E.T. Dorney / BSC Physics: PH402 Quantum

Note Title

Qualitative ~ time indep Solns of Schrö

Sherrer 3.3.

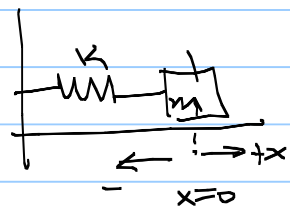
3/29/2006

→ & idea of Energy Quantization

time indep Schrö

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E_T\psi$$

2 .... hints:



$\xrightarrow{+} \sum F = ma$

$-kx = m\ddot{x}$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$

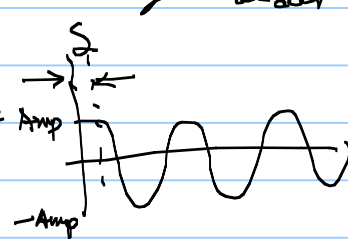
$$\omega_0 = \sqrt{\frac{k}{m}}$$

soln

Cleanest soln $\Longrightarrow$ $x(t) = \text{Amp}\cos(\omega_0 t + \delta)$

See Marion Thornton!

note 2 arb const from 2nd order diff-Q

$=$ 

Now Look @ t indep Schrö

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[E_T - V\right]\psi = 0$$

$$\psi'' + \omega_0^2\psi = 0$$

$$\psi'' + \omega_0^2 \psi = 0 \qquad \omega_0^2 = \frac{2m}{\hbar^2}\left[E_T - V\right]$$

* now if $E_T > V$
  then soln
  $= \psi(x) = \text{Amp}\cos(\omega_0 x + \delta)$

* But if $E_T < V$
  $\psi(x) =$ exponential!

2nd Hint

Further: try looking @ 2nd deriv's & concave ↑ or ↓

$$\psi'' = -\omega_0^2 \psi$$

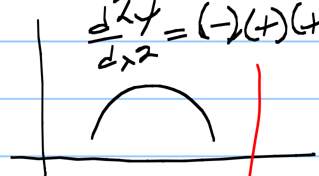
$$\frac{d^2\psi}{dx^2} = -\omega_0^2 \psi \qquad : \omega_0^2 = \frac{2m}{\hbar^2}\left[E_T - V\right]$$

$E_{TOT} > V$ — ψ curves toward axis — $\omega_0^2 = +$

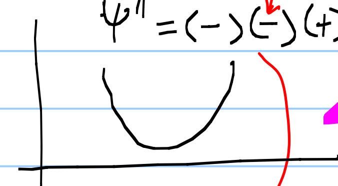
$E_{TOT} < V$ — ψ curves away from axis — $\omega_0^2 = (-)$

