

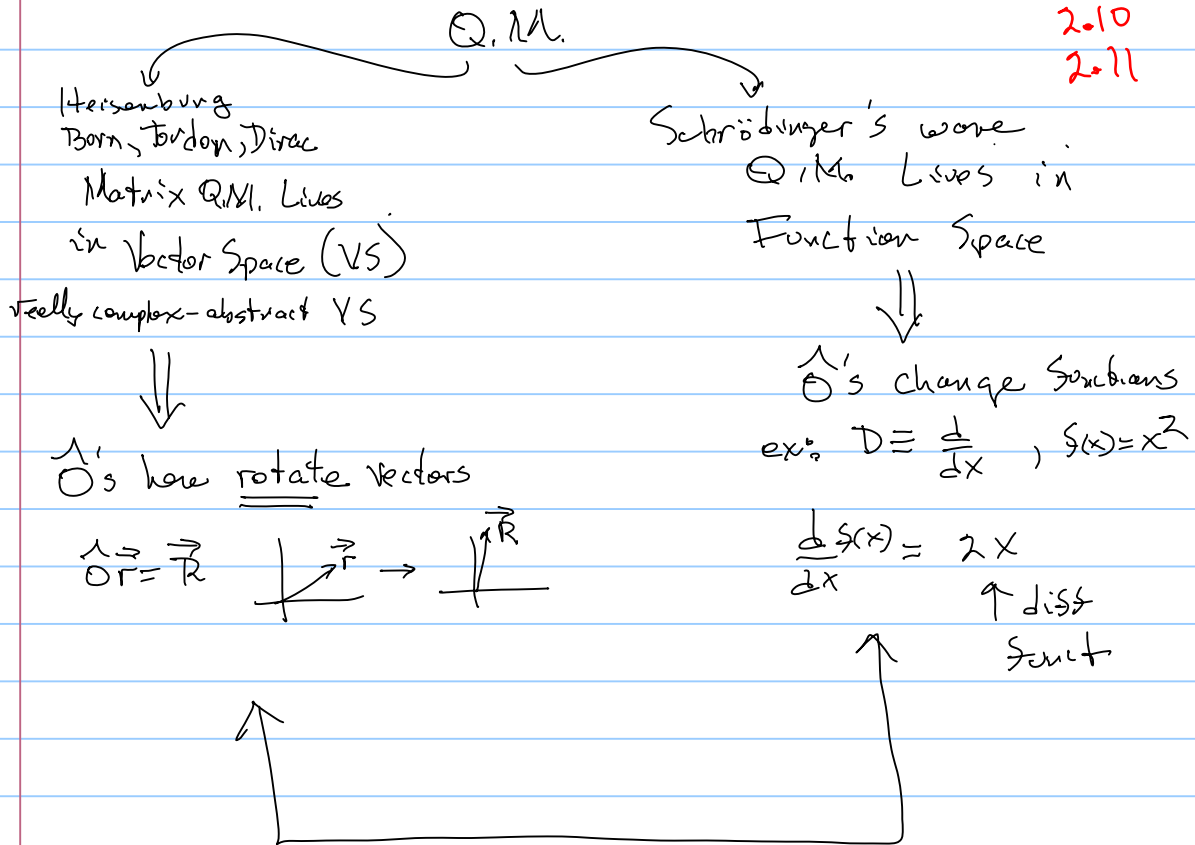
Operators are HUGE in Q.M.,

Homework
Scherrer 2.7

2.8

2.10

2.11



Gross

\Rightarrow what ties them are that
solns to Schri equations That
are Square Integrable

ie $\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = \text{Finite}$

Physical, in the
Schri-Born formalism,
 $|\Psi|^2 dx = \text{probability}$

Form a Complete, Hilbert basis in F.S.

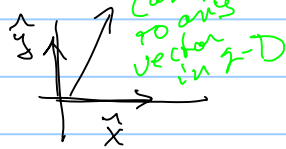
Thus
 $\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = \text{Finite}$

is Physical! because The prob of something next = 1
Thus non square integrable solns to Schri's \neq physical.

Hilbert's Matrix

V.S.

