so ... particles = particles + waves

so ↓

$$\hat{H}_{classical}\, \Psi_{(x,t)} = i\hbar \frac{\partial \Psi}{\partial t}(x,t) = \text{time dep Schrö}$$

$$(\hat{E}_k + \hat{E}_p)$$

$$\left(\frac{\hat{p}^2}{2m} + \hat{V}(x,t)\right)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$$

if $V \neq f(t)$

Great Simplification

* Sep of variables
* demand Energy eigen kets of them

$$i\hbar \frac{\partial \Psi}{\partial t} = E_n \Psi$$

time indep Schrö →

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_{(x)}}{\partial x^2} + V_{(x)}\,\psi_{(x)} = E_n\, \psi_{(x)}}$$

It truly all comes down to what $V(x)$ is to determine $\psi(x)$

Then of course

$$\Psi_{(x,t)} = \psi_{(x)}\, e^{-i\frac{E_n}{\hbar}t}$$

= stationary states

to build all answers from

why not? at Fundamental level

$$\vec{F} = -\vec{\nabla} V \quad \{\text{is conservative}\}$$

So is all about the Forces (1 E&M, 2 strong force, 3 weak nuc, 4 gravity) acting

Thus we will consider 5 Huge Cases!

Recall: Ψ not Localized & $\int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = 1$ so need to consider Ψ thru all space.

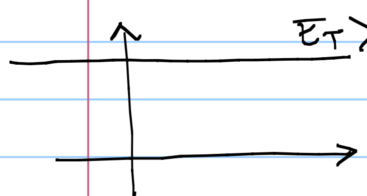
$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = E_T \psi$$

$$\psi'' + \frac{2m}{\hbar}\left(E_T - V(x)\right)\psi = 0$$

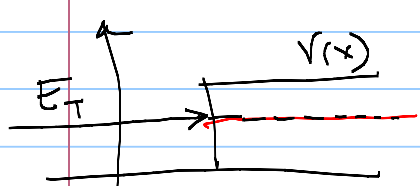
I

For problems where $V(x)=$ Constant over defined regions 4 Big Cases!

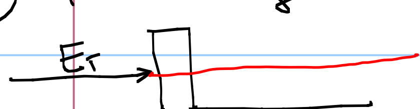
1.) FREE Particle States



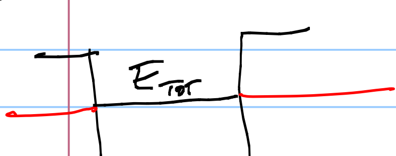
2.) Scattering States



3.) Tunneling States

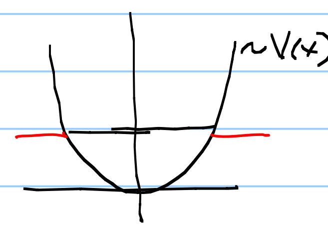


4.) Bound States



II

$\underbrace{V(x) \cong \frac{1}{2}kx^2}$ all systems near stable equi.



ie Harmonic Oscillators!

III

$V(x) =$ $\underbrace{\text{something else}}$

Ton's of Techniques WKB, Perturbation & more much Later!

I : $V(x)$ = constant over defined region → Free particle states / → Scattering states / → tunneling states / → Bound states

all solved exactly same way from same beginning:

