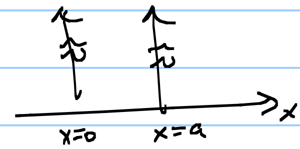
now extend to e^-

Because they too = particle + wave

So $p(x)$ for $e^- = |\Psi(x)|^2\,dx$

$\int_{-\infty}^{+\infty}|\Psi(x)|^2dx = 1$ & $|\Psi(x)|^2$ = prob density!

So we've got solns to



$$\left[\frac{p^2}{2m}+V\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$ for 1-D particle in ∞ well

$$\Psi(x,t) = \begin{cases} A\sin\left(\frac{\pi x}{a}\right)e^{\frac{-i\hbar\pi^2}{2ma^2}t} & 0 \leq x \leq a \\ 0 & x<0 \\ & x>a \end{cases}$$

(we used B.C's to get soln)

note A is arbitrary

We simply use our results

Built our results from (8) = wave + particles now extended to (e^-) = particle + waves

$|\Psi(x)|^2$ = prob density, here $\frac{\text{prob}}{\text{length}}$

$P(x) = |\Psi(x)|^2 dx$ = prob that e^- will be in bin width dx centered @ X_0

&

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$ ie the e^- must be somewhere!

Note: This "process" is called Normalization. Inherent is the idea that

$\Psi \to N\Psi$ = Both good solns.

* note: conservation of charge implied here... could not just lose charge

What? No problem. Later recall using Linear $\hat{O}$'s so $\hat{O}\Psi = O\Psi \iff \hat{O}[N\Psi] = O[N\Psi]$ same

$\bar{\Psi} = N\Psi$ for any eigenvector

So for our 1-D well let's get Ψ exactly!

(ie Normalization)

we require

Later.... just Solve for Ψ then $\Psi \to N\Psi$

so

$$\int_{-\infty}^{+\infty} |\psi_{(x)}|^2 dx = 1$$

$$\int |\Psi_{(x)}|^2 dx = N^2 \int |\Psi_{(x)}|^2 dx$$

$$\int_{-\infty}^{+\infty} \Psi^*_{(x)} \Psi_{(x)} \, dx = 1$$

$=$

so

$$N^2 = \frac{1}{\int |\Psi_{(x)}|^2 dx}$$

$$\int_{-\infty}^{-\varepsilon \to 0} \underbrace{(0)(0)}_{\to 0} dx + \int_{x=0}^{x=a} \Psi^*_{(x)} \Psi_{(x)} dx + \int_{a+\varepsilon}^{+\infty} \underbrace{(0)(0)}_{\to 0} dx = 1$$

$$1 = \int_0^a \left(A^* \sin\left(\frac{\pi x}{a}\right) e^{+\frac{i\hbar\pi^2 t}{2ma^2}}\right)\left(A \sin\left(\frac{\pi x}{a}\right) e^{-\frac{i\hbar\pi^2 t}{2ma^2}}\right) dx$$

$$\underbrace{\int_0^a |A|^2 \sin^2\left(\frac{\pi x}{a}\right) dx = 1}$$

Homework $\Rightarrow$ $\int \sin^2 x \, dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$

$$|A|^2 \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx = \left(|A|^2 \int_0^{\pi} \sin^2 u \, du\right)\frac{a}{\pi}$$

$u = \frac{\pi x}{a}$; $du = \frac{\pi}{a} dx$

$$= |A|^2 \frac{a}{\pi} \left\{ -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x \right\}\Big|_0^{\pi} = |A|^2 \frac{a}{\pi} \left\{ \frac{\pi}{2} + \overset{?}{0} \right\} = |A|^2 \frac{a}{2}$$

$$|A|^2 \frac{a}{2} = 1$$

or $|A|^2 = \frac{2}{a}$

$|A| = \sqrt{\frac{2}{a}}$ take $+\sqrt{\frac{2}{a}}$

*Note why not

$-\sqrt{\frac{2}{a}}$

well $(-1) = e^{i\pi} = \cos\pi + i\sin\pi$

so
2^{nd} soln $= e^{i\pi}\sqrt{\frac{2}{a}} = (-1)\sqrt{\frac{2}{a}}$

But not this is arbitrary phase

ultimately, The world doesn't depend on it because ultimately we do

$$\Psi^*\Psi = (\cancel{e^{-i\pi}})(\Psi)\cancel{e^{i\pi}}(\Psi)$$

& universe is invariant!

