E. F. Deveney / BSC Physics PH402 Q.M. SHERRER Chpt 1 Modern Phys

Note Title 1/23/2006

Its 1900; Keep in mind Light = wave
particles (?) = particles

About to Find:

Light = wave-like & particle-like
e^-'s, particles = particle-like & wave like

→ nice symmetry Unbelievable
unexpected!

*

This is Q.M.

later Quantum Field Theory: All Fields
Thats another story

1900 Light ⇒ Maxwell's Equations

static / dynamic

1) $\vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\varepsilon_0} + 0$

2) $\vec{\nabla}\cdot\vec{B} = 0 + 0$

3) $\vec{\nabla}\times\vec{E} = 0 + -\frac{d\vec{B}}{dt}$

4) $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}_{free} + \mu_0\varepsilon_0 \frac{d\vec{E}}{dt}$

Field | Sources

$\vec{E}$ = Electromagnetic fields

$\vec{F}_q = q\vec{E}$

$\vec{B}$ = magnetic field

$\vec{F}_q = q\vec{V}\times\vec{B}$

$\vec{\nabla}\cdot\vec{V}$ = field } span 3-D

$\vec{\nabla}\times\vec{V}$ = fields }

$\vec{F}_{Lorentz} = q\vec{E} + q\vec{V}\times\vec{B}$

(For: free space, $\rho_f = 0$ & $\vec{J}_f = 0$)

if you $\vec{\nabla}\times(\#3)$ & $\vec{\nabla}\times(\#4)$ } Then use $\vec{A}\times\vec{B}\times\vec{C} = \vec{B}(\vec{A}\cdot\vec{B}) - \vec{C}(\vec{A}\cdot\vec{B})$ rule

you get (H)

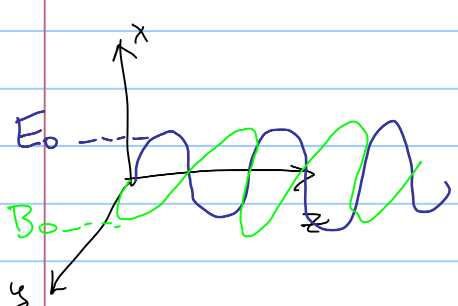
$$\frac{d^2E}{dx^2} = \mu_0\varepsilon_0 \frac{d^2E}{dt^2}$$

$$\frac{d^2B}{dx^2} = \mu_0\varepsilon_0 \frac{d^2E}{dt^2}$$

} recognized as wave equat (H.W.)

$$\frac{d^2f}{dx^2} = \frac{1}{V^2}\frac{d^2f}{dt^2}$$

{ see in free space



$$\vec{E} = E_0 e^{i(kx-\omega t)}\hat{x}$$

$$\vec{B} = \frac{E_0}{c} e^{i(kx-\omega t)}\hat{y}$$

* $k = \frac{2\pi}{\lambda}$
* $\omega = 2\pi f$ ↑freq

$V = \lambda f$

$= \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 \, m/s = c$

Wow--- Light in a vacuum = self-propagating electro-magnetic traveling wave @ c.

Light = A wave!

$$\text{Energy density} = \frac{J}{m^3} = \frac{1}{2}\left(\varepsilon_0 |E|^2 + \frac{1}{\mu_0}|B|^2\right)$$

Griffith E&M

Ah, energy just like energy of wave

$\propto |\text{Amplitude}|^2$

Now Light = particle

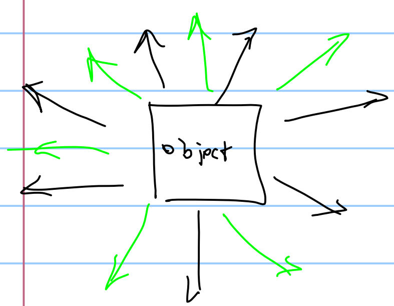
BB rad 1900

S.R 1905

Compton Scattering 1923

Chpt 1 Scherrer

Plank BB rad 1900



radiation → 1.) reflection

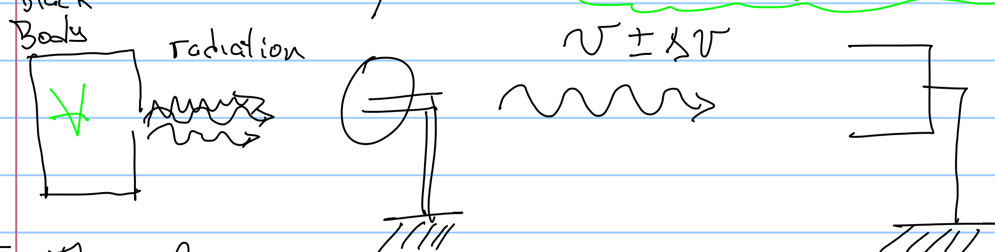
→ 2.) Internal energy (= due to temperature) on atomic scale jiggling $\propto T$

