E.F. Deveney: BSC Physics PH402 Q.M. Lecture: Chapt 2 Scherrer Complex #s

Note Title 1/26/2006

H.W. all examples in this sect + 2.1 2.3 2.4 2.6

Not the 'say all' lecture for complex #'s
But
perhaps handy .... shows usefullness.

you've got + #'s --- ie 7 cows
1) add them
2) Then subtract them need to invent (−) #'s
3) Then multiply — no new inventions ↑ rules
4) Then divide by #'s $\frac{\#}{\#}$ = rational #
invent decimal equivalents
Signature = repeating decimal

5) Then say, $5 \times 5 = 25$, so $25^{1/2} = \pm 5$ OK

But then run into trouble w/ $2^{1/2}$

$(2)^{\frac{1}{2}} = ?$ Nothing works (ie Rational #'s)

so invent $\sqrt{2}$ = The right #
such that
$(\sqrt{2})^2 = 2$

$\sqrt{2}$ = new irrational #

Signature Never repeating decimal (ie π)

So everything is cool

But can access all + #'s from $(\ )^2$
But can't get to $(-)$ #'s from this operation.

So invent new thing, why not $\sqrt{2} = \infty$ non-repeating decimal

why not i, such that $i^2 = -1$

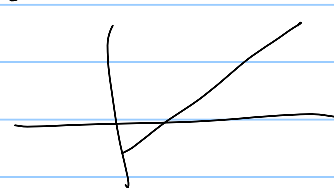
$$\therefore \sqrt{-1} = i$$

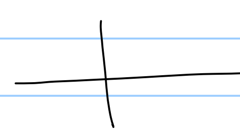
Is this useful? You betcha!
why?

Here is what Feynman calls the Jewel of mathematics!

Basic idea: Every algebraic entity has a geometric representation.

$3 \Longleftrightarrow$ 

$3x+-1 \Longleftrightarrow$  a line $mx+b$

$9+16=25$  a circle

} so on

it's, The geometric p, is use $f(x)$ --- see nature plop down math.

Ok, so now we've got i, such that $i^2 = 1$ is some imaginary #

well e^{ix} why not --- does it have geometric map?

How can you see what $f(x)$ is made of?
You got it Taylor series

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \cdots$$

for all x

$$e^{ix} = 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \frac{-x^6}{6!} + \frac{-ix^7}{7!}$$

$$e^{ix} = \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \ldots\right) + i\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \ldots\right)$$

recall

McLauren series expansion

$$\cos\theta = 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4 - \ldots$$

$$\sin\theta = \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \ldots$$

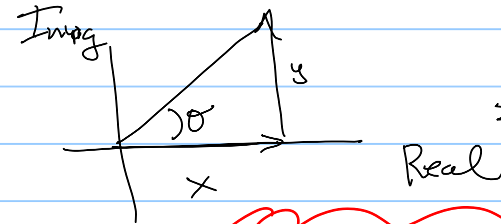
Then

$$e^{ix} \text{ or } e^{i\theta} = \cos\theta + i\sin\theta$$

WOW!

Now even complex #'s have geometrical interpretation

$e^{i\theta} = \cos\theta + i\sin\theta =$



$= x+iy \equiv Z$

so $e^{i\theta}$ = A VECTOR!

Convention from Griffs & Me!

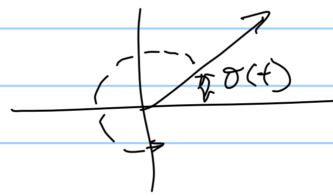
$\vec{A}$ = vector

$\underset{\sim}{A}$ = matrix

$\tilde{A}$ = complex #

is $\theta = f(t)$

$e^{i\theta} =$



Clock!

See Feynman "The Strange Theory of Light & Matter"

Very Cool!

1) Having geometric feel is key

2) Having Z is fun too. Often you must guess @ solns to problems & then fit them to work (Diffy-Q's)

Here, if guess @ real # only, might never work. But if start guessing w/ most general 'encompassing' object that can be a vector x+iy or scalar, y=0

You are much better off.

Because $z = x + iy$ is like a vector

$\uparrow$ Real $\uparrow$ imaginary parts are Linearly independent from each other

* $\vec{A} - c\vec{B} \neq 0$ for all c

$$\underbrace{(\hat{x} - 2\hat{y})}_{\vec{A}} - \frac{1}{2}\underbrace{(2\hat{x} - 4\hat{y})}_{\vec{B}} = 0$$

or $\begin{pmatrix}1 & -2\\ 2 & -4\end{pmatrix} \xrightarrow{2\times 1} \begin{pmatrix}2 & -4\\ 2 & -4\end{pmatrix}$

$2-1 \begin{pmatrix}2 & -4\\ 0 & 0\end{pmatrix} \to \frac{1}{2}$

$= \begin{pmatrix}1 & -2\\ 0 & 0\end{pmatrix}$

echelon row

for rn $= 1 \Rightarrow$

$\therefore$ $\vec{A}$ & $\vec{B}$ $\neq$ linearly indep.