doesn't have absolute

ϕ phase angle just as

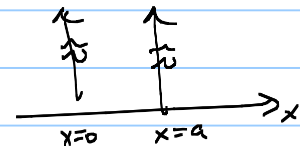
time deriv of $\mathcal{L} = 0$

Note From
A invariance $\Rightarrow$'s Conservation!
See Marion & Thornton
$x_0 \Rightarrow$ cons of $\vec{p}$
$\theta_0 \Rightarrow$ cons of $\vec{L}$
$t_0 \Rightarrow$ cons of E $\rightarrow$ not obvious see Marion & Thornton

universe is invariant to x_0, t_0 & θ_0 * Not obvious $\phi \Rightarrow$ cons of electric charge & electrical potential dissonance!

But anyway....
we've got it Finally

So we've got solns to



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cause used B.C's to get soln

note A is arbitrary

has

$$\Psi(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right) e^{\frac{i\hbar\pi^2 t}{2ma^2}} & 0 \le x \le a \\ 0 & x<0 \\ & x>a \end{cases}$$

where

$$P(x) = |\Psi(x)|^2\,dx$$

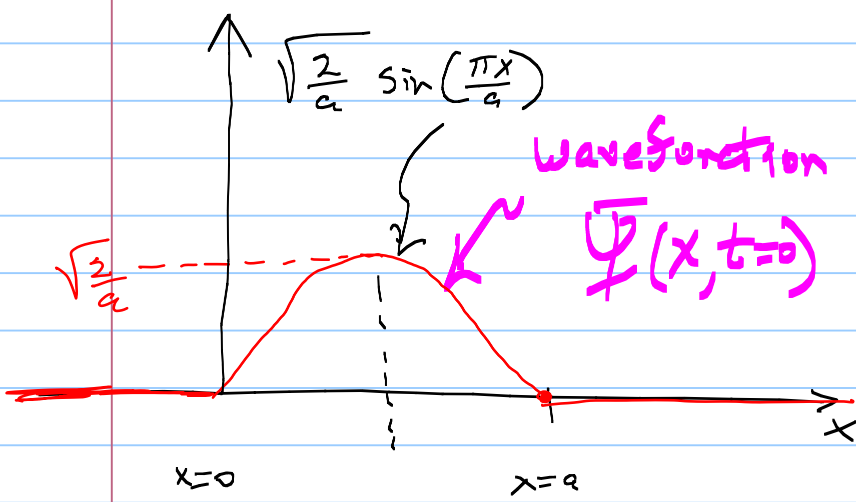
& $|\Psi(x)|^2 = \frac{\text{prob}}{\text{Length}}$ ⇒ recall $|E|^2$ = intensity of Light thru double slit

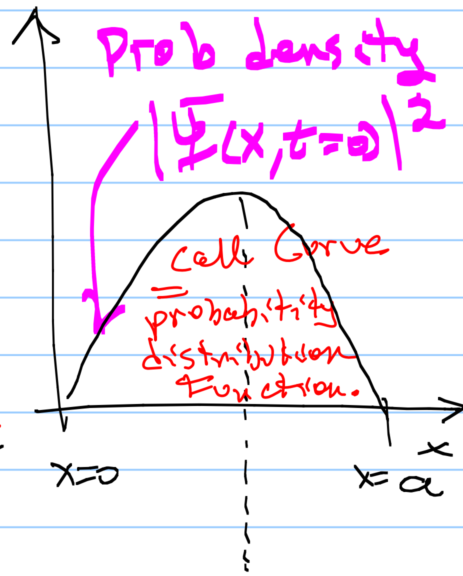
Clearly

So we are very interested to see what $|\Psi(x)|^2$ looks like!