$$\psi'' + \frac{2m}{\hbar^2}(E_T - V(x))\psi = 0$$

Keep in mind

$$\Sigma F = m\ddot{x}$$

$$(-kx) = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$

$$x(t) = Amp(\cos(\omega_0 t + \delta))$$

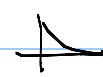
lets solve more generally

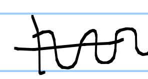
$$\frac{d^2\psi}{dx^2} + A\psi = 0$$

$\Downarrow$

typical approach of expert --- guess

recall e^{px} = most general of <u>all</u> algebraic expressions

⇒  is p = real

⇒  is p = complex

assume

$$\psi(x) = Ce^{px}$$

sub back in

$$Ce^{px}(p^2 + A) = 0$$

for $C \neq 0$; $(p^2 + A)$ must $= 0$

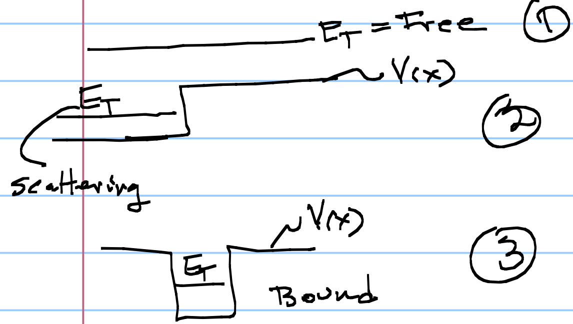
$$p^2 = -A$$

$$p = \pm\sqrt{-A}$$

So $\psi(x) = C_1 e^{+\sqrt{-A}x} + C_2 e^{-\sqrt{-A}x}$ $\Big\}$ $A = \frac{\hbar^2}{2m}(E_T - V(x))$

So is

$\boxed{E_T > V(x)}$ Then



Then $A = (+)$ so $\sqrt{-A} = i\sqrt{A}$

&

$$\psi(x) = C_1 e^{+i\sqrt{A}\,x} + C_2 e^{-i\sqrt{A}\,x} \quad (*)$$

which is exactly the same as

$$\psi(x) = D_1 \cos(\sqrt{A}\,x) + D_2 \sin(\sqrt{A}\,x) \quad (**)$$

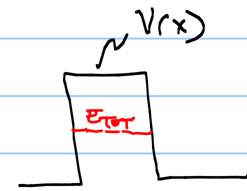
which is exactly the same as

$$\psi(x) = Amp\left(\cos(\sqrt{A}\,x + \delta)\right) \quad (***)$$

↳ Note: this soln we choose to use for ~~Lum~~ [illegible]

$x(t) = Amp\left(\cos(\omega_0 t + \delta)\right)$

$\boxed{E_T < V(x)}$ Then

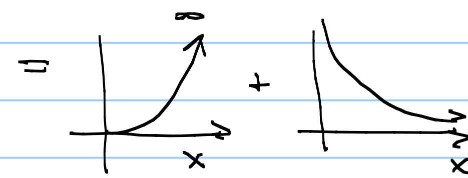


= ie tunneling states

⇓

Then $A = (-)$ so $\sqrt{-A} = \sqrt{A}$

$$\psi(x) = C_1 e^{+\sqrt{A}\,x} + C_2 e^{-\sqrt{A}\,x}$$



ie exponentials!

in regions where

$E_T < V(x)$

However, The forms of The solns $*$ & $**$ are more useful for Our Q.M. problems So we use Them!

point is all 3 are exactly the same BUT all simply different but equivalent representations of the soln

So... believe it or not...

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{dx^2} + V(x)\psi = E_T\,\psi(x)$$

has solns

$E_T > V(x)$

$$\psi(x) = C_1 e^{i\sqrt{A}\,x} + C_2 e^{-i\sqrt{A}\,x}$$

or

$$\psi(x) = D_1\cos(\sqrt{A}\,x) + D_2\sin(\sqrt{A}\,x)$$

or

$$\psi(x) = \text{Amp}\cos(\sqrt{A}\,x + \delta)$$

$E_T < V(x)$

$$\psi(x) = C_1 e^{+\sqrt{A}\,x} + C_2 e^{-\sqrt{A}\,x}$$

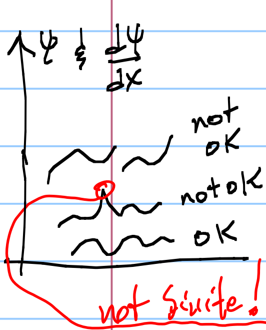
where, $A \equiv |A|$ because...

where $|A| = \left|\frac{2m}{\hbar^2}\left(E_T - V(x)\right)\right|$

magnitudes because took care of the sign of A in 2 cases!