$\psi = +$

$$\frac{d^2\psi}{dx^2} = (-)(+)(+) = -$$

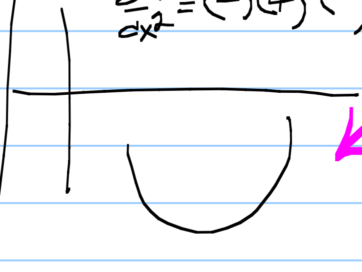


$$\psi'' = (-)(-)(+) = +$$

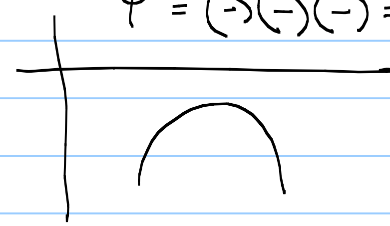


$\psi = -$

$$\frac{d^2\psi}{dx^2} = (-)(+)(-) = +$$



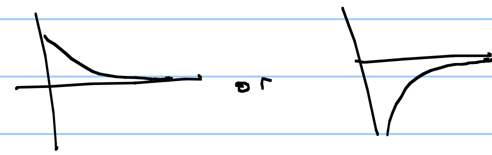
$$\psi'' = (-)(-)(-) = -$$



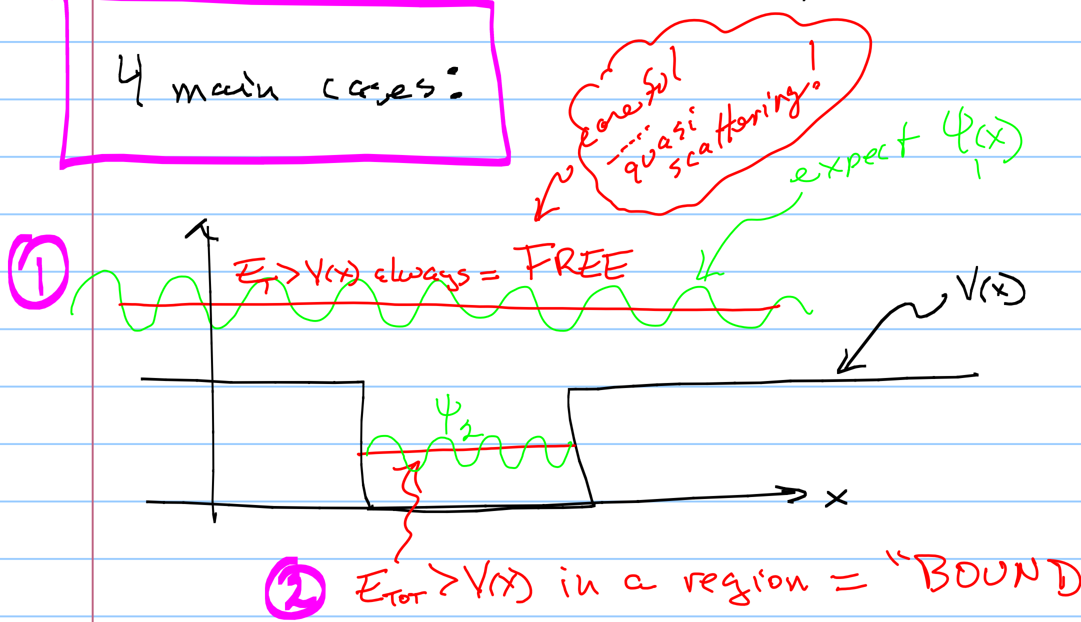
So $\quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E_T\psi$

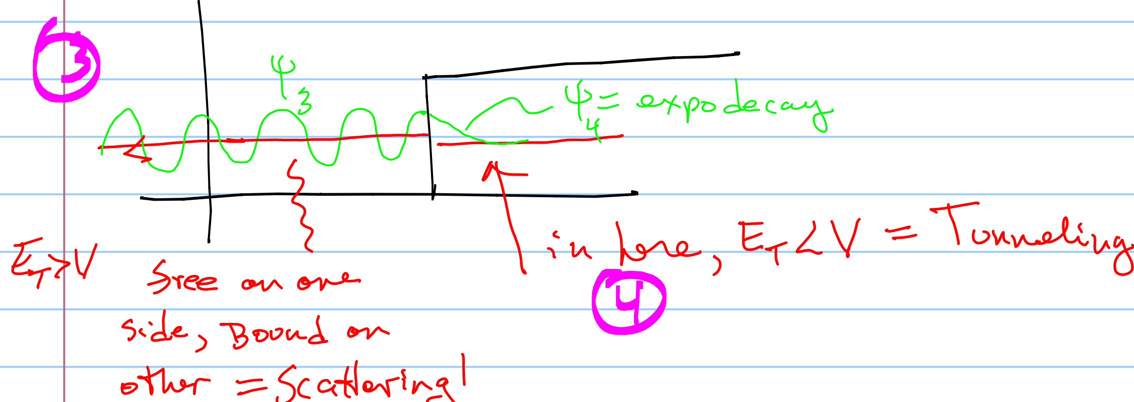
For $E_T > V$: $\psi =$ ∿∿ oscillatory
always curving toward
x-axis