Complete:



$$\vec{r} = F_x \hat{x} + F_y \hat{y}$$

\hat{x} & \hat{y} = linearly independent Basis That "span"

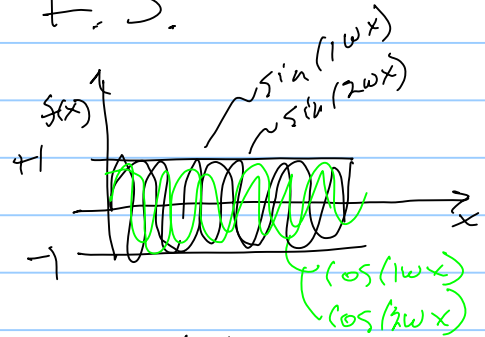
2-D space. All 2-D vectors can be built as linear combo of \hat{x} & \hat{y}

Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = row echelon reduced = 2-D

complete \Rightarrow complete spans

Schrödinger wave.

F.S.



see that

$$f(x) = \sum_{n=0}^{\infty} a_n \cos n\omega x + b_n \sin n\omega x$$

Fourier series

because

$$\cos n\omega x \text{ \& \& } \sin n\omega x$$

for $n=0 \rightarrow \infty$ = Basis that "fills" all of function space

\therefore can represent any

func as linear combo of this basis

* only dimensioned Basis set!

complete \Rightarrow is completely fills, no holes

Hilbert \Rightarrow in V.S. you have $\begin{cases} \hat{i} \cdot \hat{i} = 1 \\ \hat{i} \cdot \hat{j} = 0 \end{cases}$ ie dot prod

\Rightarrow Hilbert spaces are function spaces that

have all the properties of V.S.

$$F.S. \text{ dot prod} \equiv \int a_n a_m dx = \sum_{nm}$$

Dirac

(A.V.S) $\Phi \rightarrow |\Phi\rangle$ "kets" vectors

$\langle \Phi|$ bra vectors

$\langle \Phi|\Phi\rangle$ = dot product



rotate vectors

change functions

Schrödinger

$\Psi =$ wavefunction, $\Psi(x)$ = F.S. basis but in some vector space

We'll be doing Schrödinger-wave-1st so let's look @
function \hat{O} 's. Q.M.

A function $f(x) = x^3$ changes #'s into new #'s

$$f(2) = 8$$

↑ ↑
old new

An Operator, $D \equiv \frac{d}{dx}$ changes functions into new functions

$$\text{ex } f(x) = x^3 \leftarrow \text{old}$$

↑

$$\frac{df(x)}{dx} = 3x^2 \leftarrow \text{new}$$

Since \hat{O} 's will rotate vectors & change functions
we will only be interested in \hat{O} that
keep us in the same "space"

so

our rotations must keep us in same V.S.

our changes to functions must keep us in same F.S.

These \hat{O} 's are called Linear \hat{O} 's

Linear \hat{O} 's are those that

$$1.) \hat{L}[f(x) + g(x)] = \hat{L}(f(x)) + \hat{L}(g(x))$$

$$2.) \hat{L}[c f(x)] = c \hat{L} f(x)$$

$c = \text{constant.}$

ex: $\hat{A} \equiv [\]^2$ and $\hat{D} \equiv \frac{d}{dx}$

try 1.) $\hat{A}[f(x)+g(x)] = [f(x)+g(x)]^2$
 $= f(x)^2 + g(x)^2 + 2f(x)g(x)$

? $\hat{A}[f(x)] + \hat{A}[g(x)]$

$= f(x)^2 + g(x)^2$

No!

\hat{A} is not a linear \hat{O}

what about \hat{D}

$\hat{D}[f(x)+g(x)] = \frac{df(x)}{dx} + \frac{dg(x)}{dx} \stackrel{?}{=} \hat{D}[f(x)] + \hat{D}[g(x)]$
 $= \frac{df(x)}{dx} + \frac{dg(x)}{dx}$

yup!

$\frac{1}{c} \hat{D}[cf(x)] = c \frac{df(x)}{dx} \stackrel{?}{=} c \hat{D}[f(x)] = c \frac{df(x)}{dx}$

yup!