WITH condition that you need to consider $\psi(x)$ over all space for a given E_{TOT}

ie



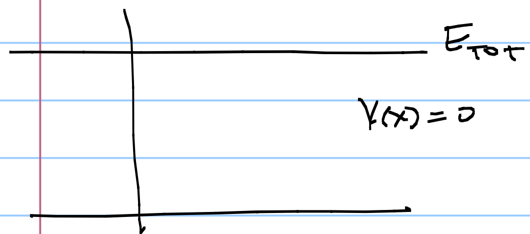
& Since $\int_{-\infty}^{+\infty}\psi^*(x)\,\psi(x)\,dx = 1$, ψ & $\frac{d\psi}{dx}$

must be well behaved... ie

$\psi(-\varepsilon+0) = \psi(0+\varepsilon)$ & $\frac{d\psi}{dx}(-\varepsilon+0) = \frac{d\psi}{dx}(\varepsilon+0)$

and FINITE as $\varepsilon \to 0$

I 1.) Free particle ---- actually maybe messiest of all



clearly
$E_{TOT} > V(x)$
so 1 of 3 *, ** & *** solns

& need $A = \frac{2m}{\hbar^2}\left(E_T - \underbrace{V(x)}_{=0}\right)$

Solution Form (*) will be most useful

$$\psi(x) = C_1 e^{i\left(\frac{\sqrt{2mE_T}}{\hbar}\right)x} + C_2 e^{-i\left(\frac{\sqrt{2mE_T}}{\hbar}\right)x}$$

why ?

Soln's are always best when you can "see" what they are:

we used $x(t) = \underbrace{Amp\cos(\omega_0 t + \delta)}_{\substack{\text{clear} = \\ \cos + \text{phase}}}$

But what is * for free particle then?

Recall
$$\Psi(x,t) = \psi(x)\, e^{-i\frac{E_T}{\hbar}t}$$

So
$$\Psi(x,t) = C_1 e^{i\left[\left(\frac{\sqrt{2mE_T}}{\hbar}x\right) - \frac{E_T}{\hbar}t\right]} + C_2 e^{+i\left[\frac{-\sqrt{2mE_T}}{\hbar}x - \frac{E_T}{\hbar}t\right]}$$

now free particle

I

$E_T = E_K = p^2/2m$

so $p = \sqrt{2mE_T}$

& $p = \hbar k$

& II $E_T = \hbar\omega$

so

$$\Psi(x,t) = \underbrace{C_1 e^{i(kx-\omega t)}}_{\substack{=\text{traveling}\\ \text{wave}\\ \text{to the} \longrightarrow}} + \underbrace{C_2 e^{-i(kx+\omega t)}}_{\substack{=\text{traveling}\\ \text{wave to}\\ \text{the} \longleftarrow}}$$

w/ $E_T = \hbar\omega$ & $p = \hbar k$ ⟷ w/ $E_T = \hbar\omega$ But $p = -\hbar k$

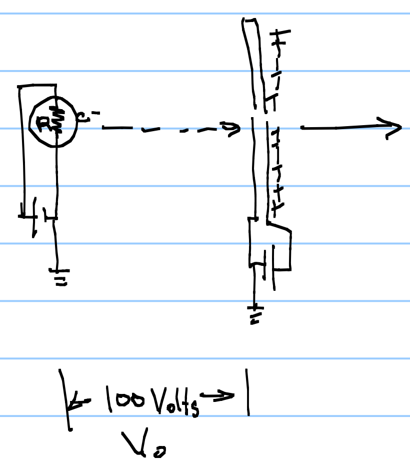
we know Ψ = energy eigen funct (definite energy) But it is not an eigen funct of $\hat{p}$ so you do not get definite p.

we Built it that way ↓ by def time Indep Schrö = energy eigenstates

Now, this true

$$\underset{\text{free particle}}{\Psi_{\text{general}}} = \underset{\text{traveling waves}}{\sum (\longrightarrow) + (\longleftarrow)}$$

