Note Title

E.F. Deveney / BSC Physics: PH402 Quantum

Qualitative ~ time indep Solns of Schrö

Sherrer 3.3.

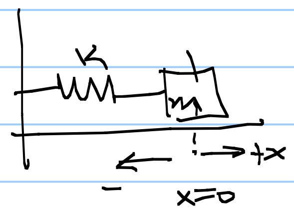
3/29/2006

→ & idea of Energy Quantization

time indep Schrö

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi=E_T\psi$$

2 .... hints:



$\xrightarrow{+} \sum F = ma$

$-kx = m\ddot{x}$

$\ddot{x}+\frac{k}{m}x=0$

$\ddot{x}+\omega_0^2 x=0$

$\omega_0=\sqrt{\frac{k}{m}}$

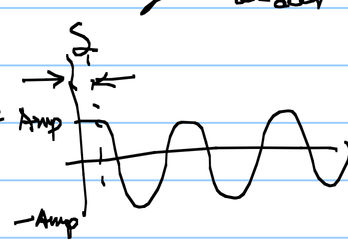
soln

Cleanest soln $\Longrightarrow$ $x(t)=Amp\cos(\omega_0 t+\delta)$

See
Marion Thornton!

note 2 arb const from 2nd order diff-Q

=



Now Look @ t indep Schrö

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}[E_T-V]\psi=0$$

$$\psi''+\omega_0^2\psi=0$$

$\psi'' + \omega_0^2 \psi = 0$ $\qquad$ $\omega_0^2 = \frac{2m}{\hbar^2}\left[E_T - V\right]$

* now if $E_T > V$
then soln
$= \psi(x) = Amp \cos(\omega_0 x + \delta)$

* But if $E_T < V$

$\psi(x) =$ exponential!

2nd Hint

Further: try looking @ 2nd deriv's & concave ↑ or ↓

$$\psi'' = -\omega_0^2 \psi$$

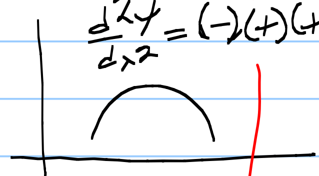
$$\frac{d^2\psi}{dx^2} = -\omega_0^2 \psi \qquad ; \omega_0^2 = \frac{2m}{\hbar^2}\left[E_T - V\right]$$

$E_{TOT} > V$ $\quad$ ψ curves toward axis

$\omega_0^2 = +$

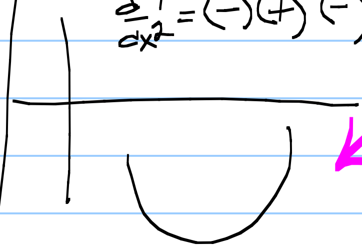
$\psi = +$

$\frac{d^2\psi}{dx^2} = (-)(+)(+) = -$



$\psi = -$

$\frac{d^2\psi}{dx^2} = (-)(+)(-) = +$

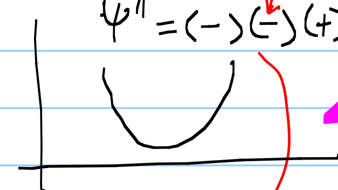


$E_{TOT} < V$ $\quad$ ψ curves away from axis

$\omega_0^2 = (-)$

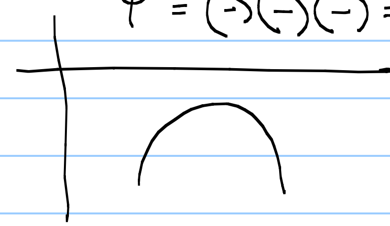
$\psi = +$

$\psi'' = (-)(-)(+) = +$



$\psi = -$

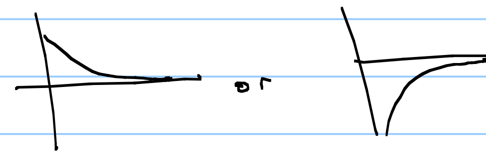
$\psi'' = (-)(-)(-) = -$



So $\quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E_T\psi$

For $E_T > V$: $\psi = $ ∿∿ oscillatory
always curving toward
x-axis

$E_T < V$: $\psi = $ Expo curving away
from axis



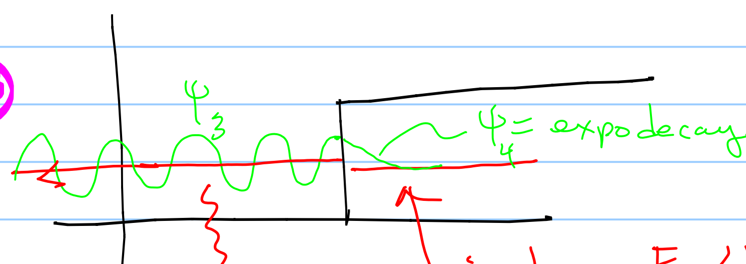
4 main cases:

one sol
quasi scattering!

expect $\psi_1(x)$

① $E_T > V(x)$ always = FREE

② $E_{TOT} > V(x)$ in a region = "BOUND



③

ψ_4 = expo decay

$E_T > V$ free on one side, Bound on other = Scattering!