Experiment: Spectral energy density $\rho(\nu) = \dfrac{(J/m^3)}{Hz}$

↑ freq

$$\rho(\nu) = \frac{\text{energy density}}{\text{frequency}}$$

* Note: in our lab we use either a prism or diffraction grating

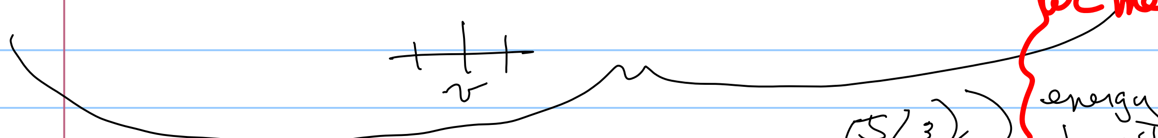


In Thermal Equ. w/ surrounding ie @ T

filters out narrow range around some central ν

Energy meter

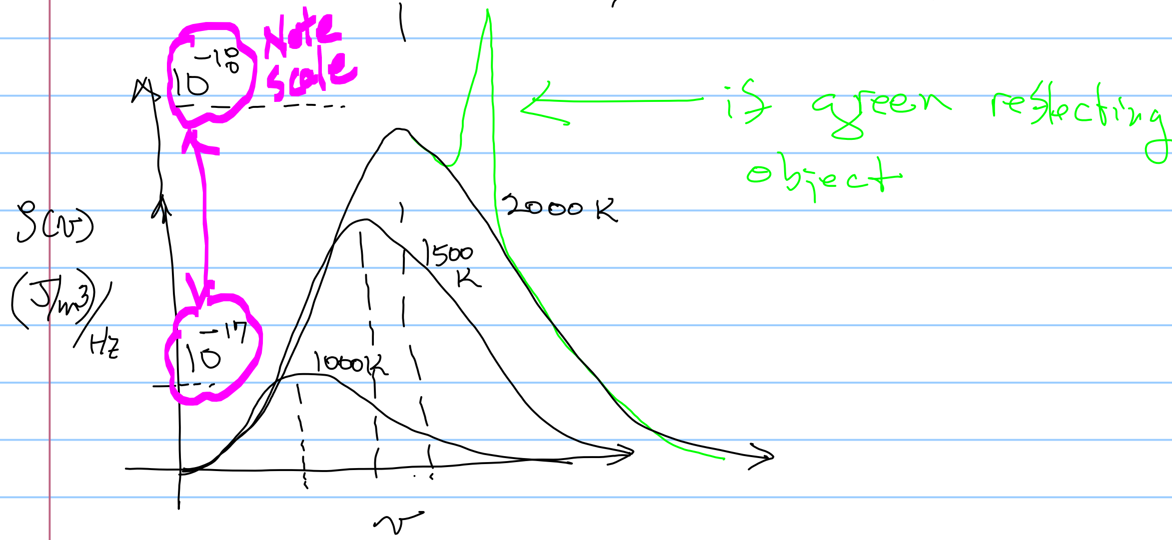
measure Joules



get J/Vol over $\Delta\nu$ or this range $\dfrac{(J/m^3)}{Hz}$

we measure energy density per freq range

New data: object rad $\begin{cases} \to \text{reflection} \\ \to \text{internal Energy} \end{cases}$



So: Black bodies do not reflect: ∴ Spectrum is due to internal energy $\propto T$

BB rad = radiation due to Temp alone!

* 1879: J. Stefan: $P_{TOT} = \sigma A T^4$

A (area) (m^2)

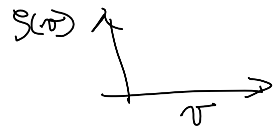
σ: Stefan-Boltzman const

$\sigma = 5.67 \times 10^{-8} \frac{J}{s \cdot m^2\, K^4}$

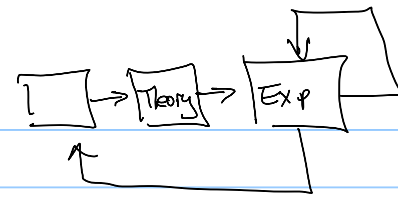
* Wien's displacement Law

clearly $\nu_{peak} \propto T$

specifically, $\lambda_{peak} = \frac{w}{T}$; $w = 2.90 \times 10^{-3}\, m \cdot K$

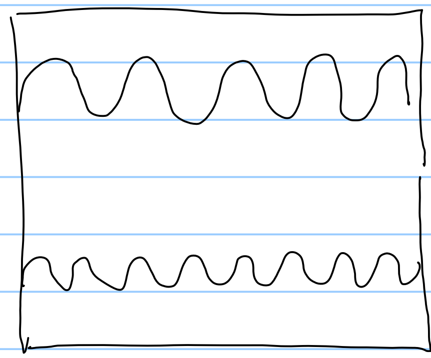


So we've got experiment: S.M.



get Theory

here is our B.B.

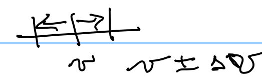


assume in equilib, we have standing waves

We need to know

$$\rho(\nu) = \frac{\left(E/m^3\right)}{Hz} = \frac{J/m^3}{\delta\nu}$$

strategy.



1) Count how many standing waves w/in $\nu \pm \Delta\nu$

scherrer 1.3

Harris: Append C: $n(\nu) = \left(\frac{\#\text{ of standing waves}}{m^3}\right) \Big/ Hz = \frac{8\pi}{c^3}\nu^2$

2) Compute average energy of standing waves in this range $\equiv \bar{E}$

3) Then

$$\rho(\nu) = \bar{E}\, n(\nu)$$

OK:

$$\rho(\nu) = \bar{E}\, n(\nu) = \frac{(J/m^3)}{Hz} = (J)\frac{(\#/m^3)}{Hz} \Rightarrow \text{compare to Exp.}$$

$\bar{E}_{classically} = k_B T$; k_B = Boltzman constant

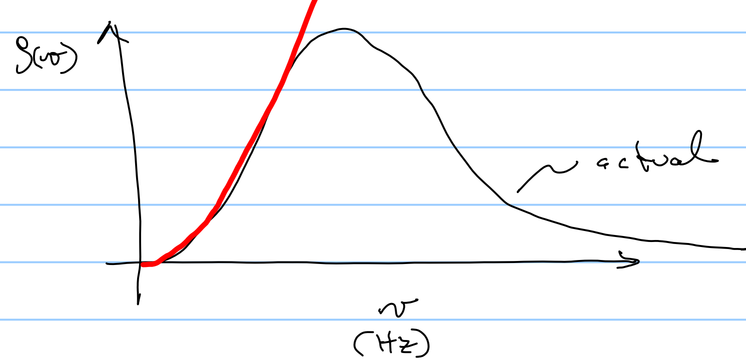
w/ 100% correct Thermo, stat mech.

assumes

$$u_{E\&M} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$ = continuous amplitudes

get

$$\rho_{classical}(\nu) = \left(\frac{8\pi\nu^2}{c^3}\right) k_B T$$



Yikes: ultra violet catastrophe

$\Rightarrow$'s sit in front of fire place & die of X-ray radiation!