But if you know Physically this is not the case



$\frac{1}{2}mv^2 = qV_0$ to $\longrightarrow$

Then let $C_2 = 0$

&

$$\boxed{\Psi(x,t) = C_1 e^{i(kx-\omega t)} \quad \text{free particle to} \longrightarrow}$$

Funny-Hah-Thing $\Longrightarrow$

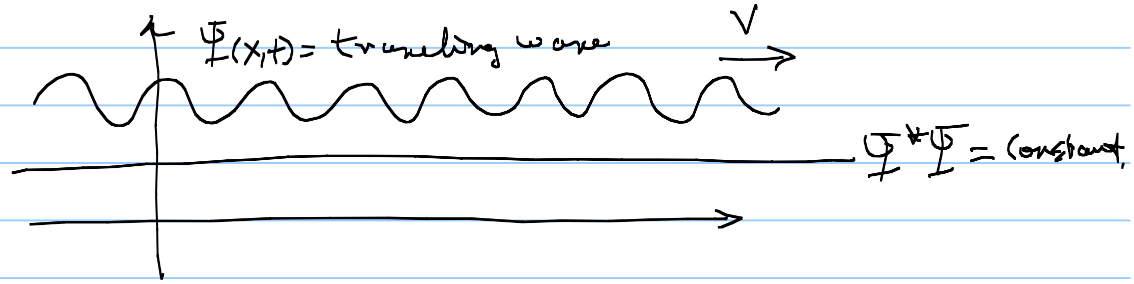
ask where is our free particle traveling $\rightarrow$?

well need $\Psi^*\Psi$

$$= C_1^* e^{-i(kx-\omega t)} C_1 e^{+i(kx-\omega t)} = C_1^* C_1$$

$$= \text{Constant}$$

i.e



WHAT? Here is the problem.
Heisenberg Uncertainty Princ.
(see attached Heis Uncert Lect)

Since 1.) time indep Schrö $\Rightarrow$'s Energy Eigenkets

$\Downarrow$'s

Definite energy

$\Downarrow$

$\langle E \rangle \pm \Delta E$

$\langle E \rangle \pm 0$

2.) let $C_2 = 0$; $\therefore \Psi =$ momentum eigenstate too

$\therefore \langle p \rangle \pm 0$

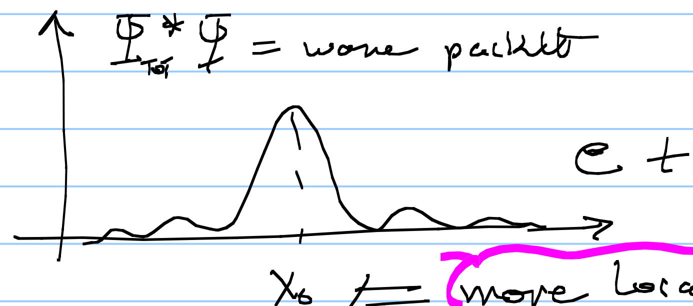
So $\Delta p \Delta x \geq \frac{\hbar}{2} \Rightarrow$'s $\Delta x = \infty$ uncertainty in position $= \infty$ could be anywhere

$\Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow$'s $\Delta t = \infty$ uncertainty in time $= \infty$ could be @ any time!

That's weird.... But note it is not practicle
you will never know $\Delta p = 0$ (ie Δk, $p = \hbar k$)
in other words won't ever have exact
p eigenstate $p = \hbar k$

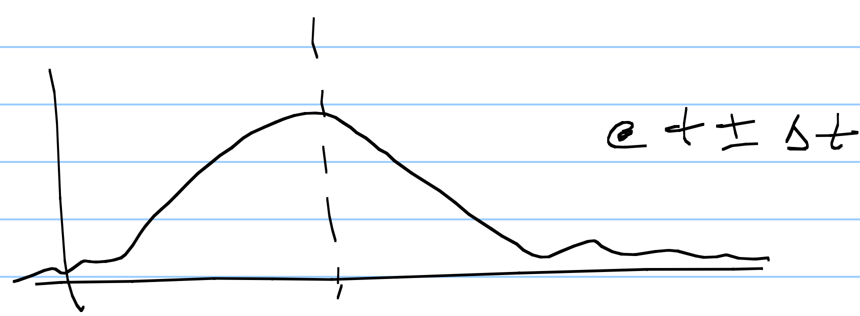
Thus $\Psi_{tot}(x,t) = \sum_{\Delta k} \Psi_k(x,t)$ = sum of Ψ's over range of Δk's about k_0