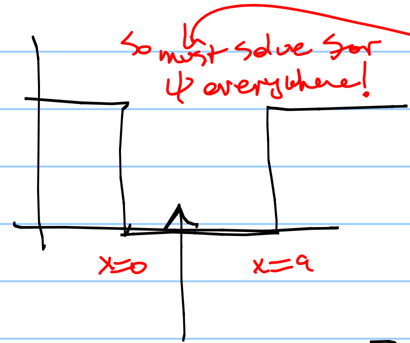
in here, $E_T < V$ = Tunneling

④

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E_T\psi$$

Further .... have a feel for ψ, but what about E

well $E_T > V$ we know curves toward axis & oscillatory



& ψ_{TOT} must be continuous

$\psi(x<0) \Rightarrow \psi(x=0) \Rightarrow \psi(x>0)$ continuously

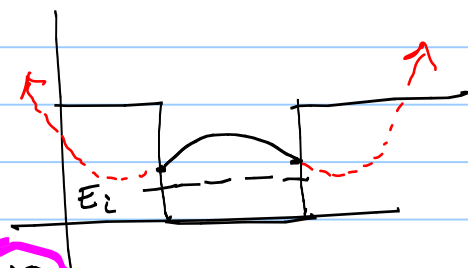
$\frac{d\psi}{dx}(x<0) \Rightarrow \frac{d\psi}{dx}(x=0) \Rightarrow \frac{d\psi}{dx}(x>0)$ continuously

& same for $x=a$

what E's work?

Start from $E = 0$ & $\uparrow$

For lowest energies, start w/ least # of oscillations



For this E_i, ψ acts ok in all regions but cause $\to \infty$

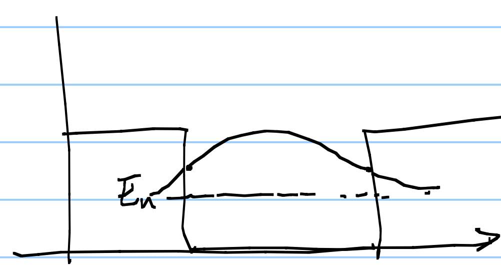
$$\int_{-\infty}^{+\infty} \psi^*\psi\,dx \to \infty$$

Non normalizable!

clearly can't be normalized

so that $E_i \neq$ physical soln to Schrö

but jiggle a little more E_n



& ψ act's normally in all regions & is finite in tunnel regions $\Longrightarrow$

That means, not only is ψ_n a soln to Schrö's

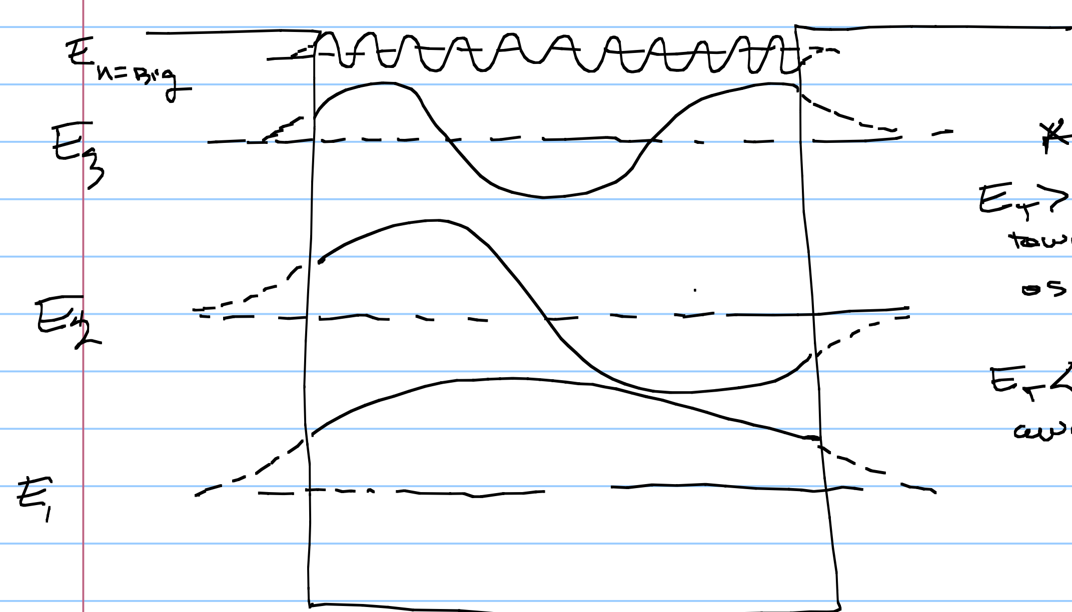
But that ψ_n & its eigen energy E_n can represent <u>physical</u> states of real particles!

NOTE what this $\Rightarrow$'s

Q.M.'s says not <u>all</u> E_T's work

Only certain ones keep ψ normallizable & only these = physical real states!

Thus Q.M. $\Rightarrow$'s QUANTIZED ENERGY'S ONLY!



*NOTE

$E_T > V$ = curve toward axis oscillatory

$E_T < V$ = curve away expo