Kind of wrong:

$$\left.\begin{matrix} \text{Theory} = \infty \\ \text{exp} = 0 \end{matrix}\right\} @ \text{Big } \nu$$

Ultimate failure ---- Theory $\neq$ Exp.

$\rho(\nu) = \bar{E}\, n(\nu)$

got to get $\rho(\nu)$ right?
out of despiration, enter Plank 1905

If
$$\bar{E} = \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

Then $\rho(\nu) = \bar{E}\, n(\nu) = \left(\frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}\right) \frac{8\pi}{c^3}\nu^2$

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)}$$

& get ....

PERFECT MATCH!

So, the question is, how did Plank get $\bar{E} = \frac{h\nu}{e^{h\nu/k_B T} - 1}$ instead of $\bar{E} = k_B T$?

well, whatever $\bar{E}$ he got, he knew in Limit of <u>small ν</u>, you better get $K_B T$

because original, classical, theory worked there

...

$$\frac{h\nu}{e^{\frac{h\nu}{K_B T}}-1} \quad ; \quad e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

if x is small

$$e^x \cong 1 + x$$

Low ν Limit $\Rightarrow$ x is small, so

$$\bar{E}(\text{Low } \nu) \cong \frac{h\nu}{(1+x)-1} = \frac{h\nu}{x} = \frac{h\nu}{\frac{h\nu}{K_B T}} = K_B T$$

Cool ... $\bar{E}_{\text{Plank}}$ contains low ν, Limit Looks good

<u>But</u> where did Plank get $\bar{E}$ from?

well Max

$$u_{EM}(\nu) = \frac{1}{2}\left(\varepsilon_0 |E|^2 + \frac{1}{\mu_0}|B|^2\right) = \text{Continuous amplitudes Classically}$$

But

$= \underbrace{n}\, h\nu$ by Plank

Amplitudes are Quantized packets of energy

Stat mech of this leads to $\bar{E} = \dfrac{h\nu}{e^{h\nu/k_BT}-1}$

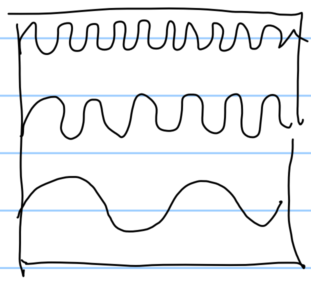
$nh\nu$ = discrete packets of energy, not continuous
discrete lumps. all or Nothing

$n = 0, 1, 2, 3 \ldots$
But never in between

How does this solve BB rad?

Well say

$E_{TOT} = 15$ intern



16 Hz, $|E(16)|^2 = 3$

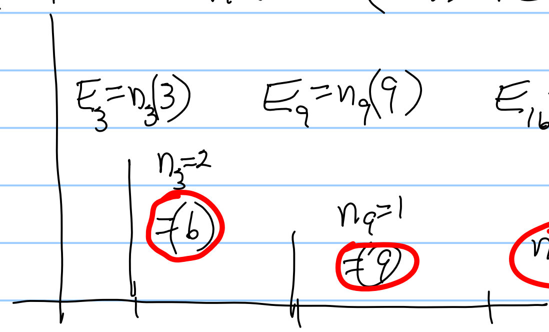
9 Hz, $|E(9)|^2 = 6$

3 Hz, $|E(3)|^2 = 6$

internal energy shared equally w/ @ frequencies

Plank : $E = nh\nu$ (assume $h=1$)

$E_3 = n_3(3)$ $E_9 = n_9(9)$ $E_{16} = (n_{16})(16)$

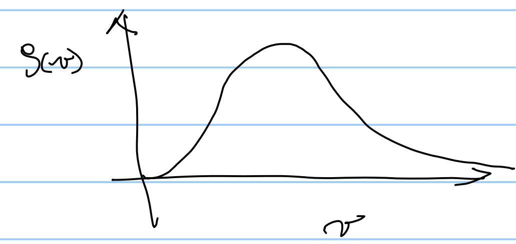


is $E_{TOT} = 15$

could never excite 16 Hz

So Spectral energy density $\rho(\nu)$

$$\rho_{\text{plank}}(\nu) = \bar{E}\, n(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} = (J/m)/Hz$$

= 

✱ @ Big ν's don't get ultra-violet catastrophy

So: 1) Experiment = Theory = $\rho_{\text{plank}}(\nu)$

WORKS

2) From $\rho_p(\nu)$

Can derive S-B Law $\left(\begin{matrix}\text{Energy}\\ \text{form}\end{matrix}\right)$

$$E_{TOT}/m^3 = \rho = \frac{8}{15}\pi^5 \frac{k_B^4}{c^3 h^3} T^4$$

compare to 1.2 Scherrer

from $\rho = \int_0^\infty \rho(\nu)\, d\nu = $ pg 9 Scherrer

TRY IT

3.) From $\rho_p(\nu)$ can derive

Wien's Law: $\nu_{max} = 2.8 \frac{k_B T}{h}$

How? $\frac{d\rho(\nu_{max})}{d\nu} = 0$ pg 9 Scherrer

try it!

So Hugely successful....

This is Science!