$\Psi(x) =$ @ $t=0$

$|\Psi(x)|^2 = \frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right)$





at $x = \frac{a}{2}$

$\sin\left(\frac{\pi \cancel{a}}{2\cancel{a}}\right)$

$= \sin(\pi/2) = \max = \sqrt{\frac{2}{a}}$

@ $x = a/2$

$\frac{2}{a}\sin^2\left(\frac{\pi \cancel{a}/2}{\cancel{a}}\right)$

$$\underbrace{= \frac{2}{a}}$$

Note here

$$\int_{-\infty}^{+\infty} |\Psi(x,t=0)|^2\,dx =$$

$$\underbrace{\int_0^a \frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right)dx = 1}$$

H.W.

That $|\Psi(x)|^2 =$ prob density $\equiv$ Born Interpretation of Schrö Equat & Ψ.

Schrö-Born Probabilistic Interp of Q.M.

Schrö Equat

what is Ψ? don't know.

what is $\sqrt{\text{probability}}$

don't know that either.

So accepted interpretation is

Ψ = wavefunct, then don't know

but

$\Psi^* \Psi$ = probability density.

1954 Nobel Prize

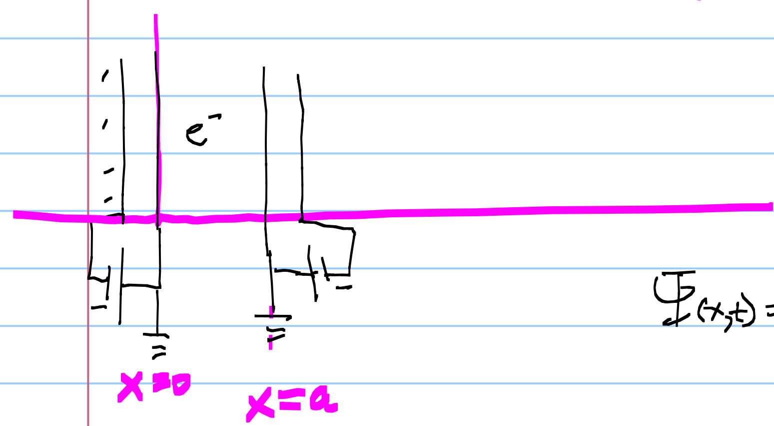
Olivia's Grand Dad.

See AJP article

73, (11) 999-1008

(2005)

So what do we do? w/ prob interp?



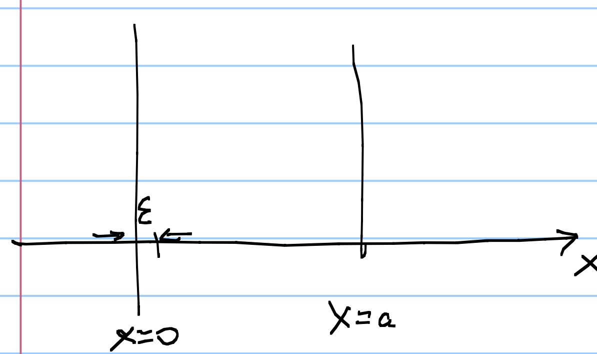
$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

Solve

get

$$\Psi(x,t) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)e^{-\frac{i\hbar\pi^2 t}{2ma^2}} & 0\le x\le a \\ 0 & \text{otherwise} \end{cases}$$

Then ask .... what is the prob that upon measuring the result you be that the e^- was within distance, ε, of the left hand wall?