$\equiv$ A wave packet
That is well, better, localizes



$\Leftarrow$ more localized Free particle

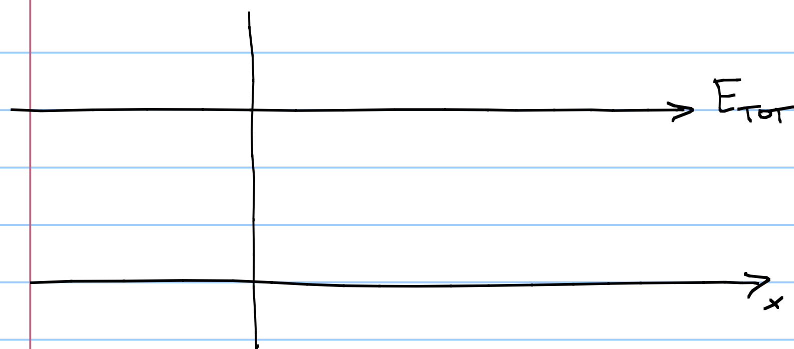
But this wave packet spreads over time



E. F. Deveney / BSC Physics PH402 Quantum: Wave Packets Griffiths chpt 2

Note Title 4/12/2006

For free Particle state we found...



$$\hat{H}\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$\downarrow$ energy eigenstates

$$\hat{H}\psi_n = E_n \psi_n$$

$$\psi'' + A\psi = 0 \qquad A = \frac{2m}{\hbar^2}(E_T - V)$$

$$E_T > V \qquad \& \quad \sqrt{A_i} = K_i$$

$\therefore$

$$\psi(x) = C_1 e^{iK_1 x} + C_2 e^{-iK_1 x}$$

Then, $\Psi_{TOT}(x,t) = e^{-i\frac{E_T t}{\hbar}}\psi$

$$\Psi_{TOT} = C_1 e^{-i(K_1 x - \omega t)} + C_2 e^{-i(K_1 x - \omega t)}$$

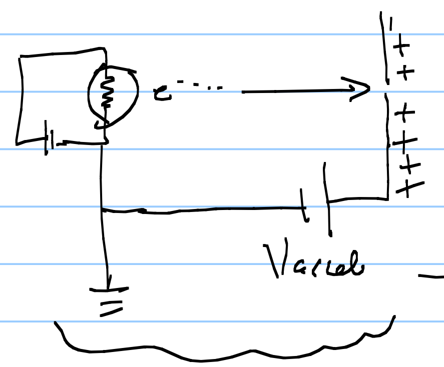
which we recognize $\longrightarrow$ traveling plane wave $+$ $\longleftarrow$ traveling plane wave

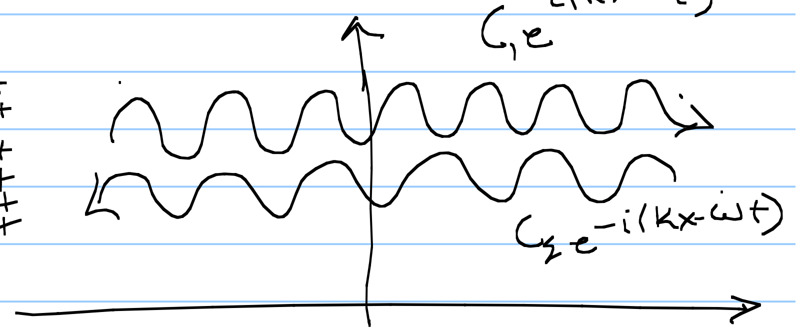
& cannot be normalized!