$\hat{D} = \text{Linear } \hat{O}$

Now, we are going to be interested in
very particular functions
or vectors

It's for those functions & vectors.
Linear because they keep us in the same
"space"

Eigen Functions (subscribe eigen vectors is in V.S.)
Eigen values

Why? Eigenvalues of Eigen functions = REAL } good! there's we measure real

OF HERMITIAN O'S

See
Griffiths or
Townsend

2) Eigen functions Form a Complete
basis in that function or
vector space

Key! You are guaranteed that
Solutions to your problems
can be built from these!

ex: in V.S.

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

Basis

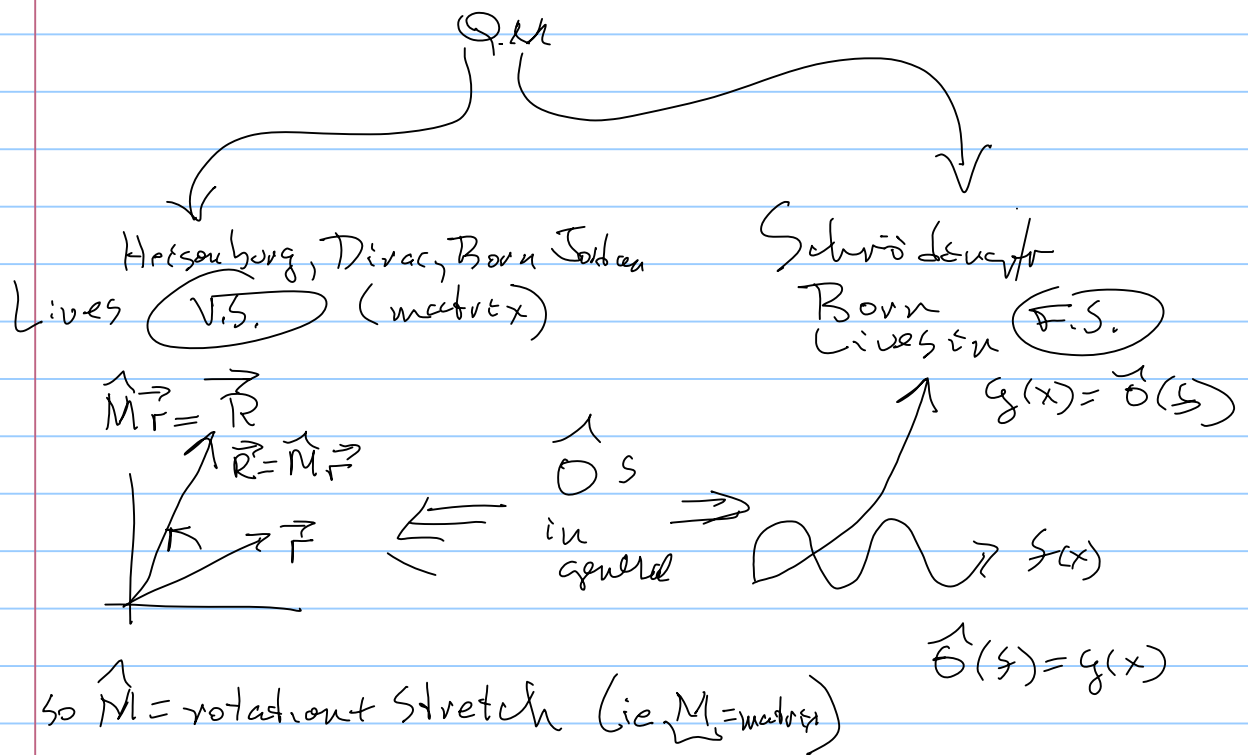
or
F.S.

$$f(x) = \sum_{n=0}^{\infty} \left[a_n (\cos(n\omega x)) + b_n (\sin(n\omega x)) \right]$$

So: Always work w/ Eigenvalues & functions
w/ Linear O'S!

Hermitian

So what are eigenfunctions (vectors) & values?



Now there may
be some vectors or functions
that are special

$$\hat{M} \vec{r} = c \vec{r} \quad \text{and} \quad \hat{O}(s) = c s(x)$$

ie that the \hat{O} 's
did not or
change the
the vector
dir form

But all it did
was change the
length or multiplicative
constant

In general real or complex.