$\boxed{\text{Theory}} \longrightarrow \boxed{\text{experiment}}$

OK: Plank: Light = quantized in
energy bundles
$= nh\nu$

1905: Albert Einstein Special Relativity

$E_T^2 = p^2c^2 + \cancel{m_0^2}c^4$ from studies of light

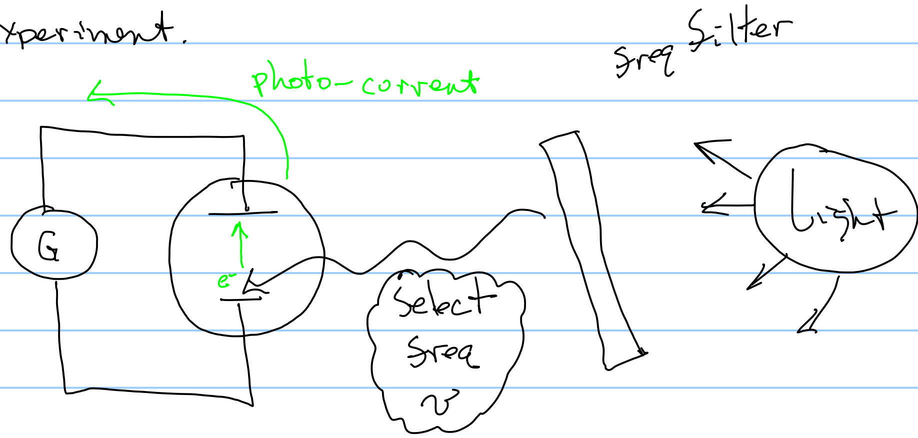
Light is $m_0 = 0$

so $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

Wo- ah... Light = wave
has $\left.\begin{array}{l} \text{1) Energy} \\ \text{2) momentum} \end{array}\right\}$ very-particle like

The Photo-Electric Effect. 1905

Experiment.



Light in measured in $\underbrace{\text{intensity}}_{\text{what you feel}} = \frac{P}{m^2} = \frac{E/s}{m^2} = \frac{E}{s \cdot m^2}$

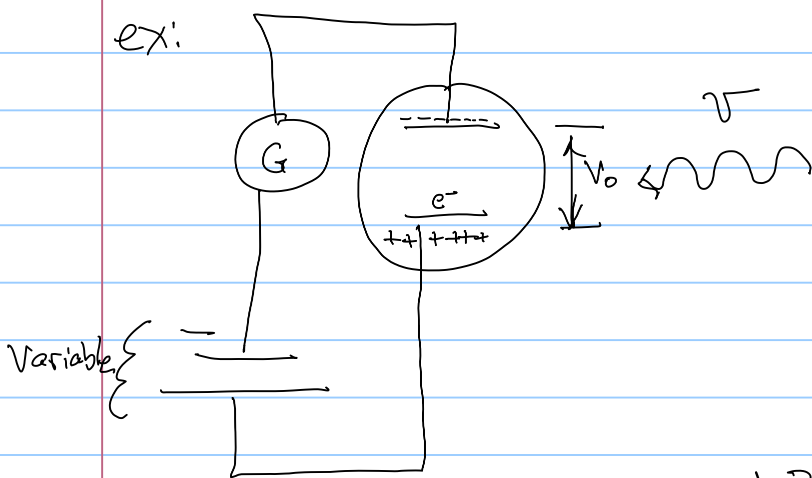
↳ need to consider time & Area energy is spread.

$$I = \frac{(n)\,h\nu}{\underbrace{(s)(m^2)}_{\text{plank}}} \quad \text{or} \quad = \frac{\frac{1}{2}\left|\varepsilon_0 E^2 + \frac{1}{\mu_0}|B|^2\right|}{\underbrace{(s)\,(m^2)}_{\text{Classical}}}$$

So @ a given frequency can turn up I by

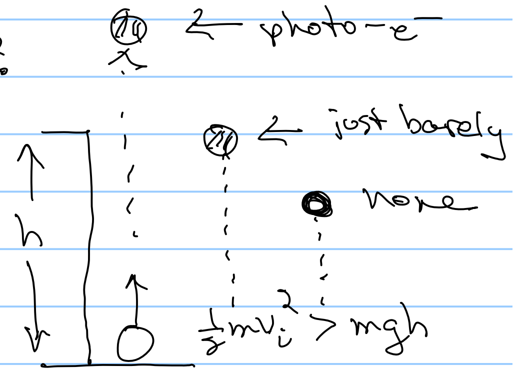
more photons all of same energy $h\nu$ ⟷ turning up Amplitude giving more energy to the wave.

ex:



@ given ν, lower V_0 until just stop photo-e^-'s

why?

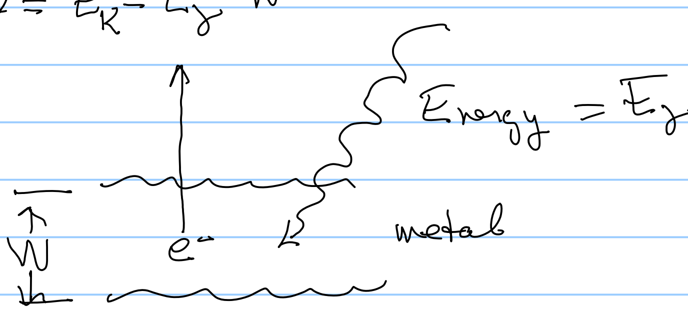


$$\frac{1}{2}mv_i^2 = mgh$$

$$\frac{1}{2}mv_i^2 \leq mgh$$

ideas:

$$\frac{1}{2}mv^2 = E_K = E_\gamma - W$$



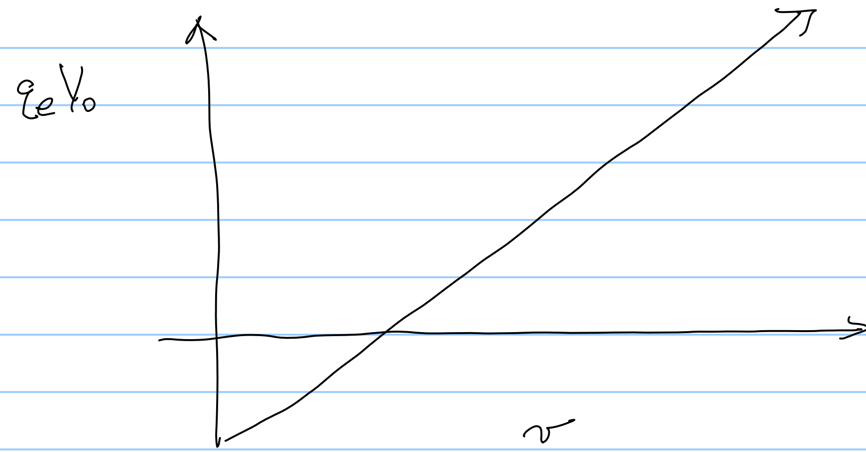
Now when $eV_0 =$ Electrical potential of e^- in V_0 $= E_K$ of e^-