w/ veloc

$e^{iK(x - \frac{\omega}{K}t)}$ $\qquad v = \frac{\omega}{K} = \frac{2\pi/T}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T}$ $\qquad \lambda = \frac{h}{p}$

So, if we set up this experiment

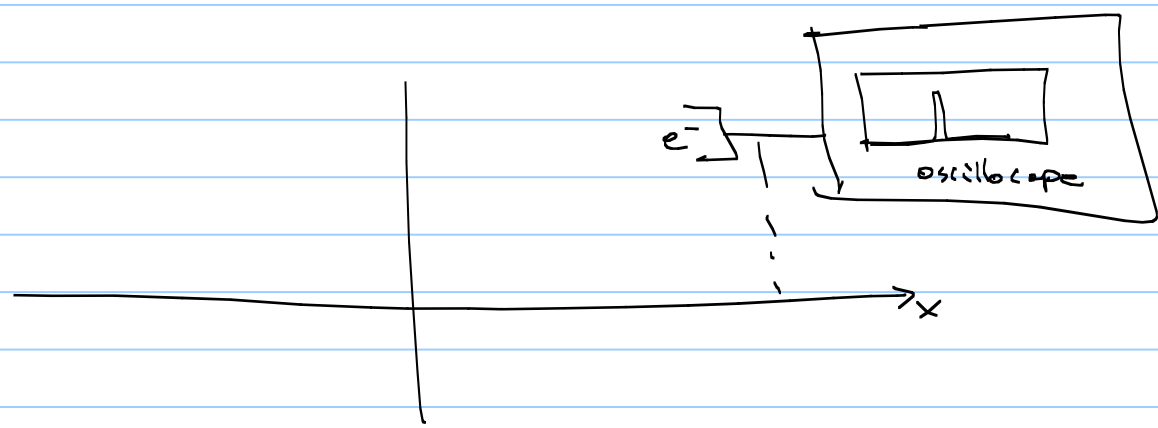




$$\frac{p^2}{2m} = qV_a$$

$$p = \sqrt{2mqV_a} = \text{well defined}$$

$$\lambda = \frac{h}{p}$$



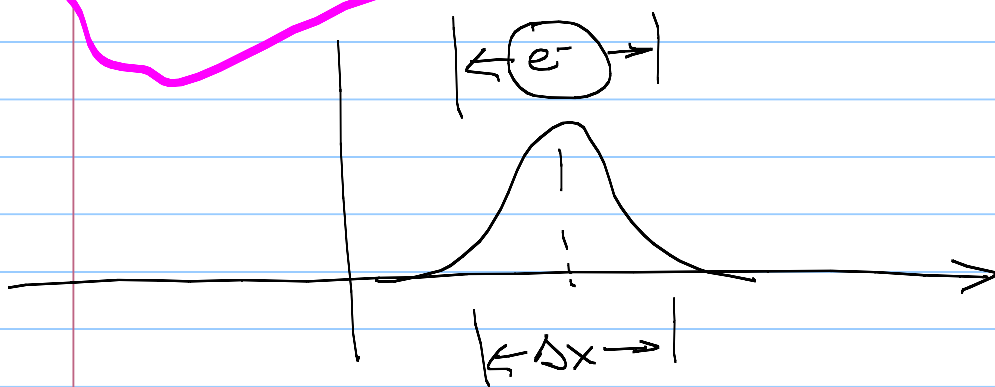
Now how can our particle be ⟶ + ⟵ ?

recall the uncertainty principle

$$\Delta x \, \Delta p \geq \hbar/2$$

As long as we don't ask where e^- is along the way, these well defined $\vec{p}$ states have unrestricted Δx so these traveling waves over all x are OK! Don't ask where the e^- is along the way and this description is fine!

However, if doing an experiment and want to follow the particle, know where Δx is



This means that you have to give up info on the well defined $p = \hbar k = \hbar \frac{2\pi}{\lambda}$ according to

$$\Delta x \, \Delta p \geq \hbar/2$$

So now our well defined momentum particle to know Δx, must $\therefore$ spread out in momentum.

