\[
\frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi
\]

2. ... hints:

\[
\begin{align*}
\frac{k^2}{m} &= E/m \Rightarrow \\
-x/kx &= m^2 \\
\Rightarrow \quad x^2 + \frac{k^2}{m} x &= 0 \\
\Rightarrow \quad x^2 + \omega_0^2 x &= 0 \\
\omega_0 &= \sqrt{k/m} \\
\end{align*}
\]

Solve

Cleanest soln \Rightarrow \quad x(t) = \text{Amp} \cos (\omega_0 t + \phi)

see Marian Thornton!

New look @ time dep. solns:

\[
\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V] \psi = 0
\]

\[\psi'' + \omega_0^2 \psi = 0\]
\[ \psi^n + \omega^2 \psi = 0 \quad \omega^2 = \frac{2m}{\hbar^2} [E_T - V] \]

If \( E_T > V \) then soln

\[ \psi(x) = A \text{exp} (\cos (\omega_0 x + \delta)) \]

But if \( E_T < V \)

\[ \psi(x) = \text{exponential!} \]

Further: try looking at 2nd deriv's & concave up!

\[ \psi'' = -\omega_0^2 \psi \]

\[ \frac{d^2 \psi}{dx^2} = -\omega_0^2 \psi \quad \omega_0^2 = \frac{2m}{\hbar^2} [E_T - V] \]

**E > V**  
\[ \psi \text{ curves toward axis} \quad \omega_0^2 = + \]

\[ |\psi| = + \]

\[ \frac{d^2 \psi}{dx^2} = (-)(+) = - \]

\[ |\psi''| = + \]

**E < V**  
\[ \psi \text{ curves away from axis} \quad \omega_0^2 = - \]

\[ |\psi| = - \]

\[ \frac{d^2 \psi}{dx^2} = (-)(-) = + \]

\[ |\psi''| = - \]
So \[ -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \]

For \( E > V \): \( \psi = \text{oscillatory} \)
always curving toward x-axis

\[ E < V : \psi = \text{Exp curving away} \]
from axis

\[ \psi \text{ mean cases:} \]

1. \( E > V \) in a region = "FREE"
2. \( E_{\text{free}} > V(x) \) in a region = "BOUND"
3. \( E > V \) Sine on one side, \( E \leq V \) = Tunneling
4. \( E < V \) Sine on one side, Bound on other = Scattering!
\[-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi = E \psi\]

Further ... have a seed for \(\psi\), but what about \(E\) will \(E > V\) we know curves toward axis & oscillatory

so must solve for \(\psi\) everywhere!

what \(E\)'s work?

Start from \(E = 0 \frac{d}{dx}\)

For lowest energies, start with least # of oscillations

For \(E_i\), \(\psi\) acts ok in all regions but cause \(\rightarrow 0\) clearly can't be normalized:

\(\int |\psi|^2 dx \rightarrow \infty\)

so that \(E_i \neq \text{physical solvent to Schrödinger}\)

but juggle a little more \(E_n\)

\(\frac{d}{dx}\) \(\psi\) acts normally in all regions & is finite in tunnel regions
That means, not only is $\psi_n$ a solution to Schrödinger's equation $E_n + \psi$, its energy $E_n$ can represent physical states of real particles!

**Note what this $\Rightarrow$ signifies:**

QM's says not all $E_n$'s work.

Only certain ones keep $\psi$ normalizable.

Only these $\Rightarrow$ physical real states!

Thus QM $\Rightarrow$ QUANTIZED ENERGY'S ONLY!

[Diagram showing energy levels $E_1$, $E_2$, $E_3$, $E_n$, with notes: $E_n > V$ = curve toward axis oscillating, $E_n < V$ = curve away from axis.]