photo current just stops

$$E_p(e^-) = E_k(e^-) = E_k(max)$$

$$q_e V_0 = E_\gamma - W_{metal}$$

Now if plot V_0 vs ν get



Experiment

or

$$E_k(max) = m\nu + b$$

$$E_k(max) = h\nu - W$$

Find
or that

$E_\gamma = h\nu$
same as

Plank!

Quantum challenges
Scully show's this can be done w/ wave picture of E&M

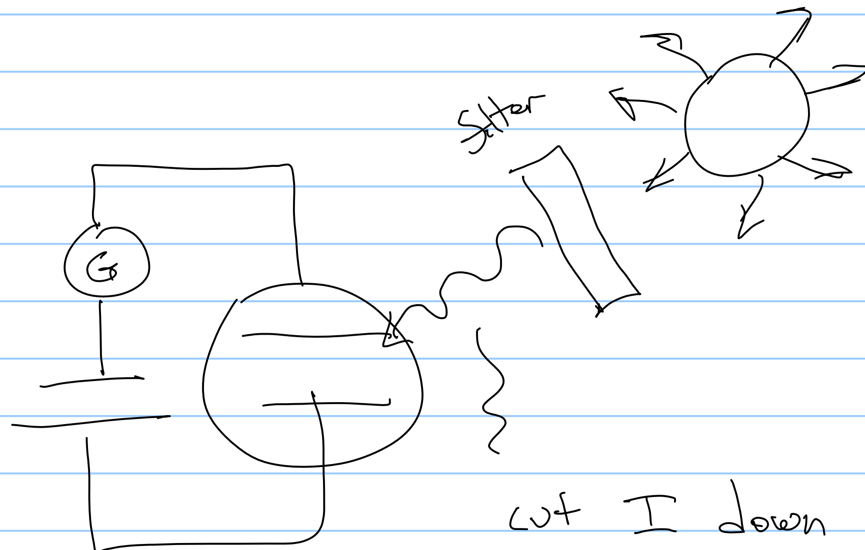
& γ & e^-'s completely acting like particles

Now $I = \dfrac{n h\nu}{(s)(m^2)}$ or $\underbrace{\dfrac{\frac{1}{2}\left|\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right|}{(s)(m^2)}}$

I from n photons all of same energy!

I from wave getting more & more energy (amplitude)

So is



cut I down
down
down
down

until
just barely

See photo current & still get

$= h\nu - W$

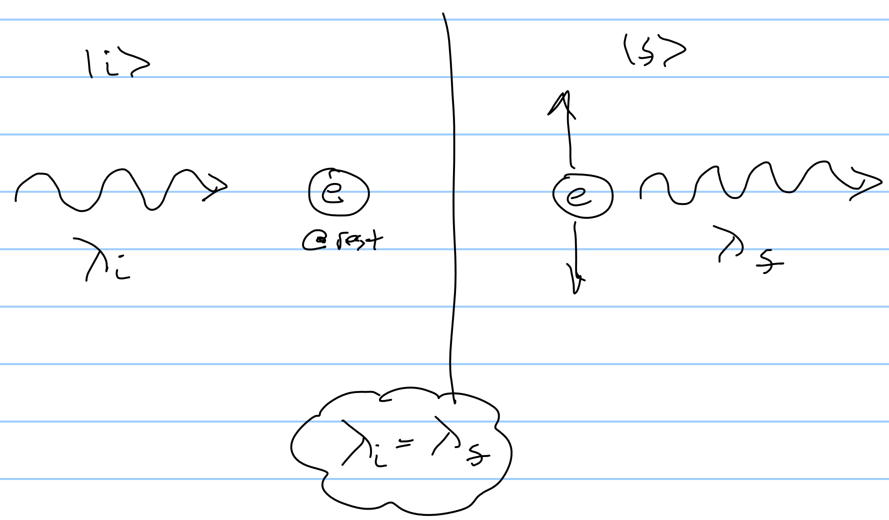
Had $I \propto (\text{amp})^2$ would have been able to impart more & more energy to a single photo e^-.

Not what you see!

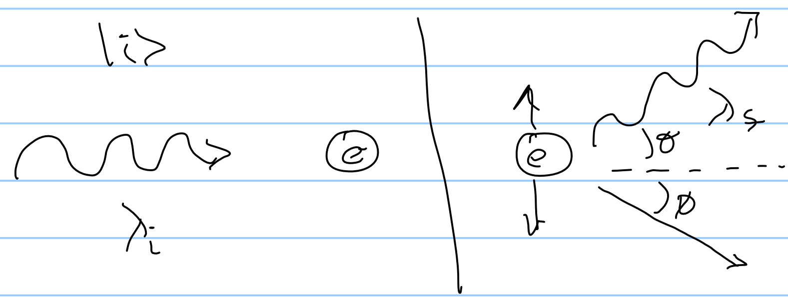
Finally Compton Scattering (1923)

http://focus.aps.org/story/v13/st8

Classical wave interaction w/ particle:



But do real experiment: Very different Result



where $\lambda_f > \lambda_i$?