Thus our Ψ_{TOT} must be summed over a range of momenta continously

$$\Psi(x,t) = e^{-i\frac{E_{TO}}{\hbar}t}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\phi(k)e^{ikx}\,dk\right]$$

recognize as inverse Fourier transform

Also note: a Fourier series

$f(x) = \sum_n [a_n \cos n\omega x + b_n \sin n\omega x]$ = finite sum that can make periodic waves

But not a single pulse @ x_0

This is called a wave packet

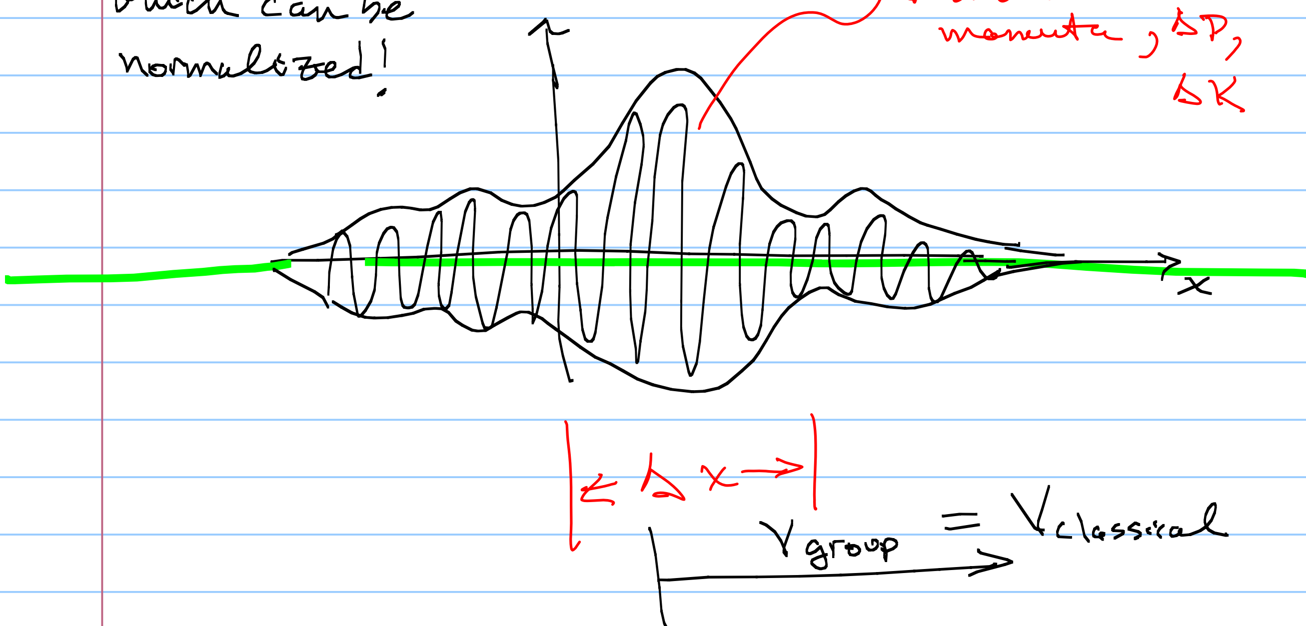
where $\phi(k)\,dk = \dfrac{\text{prob}}{\text{wave }\#}$ on projection onto traveling wave basis of STATIONARY STATES

(remember $\int dk$ is $\int$ over p as $p = \hbar k$)

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k)\, e^{i(kx-\omega t)}\, dx$$

which can be normalized!

different momenta, Δp, Δk



Griffiths can show

$$V_{group} = \frac{d\omega}{dk}$$

$V_{phase} = \omega/k$ = veloc of each traveling wave (k_i), ie each stationary state

& $V_{classical} = V_{group} = 2\,V_{phase} = \left(\frac{\omega}{k_0}\right)$

central k_0, p_0