So there might be

1.) V.S. $\hat{M} \vec{r} = c \vec{r}$

2.) F.S. $\hat{O} \psi(x) = c \psi(x)$

so \vec{r} = eigen vector

$\psi(x)$ = eigen function

c = eigen value

Our goal will be to find Eigenvalues & Eigen vectors
of Hermitian linear O's
because they are Physical!

V.S.

$$\hat{M} \vec{r} = c \vec{r}$$

F.S.

$$\frac{d}{dx} (e^{mx}) = m e^{mx}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} r_x \\ r_y \end{pmatrix} = c \begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

ex: 2.5 Scherrer: Eigenfunctions & values of $\hat{D} \equiv \frac{d}{dx}$

Find eigenfunctions of \hat{D}

Looking for

$$\hat{D}[g(x)] = c(g(x))$$

or

$$\frac{d(g(x))}{dx} = c(g(x))$$

Can separate variables $\rightarrow g(x) = \text{entire variable}$

$$\frac{d(g(x))}{g(x)} = c \, dx$$

$$\int \frac{dg(x)}{g(x)} = \int c \, dx$$

* ind. of integral
so need to keep
integration
constant

$$\ln g(x) = cx + C_2$$

$$e^{cx + C_2} = g(x)$$

$$g(x) = e^{cx} \underbrace{(e^{C_2})}_{\substack{\text{constant} \\ \text{call it} \\ A}} = A e^{cx}$$

So: all functions of form $A e^{cx}$ = eigenfunctions
of \hat{D}

There are
an ∞ of
solutions of
this form.
 $\infty \neq E, F, E!$

* seems trivial, but in fact not all $f(x)$'s are eigenfunctions

ex:

$$\hat{D}(\sin(x)) = \cos(x) \neq c \sin(x)$$

$$\hat{D}(x^2) = 2x \neq cx^2$$

$$\hat{D}[\ln(x)] = \frac{1}{x} \neq c \ln(x)$$

∴ Eigen Functions ARE special!

again Q.M. Based on

EF

↳ EV of Hermitian Linear \hat{D} 's

Ex 2.6 in Scherrer: Find EF & EV of parity $\hat{O} \equiv \hat{\Pi}$

HUGE! in Fund Physics
↳ search for e^- EDM

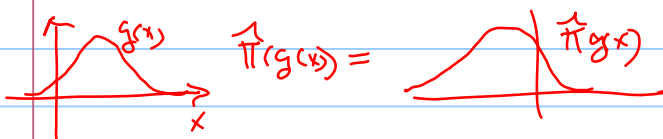
Violations of $\hat{\Pi}$ come only as a result of weak nuclear Force & Matter to distinguish if physics is being seen in a mirror or not!

idea: $\hat{\Pi}[g(x)] = g(-x)$

$\hat{\Pi}$ reflects the function about $x=0$

this is

the operation



So looking for

$$\hat{\Pi}[g(x)] = g(-x) = \underbrace{C g(x)}$$

don't know how
to solve this

easily so TRICK!

$$\text{try } \hat{\Pi}^2[g(x)] = \hat{\Pi}\{\hat{\Pi}(g(x))\} = \hat{\Pi}(C g(x))$$

$$\hat{\Pi}(g(-x)) = C \hat{\Pi}g(x)$$

$$g(x) = C^2 g(x)$$

Note: even when an EF of a \hat{L} is multiplied by an arbitrary constant, it remains an EF w/ the same EV.

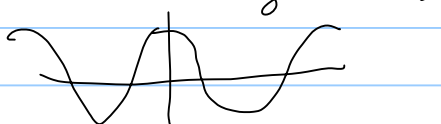
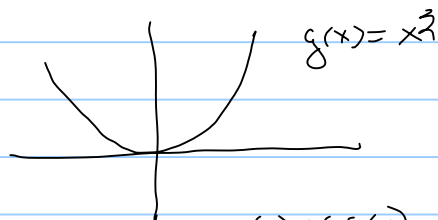
Now can solve this

$$C^2 = 1 \text{ or } C = \pm 1 \} \text{ 2 Discrete eigenvalues}$$

Now the eigen functions:

$$\text{for } C_1 = 1 = \text{all even functions}$$

$$g(x) = g(x)$$



$$C_2 = -1 = \text{all odd functions}$$

$$g(x) = -g(x)$$