Looks like a collision $\Longrightarrow$

But now we know how to do physics w/ relativistic objects:

1) $E_{tot} = \gamma m_0 c^2$
2) $\vec{p} = \gamma_u m_0 \vec{u}$

$\left.\right\}$ These are conserved

Note: this is how SR is used in Q.M. Not about diff ref frames But more using the Lorentz Frame invariant "correct" forms of E & $\vec{p}$.

OK: But what about E_T & $\vec{p}$ for Light? a photon γ

we know

see Modern Book

$$E_T^2 = (\gamma m_0 c^2)^2 = p^2 c^2 + m_0^2 c^4$$

Light is massless travel @ c

$$E_T^2 = p^2 c^2$$

so

from Plank

$$p_\gamma = \frac{E_T}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = |P_\gamma|$$

ok then $E_T = pc = \frac{hc}{\lambda} = |E_{T\gamma}|$

So now try to tackle Compton Scattering

$|i\rangle$

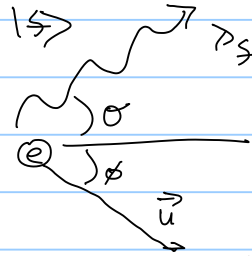
$E = \dfrac{hc}{\lambda_i}$

$P_i = \dfrac{h}{\lambda_i}$

$E_T = m_e c^2$

$P = 0$

$|f\rangle$ $\lambda_f \neq \lambda_i$



$E = \dfrac{hc}{\lambda_f}$

$P = \dfrac{h}{\lambda_f}$

$\vec{P} = \gamma_u m_e \vec{u}$

$E_T = \gamma_u m_e c^2$

Now Lorentz invariant E & $\vec{P}$ must be conserved

so

E: $|i\rangle$

$$\frac{hc}{\lambda_i} + m_e c^2 = \frac{hc}{\lambda_f} + \gamma_u m_e c^2$$

$\vec{P}$:

$$x: \quad \frac{h}{\lambda_i} + 0 = \frac{h}{\lambda_f}\cos\theta + \gamma_u m_e u \cos\phi$$

$$y: \quad 0 + 0 = \frac{h}{\lambda_f}\sin\theta - \gamma_u m_e u \sin\phi$$

Use these three pg (15) eq 1.12 – 1.16

$$\lambda_f - \lambda_i = \lambda_c (1 - \cos\theta) \quad \therefore \lambda_c = \frac{h}{m_e c} = \text{compton } \lambda$$

$$= 2.4 \times 10^{-12}\,m$$

or

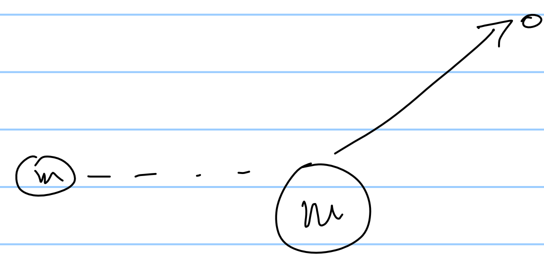
$$\lambda_f = \lambda_c (1 - \cos\theta) + \lambda_i \quad \therefore \lambda_f > \lambda_i$$

since $E_\gamma = \dfrac{hc}{\lambda}$

The outgoing γ is lower energy.

ξ λ_s max = when $\theta = 180^\circ$ = Backscattering

All looks like



particles scattering Not waves!

γ $\Rightarrow$ acting like waves!

Is Light a particle or wave?

See "Quantum Challenges" Art Zajonc
George Greenstein.

Suffice now wave-particle duality of Light $\leftrightarrow$ photon

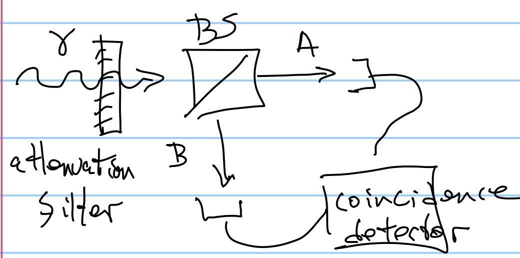
Forshadowing $e^- =$ particle... or is it?

AJP: 1) "Observing Quantum Behavior of Light in an Undergrad lab"
J.J. Thorn et al., AJP 72 (9) 1210–1219 (2004)

2) Interference w/ Correlated photons: Five Quantum mechanics experiments for undergraduates"
E.J. Galvez et al., AJP 73 (2) 127–140 (2005)

Wheeler Which Way experiment
from Quantum Challenges

1)

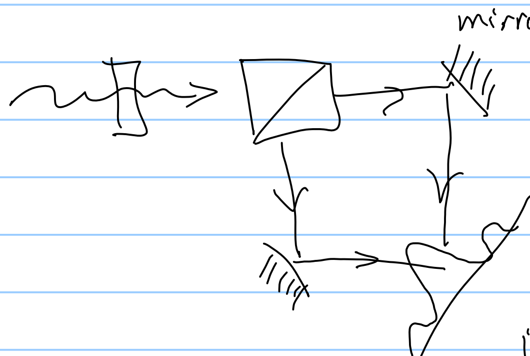


attenuate until 1 γ @ a time

Find 100% anticoince means γ, 1, goes A or B but not both!

Not @ all wavelike behavior
can't be explained w/ waves

2.)



attenuate down to 1 photon limit

& find you get interference pattern build up ... as if each photon interferes w/ itself = wave like!