$E_T < V$: $\psi =$ Expo curving away
from axis



$\boxed{\text{4 main cases:}}$

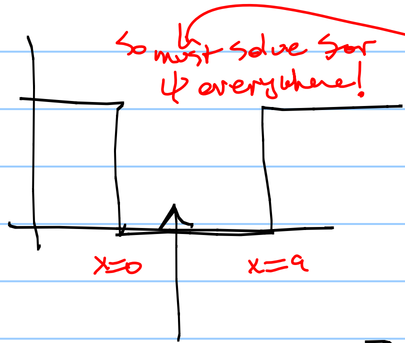




$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E_T\psi$$

Further .... have a feel for ψ, but what about E

well $E_T > V$ we know curves toward axis & oscillatory



& ψ_{TOT} must be continuous

$\psi(x<0) \Rightarrow \psi(x=0) \Rightarrow \psi(x>0)$ continuously

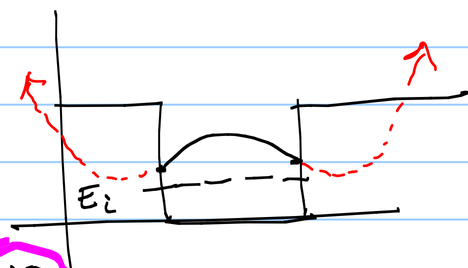
$\frac{d\psi}{dx}(x<0) \Rightarrow \frac{d\psi}{dx}(x=0) \Rightarrow \frac{d\psi}{dx}(x>0)$ continuously

& same for $x=a$

what E's work?

Start from $E=0$ & $\uparrow$

For lowest energies, start w/ least # of oscillations



For this E_i, ψ acts ok in all regions but cause $\to \infty$

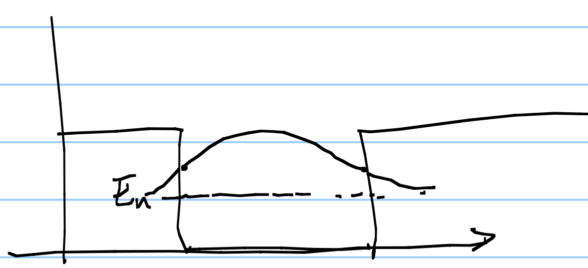
$$\int_{-\infty}^{+\infty} \psi^*\psi\, dx \to \infty$$

Non normalizable!

clearly can't be normalized

so that $E_i \neq$ physical soln to Schro.

but jiggle a little more E_n } & ψ act's normally in all regions & is finite in tunnel regions $\Rightarrow$



That means, not only is ψ_n a soln to Schrö's

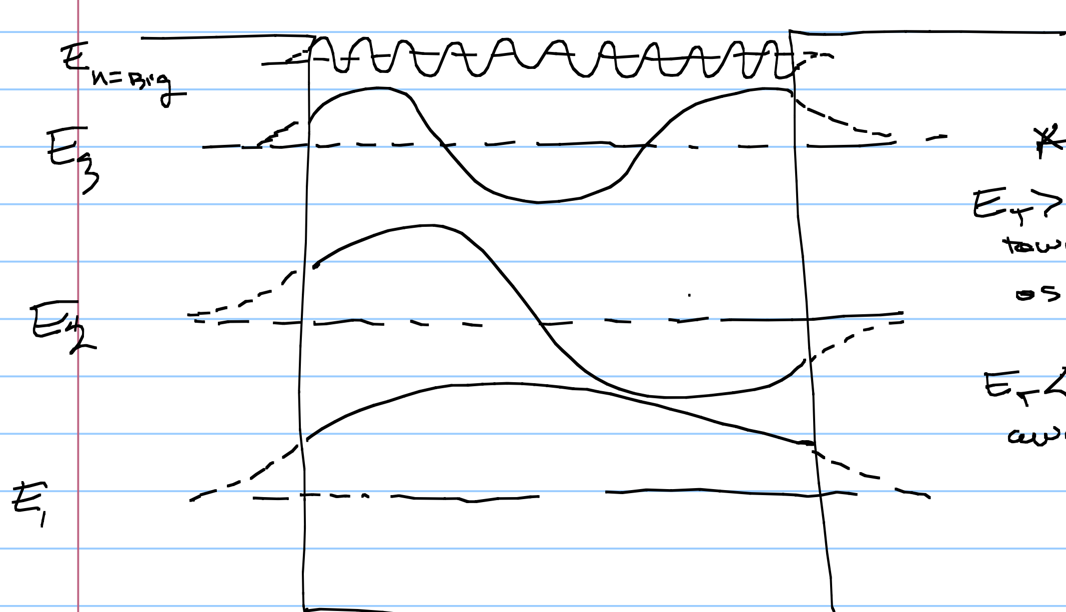
But that ψ_n & its eigen energy E_n can represent physical states of real particles!

NOTE what this ⇒'s

Q.M.'s says not all E_T's work

Only certain ones keep ψ normallizable & only these = physical real states!

Thus Q.M. ⇒'s QUANTIZED ENERGY'S ONLY!



*NOTE

$E_T > V$ = curve toward axis oscillatory

$E_T < V$ = curve away expo