So being linearly independent you can do cool tricks

ex: A apples + B bananas = x apples + y bananas

app & ban are linearly indep

$\therefore$ it is TRUE that

$x = A$

& $y = B$

So, if you have a tough problem w/ lots of sins & cos ie mult & ÷ & so on ...

Then Let all your variable go complex!

elevate them to the complex plane

see electronics!

E&M I did it Griffiths.

Next 2 pages!

ex! if you have $\cos\theta \rightarrow Re(e^{i\theta})$ then

This page & next from p#458, E&M2 griffith chpt 9, waves2 lect

Now we have to have a complex algebra break.

Say you have $s_1 = \cos(.7\pi + x)$

$s_2 = \cos(.7\pi + y)$

$s_3 = s_1 + s_2 = ?$ yikes you'd have to do lots of trig identities

Heres the idea:

elevate s's $\rightarrow \tilde{s}$ complex (you've added imaginary part But thats OK, it's lineauly indep)

so $s_1 \rightarrow \tilde{s}_1 = e^{i(.7\pi + x)}$

$s_2 \rightarrow \tilde{s}_2 = e^{i(.7\pi + y)}$

Then $\tilde{s}_1 = e^{ix} e^{i.7\pi}$

$\tilde{s}_2 = e^{iy} e^{i.7\pi}$

$\tilde{s}_3 = \tilde{s}_1 + \tilde{s}_2 = \underbrace{(e^{ix} + e^{iy})}\, e^{i.7\pi}$

$= \tilde{s}\, e^{i.7\pi}$

$|\tilde{s}|$

recall our 3 notation for 3 algebra's

1.) vector $\vec{A}\vec{B}$ → $\vec{A} \cdot \vec{B}$, $\vec{A} \times \vec{B}$

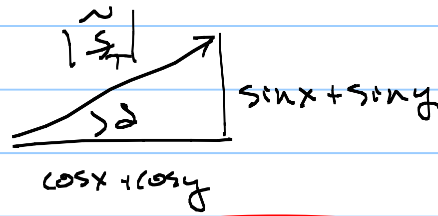
2.) Tensor $\vec{r}\,' = \underline{A}\,\vec{r}$

recall 1 & 2 = matrix algebra

3.) complex $\tilde{C} = \tilde{A}\tilde{B}$

$$(\cos x + i\sin x) + \cos y + i\sin y$$

$$(\cos x + \cos y) + i(\sin x + \sin y)$$



$$\delta = \tan^{-1}\left(\frac{\sin x + \sin y}{\cos x + \cos y}\right)$$

$$S_T = |\tilde{S}_T| = \sqrt{(\cos x + \cos y)^2 + (\sin x + \sin y)^2} = \tilde{S}_T^* \tilde{S}_T$$

$$\tilde{S}_3 = \tilde{S}_1 + \tilde{S}_2 = \tilde{S}_T e^{i.7\pi}$$

$$= S_T e^{i\delta} e^{i.7\pi} = S_T e^{i(.7\pi + \delta)}$$

$$\tilde{S}_3 = \; = S_T\left[\cos(.7\pi + \delta) + i\sin(.7\pi + \delta)\right]$$

Now wanted $S_3 = S_1 + S_2$ so take real parts

$$S_3 = Re(\tilde{S}_3) = S_T \cos(.7\pi + \delta)$$

$\Longrightarrow$ But other part, from linearly indep complex part is also soln.

? Since you elevated it to 2-D you expect 2 solns

So, ① when you see messes w/ sin & cos, think making your life easier by → elevating to complex plane

↕

doing your work

answer

Bring back to Real answers using Linearly indep prop.

Note: Bonus is you might get 2 good answers!

In this case Complex #'s are simply a Book Keeping tool to keep independ solns apart from eachother

② or have messy diffy-Q that you dont know the answer too, guess

$$\frac{df}{dx} = [\;] \quad f(x) = e^{px}$$

if p = complex

$$\text{Then } f(x) = e^{ipx} = \underbrace{\cos px + i \sin px}_{\text{vector!}}$$

Here complex #'s arise because vectors are solns!

But again vectors are Linearly indep solns, so kind of same thing!

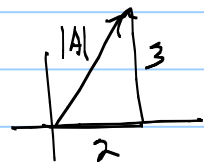
But this is Big idea.

Multiplication of complex #'s

& complex conjugating!

ex: $\underset{\tilde{A}}{(2+3i)}\underset{\tilde{B}}{(1-i)} = 2 + -2i + 3i - 3 = -1 + i$

or:

$\tilde{A} = 2+3i =$ 

; clear that $|A| = \sqrt{2^2+3^2} = \sqrt{13}$

but we now need a trick to find complex # magnitudes

#→ don't worry, we have history of this....