So $|\Psi|^2 = \frac{\text{prob}}{\text{length}}$

$|\Psi(x)|^2 dx =$ prob in bin size dx

So

$$P(0\le x\le\varepsilon) = \int_{x=0}^{\varepsilon}\Psi^*(x)\Psi(x)\,dx$$

$$= \int_0^{\varepsilon}\frac{2}{a}\sin^2\frac{\pi x}{a}\,dx$$

$$= \frac{\varepsilon}{a} - \frac{1}{2\pi}\sin\left(\frac{2\pi\varepsilon}{a}\right)$$

Now if $\varepsilon \ll a$

Then $\sin(x) = 0 + x + 0 - \frac{x^3}{3}$

So $\sin\frac{2\pi\varepsilon}{a} = +\frac{2\pi\varepsilon}{a} - \left(\frac{2\pi\varepsilon}{a}\right)^3\frac{1}{3}$

&

$$\text{Prob} = \frac{\varepsilon}{a} - \frac{1}{2\pi}\left(\frac{+2\pi\varepsilon}{a} - \left(\frac{2\pi\varepsilon}{a}\right)^3\frac{1}{3}\right) = \cancel{\frac{\varepsilon}{a}} - \cancel{\frac{\varepsilon}{a}} + (2\pi)^2\left(\frac{\varepsilon}{a}\right)^3\frac{1}{3} \simeq \frac{(2\pi)^2}{3}\left(\frac{\varepsilon}{a}\right)^3$$

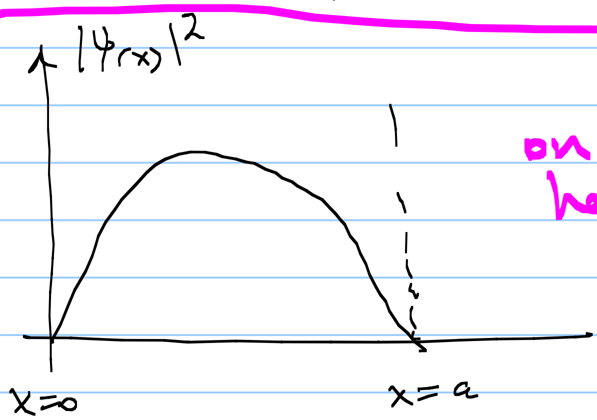
WORKS perfectly!

Brief.....

talk about pre-measurement

Schwerer waits until Sect 8.8 on measurement theory here (pg 187)

I'll do a bit here



Quantum mechanics "says" before measurement, all you can say is that

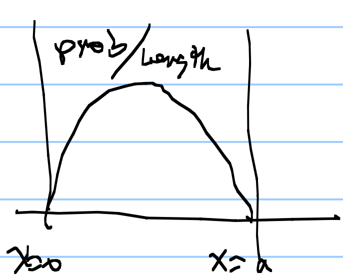
This is the probability distribution function for where the particle is!

That's it. Q.M. says it is in superposition of this entire distribution function

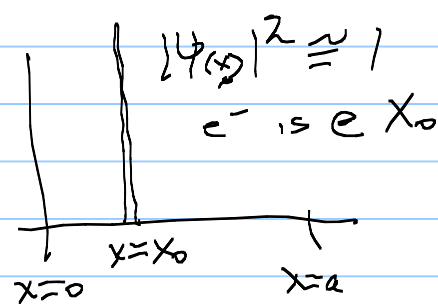
Until the measurement!

Once a measurement is made---- and it is where it is, It is said that the wavefunction instantly collapses into the state to which it was measured!

Before measurement



after measurement



This is called the collapse of the wavefunction!

$$\boxed{\psi^*_{\text{all } x}\,\psi_{\text{all } x} = \text{Everything}} \xrightarrow[\text{measure}]{} \psi^*_{(x_0)}\,\psi_{(x_0)}$$

its @ x_0.

Idea:

Spin $= \uparrow$ or $\downarrow$

$$\psi_{spin} = |\uparrow\rangle + |\downarrow\rangle \xrightarrow[\text{measure}]{} |\uparrow\rangle \text{ or } |\downarrow\rangle$$

Schrödinger's Cat

$$|\text{dead}\rangle + |\text{alive}\rangle \xrightarrow[\text{measure}]{} |\text{dead}\rangle \text{ or } |\text{alive}\rangle$$

ALL called Copenhagen Interpretation of Quantum Mechanics!

This is EARTH Shattering!

Classical $\Rightarrow$ deterministic

Q.M. $\Rightarrow$ replace w/ probability!

* Recall Classical can be derived from Q.M.