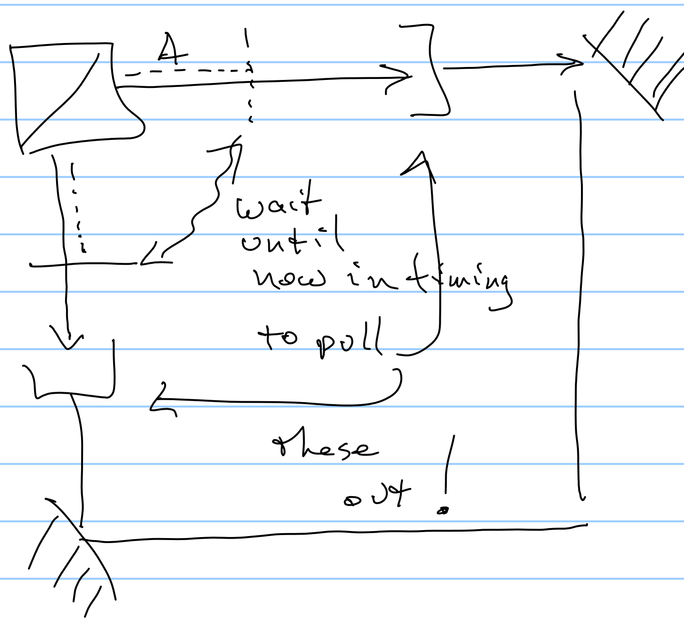
3) which way?

Seems identical exp. only diff is that we decided to measure (particle or wave)

So trick into thinking going to look for particle like behaviour and wait long enough for 1 (γ) to go either A or B as particle

But then pull particle detector out!

Had left them in --- would have rung particles

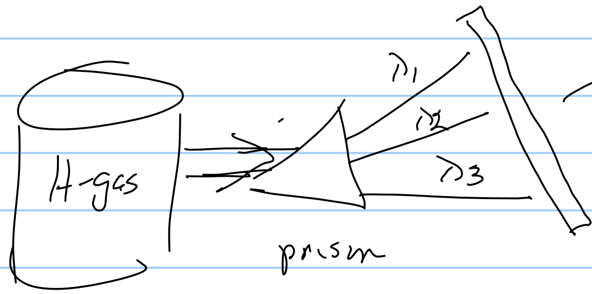


But then you get wave interference

Light $\Leftrightarrow$ photon wave particle duality.

Fine what about particles?

1900: Johann Balmer



~ discrete Lines!

he found experimentally ... a pattern based on whole #s

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

where $m = 1, 2, 3, 4 \ldots$

& $n = 2, 3, 4$

for $n > m$ & $R =$ Rydberg Constant

$$= \frac{1.097 \times 10^7}{m}$$

purely empirical result! (discovered no theory)

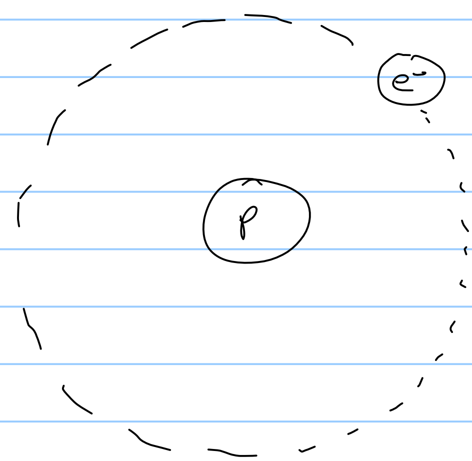
* $n \rightarrow m=1 \Rightarrow$ Lyman (ultra violet)

$n \rightarrow m=2 \Rightarrow$ Balmer (visible)

$n \rightarrow m=3 \Rightarrow$ Lyman (infrared)

This could not be explained by all physics ~1900
Newtons Laws
Maxwell
Max-Boltz

ex: H-atom



accelerating charges radiate!

radiation = energy.

∴ e⁻ accelerating in orbit should continuously loose energy to radiation

& as it does so, it should spiral into the proton radiating more & more energy!

2nd version of ultraviolet catastrophy!

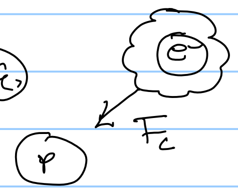
1913 Bohr came up w/ a solution.

Bohr atom:

1) Angular momentum of orbits quantized!

$$\vec{L} = \vec{r} \times m\vec{v} = rmv = n\hbar, \quad n = 1, 2, 3 \ldots$$

Note de Broglie.... matter waves not until 1924

2) a) 

$$\sum F_c = \frac{-m_e v_e^2}{r_e}$$

$$\frac{-ke^2}{r^2} = \frac{-m_e v_e^2}{r_e}$$

b) $r_n m v = n\hbar$

c) $E_{TOT} = E_K + E_p = \frac{1}{2} m_e v_e^2 + \frac{-ke^2}{r}$

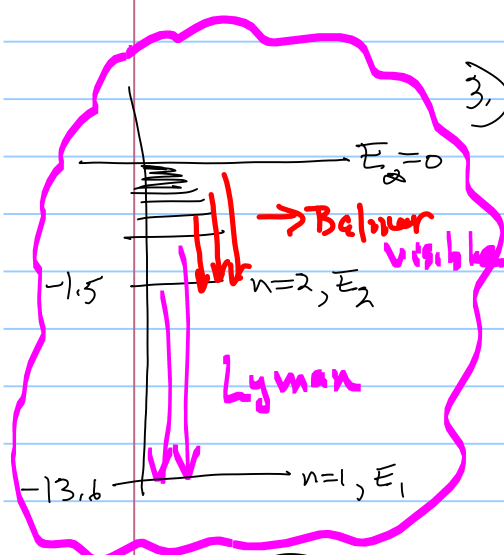
3) Combine to get

Quantized L ⇒ Quantized r_n & E_n

$$r_n = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} n^2, \quad n = 1, 2, 3 \ldots$$