1) $|\vec{A}|^2 = \vec{A}\cdot\vec{A} = A_x^2 + A_y^2$ "dot" rule

2) in Q.M. we will want $|\psi|^2 = $ prob: $\underbrace{\langle\psi|\psi\rangle}_{} = |\psi|^2$

Bra-Ket notation of "inner product" or dot product

Born & Jordan + Dirac recognized this from Heisenberg. ⟹ $= (A_x \; A_y)\begin{pmatrix} A_x \\ A_y \end{pmatrix} = A_x^2 + A_y^2$

= matrix Multiplication!

3) & in S.R. we had 4-vector dot prod: $x^\mu = (ct, \vec{x})$

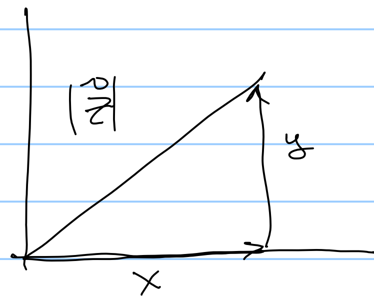
then $x_\mu = g_{\mu\nu} x^\nu$ ($g_{\mu\nu}$ = metric Tensor from Einstein equation

so $I^2 = x_\mu x^\mu = (ct^2 - |x|^2)$

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ = Minkowski - flat - space metric

So -- we have history of making things, rules, up.

here



$$|\tilde z|^2 \overset{?}{=} (x+iy)(x+iy)$$

Lets see

$$x^2 + ixy + ixy - y^2$$

$$= x^2 - y^2 + 2ixy$$

Nope?

Try $|\tilde z|^2 = z^* z$ where $* \equiv$ "taking the complex conjugate" rule where ever

$$+i \to -i$$
$$-i \to +i$$

So here

$$z^* z = (x-iy)(x+iy)$$

$$= x^2 + \cancel{xiy} - \cancel{xiy} - i^2 y^2 = x^2 + y^2$$

That's it .... if you have a complex #, then mag of complex # is

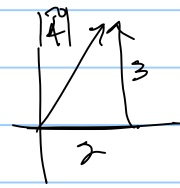
$$|\tilde z| = \sqrt{z^* z}$$

just as $|\vec A| = \sqrt{\vec A \cdot \vec A}$

But back to our simpler problem

$\tilde{A}\tilde{B} = ?$ $\qquad \tilde{A} = 2+3i$

$\qquad\qquad\quad\ \tilde{B} = 1-i$

$\tilde{A} =$ 

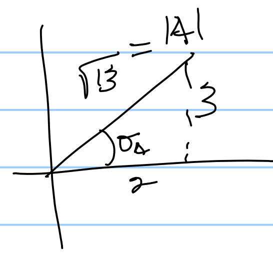
now $|\tilde{A}| = \sqrt{A^*A} = \sqrt{(2-3i)(2+3i)}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$|A| = \sqrt{13}$

so

$\tilde{A} =$ 

$\theta = \tan^{-1}\frac{3}{2}$

$\tilde{A} = |\tilde{A}| e^{i\theta_A}$ because $= |\tilde{A}|\cos\theta_A + i|\tilde{A}|\sin\theta_A$

$= 2 + i3$

$\therefore\ \tilde{A}\tilde{B} = (2+3i)(1-i) = -1 + i$

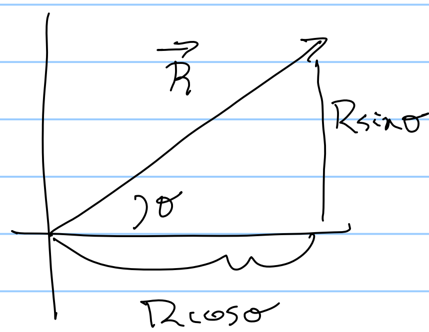
$\text{or} = |\tilde{A}| e^{i\theta_A} |\tilde{B}| e^{i\theta_B}$

$= \sqrt{13}\, e^{i\tan^{-1} 3/2} \sqrt{2}\, e^{i\tan^{-1}\frac{-1}{1}}$

Got a bit messy there, but back up to early pt

complex # = geometric vector

so

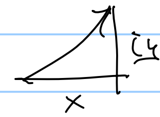


$$Re^{i\theta} = R\cos\theta + i\,R\sin\theta$$

way easier
to do math
mult, div &
Calculus (differentiate &
Integrate)

w/ exponentials than
w/ vectors!

Another Big Point-----

we know that $x+iy$ = vector  $= \tilde{z}$.

But what is

$$\frac{a+ib}{c+id}?$$

no clue so always do this

$$\frac{a+ib}{c+id} * \frac{c-id}{c-id} = \frac{(a+ib)(c-id)}{c^2+d^2}$$

$$= \frac{ac+i(bc-da)+bd}{c^2+d^2} = \frac{1}{c}\left(x+iy\right)$$

const

good vector again from