$$= a_0 n^2 \quad ; \; a_0 = 0.0529 \text{ nm}$$

d atom = .1 nm = 1 Å



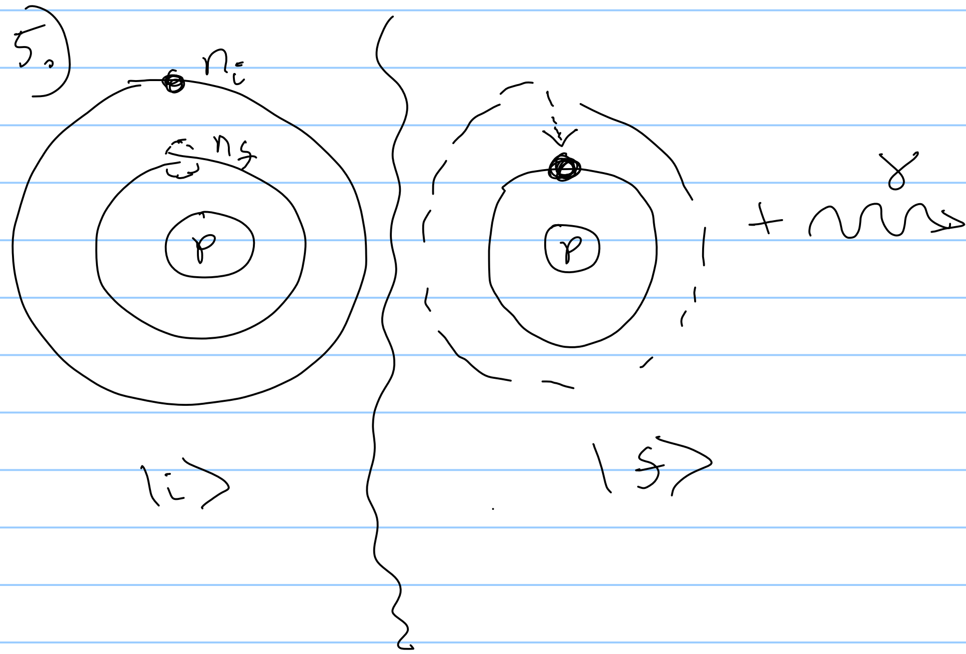
$$E_n = \frac{-m}{2\hbar^2} \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{1}{n^2} \quad ; \; n = 1, 2, 3 \ldots$$

$$= \frac{-13.6 \text{ eV}}{n^2}$$

* middle of visible ≅ 2 eV

4) e^-'s stay in stable orbits w/o radiating (?)

5.)



$$E_i = E_f + E_\gamma$$

$\therefore$

$$E_\gamma = E_i - E_f$$

$$E_\gamma = -13.6\,eV\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E_\gamma = +13.6\,eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

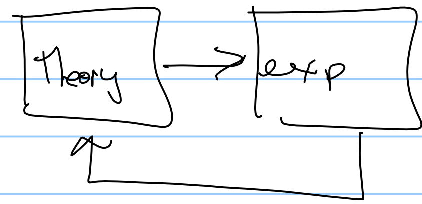
$$\frac{hc}{\lambda} = 13.6\,eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{13.6\,eV}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\boxed{\frac{1}{\lambda} = \frac{1.097\times10^7}{m}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

WOW!
Bohr Theory derived Balmer empirical relation!

So again



Bohr atom, new physics, quantized orbits $\Rightarrow$

- quantized radii
- energy
- All Spectroscopy

where no other classical Theory could!

died 1987, @ 95 years old

1924: Louis de Broglie
perhaps thinking of Bohr Atom
& why Quantized orbits?

maybe seeing Light = particle-wave
+ Quantized

Said e^- = particle-
Quantized
WAVE?

We ⊗ Light

$$E^2 = p^2c^2 + m_0^2c^4 \quad (m_0 \to 0)$$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

so

maybe true in general

$$\lambda = \frac{h}{p}$$ For everything!

λ_{me} @ 1 m/sec walk
$\cong 10^{-35}$ m

$\vec{p} = m\vec{u}$ or $\vec{p} = \gamma_u m\vec{u}$? yes! pg 16 Scherrer

* NOTE: See λ-Like nature need
λ & expriment of same order.

So Non relativistic $e^- \sim 10^6$ m/s

$\lambda_e \cong 10^{-10}$ m } where could you get ...

*Note: See λ-Like nature need λ & expiment of same order.

So Non relativistic $e^- \sim 10^6$ m/s

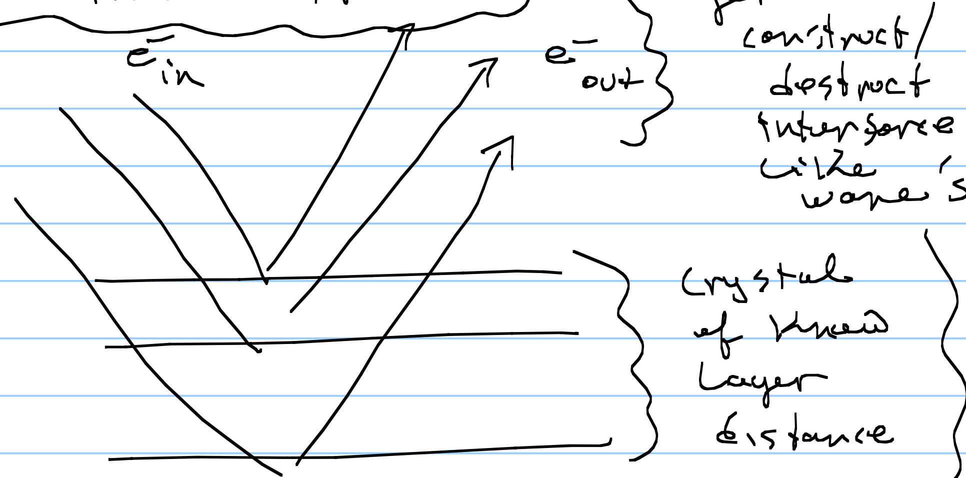
$\lambda_e \cong 10^{-10}$ m } where could you get —...

exper $\sim 10^{-10}$ m

atom sized or atom spacing

At in a crystal

Davisson-Germer experiment



See $\lambda_e \sim 10^{-10}$ m!