E. Tr. Deveney / BSC Physics PH460 QM II : QFT - Dirac Equ.

Note Title 1/24/2007

A flavor of $\underbrace{\text{Relativistic Quantum Field Theory: rQFT}}_{\text{most successful theory known to man}}$

photon's = "Quanta" particles of $F^{\mu\nu}$ field

e^-'s = "Quanta" particles of Ψ field.

Everything = Fields
particles = Quanta of fields!

See "Teaching Quantum Physics w/o Paradoxes"
Art Hobson, TPT Vol. 45 Feb 2007
pg 96 + 99

and "Electrons as field quanta: A better way to teach quantum physics in intro gen phys courses"
Art Hobson, AJP 73, 630-634
July 2005

Normally: 1.) Start w/ $\mathcal{L}$
2.) use E-L equat of motion
3.) get Dirac equat

all 4-vectors.

Here just get to it from Dirac.

Note: Natural units: **Side Note**

$$E_T^2 = p^2c^2 + m_0^2c^4$$

so $\boxed{mc^2 \text{ units of energy}}$

now $\underbrace{[\hbar]}_{\text{dim}} = \text{energy} \cdot \text{sec} \equiv 1$

$\therefore [\text{sec}] = \dfrac{\hbar}{mc^2}$

$[d] = [c][t]$; $[d] = c\dfrac{\hbar}{mc^2} = \dfrac{1}{m}\left(\dfrac{\hbar}{c}\right)$

So

m = Fund unit

$\hbar \equiv c \equiv 1$

m

$E = (m)(c^2)$ $\qquad t = \dfrac{1}{m}\left(\dfrac{\hbar}{c^2}\right)$ $\qquad d = \dfrac{1}{m}\left(\dfrac{\hbar}{c}\right)$

if m is given in energy units (ie MeV or GeV)

m (MeV, GeV) $\equiv m_{\text{Energy}}$

$E = m_E$ $\qquad t = \dfrac{\hbar}{m_E}$ $\qquad d = \dfrac{c\hbar}{m_E}$

From Rolnick: Intro to Particles & Fields.

$$\hat{H}\Psi = i\hbar \frac{\partial \Psi}{\partial t} \equiv \text{Equat of Motion (EOM)}$$

use Dirac idea: Force $E_T = \left[p^2c^2 + m_0^2 c^4\right]^{1/2}$

to

$$E_T = c\underset{\sim}{\vec{\alpha}} \cdot \vec{p} + \underset{\sim}{\beta}\, m_0^2 c^4$$

natural units $c = \hbar = 1$

$$\hat{H} = E_T = \underset{\sim}{\vec{\alpha}} \cdot \vec{p} + \underset{\sim}{\beta}\, m^2$$

$\vec{p} = i\hbar\vec{\nabla}$, $i\vec{\nabla}$

$$\left(\underset{\sim}{\vec{\alpha}} \cdot \vec{p} + \underset{\sim}{\beta}\, m^2\right)\Psi = i\frac{\partial \Psi}{\partial t}$$

?(-)

$$\left[i\frac{\partial}{\partial t} + i\underset{\sim}{\vec{\alpha}} \cdot \vec{\nabla} + \underset{\sim}{\beta}\, m^2\right]\Psi = 0$$

recall 4-vect, space time deriv

$$\partial_\mu \equiv \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$ ---- maybe rewrite all w/ 4-vectors. is there another?

so maybe try

$$\beta \left(\text{everything}\right)$$

Convention in QFT

$$\left[i\beta\frac{\partial}{\partial t} + i\beta\underset{\sim}{\vec{\alpha}} \cdot \vec{\nabla} + \beta^2 m\right]\phi = 0$$

New new 4-vector $\underset{\sim}{\gamma}^{\mu} \equiv (\beta, \beta\alpha^i) = (\gamma^0, \gamma^i)$

Dirac
or
Gamma matrices

So γ^{μ} = 4-vector just as

$$x^{\mu} = (x^0, x^i) = (ct, \vec{x})$$

* convent: μ, ν, γ
run over 4-space-time
coords

i, j, k run over 3-space
coords

so γ^0 = scalar
$\gamma^i = \gamma^{1,2,3}$ = vector

$$\gamma^{\mu} \text{ turn out } = \underset{(\beta,\, \beta\alpha^i)}{(\gamma^0, \gamma^{1,2,3})} = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \beta\sigma^1, \beta\sigma^2, \beta\sigma^3 \right)$$

$\sigma^1, \sigma^2, \sigma^3$
= Pauli Spin
matrices.

then rewrite as
entire Dirac Equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\phi = 0$$

And

$(i\gamma^\mu \partial_\mu - m)\phi = 0$ ≡ relativistically covariant Dirac Equation.

Further: Feynman:

$\gamma^\mu (\text{anything})_\mu$ = 4-vector dot prod w/ γ^μ

$\equiv \cancel{(\text{anything})}$ = "Slash"

so

Dirac Equation

$$(i\cancel{\partial}_\mu - m)\phi = 0$$

* Story: Bethe (I think) said after Conference w/ Schwinger & Feynman that he (Tried!) worked on Schwinger's Formalism but tough & all could rember of Feynman's Talk was introducing Funny diagrams & some slash notation!

Since $\not\partial_\mu$ built from $\gamma^\mu = \left(\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}, B\sigma^i \right)$

it forces

$$(i\not\partial_\mu - m)\phi = 0$$

$$\phi \Rightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

ie 4 solution's

$$\begin{pmatrix} \phi_1 & \\ & \text{w/ } +E \\ \phi_2 & \\ & \\ \phi_3 & \\ & \text{w/ } -E \\ \phi_4 & \end{pmatrix} \Rightarrow \begin{pmatrix} e^-, \uparrow \\ e^-, \downarrow \\ e^+, \uparrow \\ e^+, \downarrow \end{pmatrix}$$

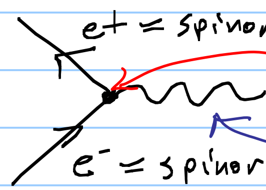
ϕ's = Soln to Dirac equat ≡ "SPINORS"
carries spin

Can see: SPIN = ? don't know, But relativity demands it
(Schrö ≠ relativistic) had to add spin in

Feynman diagrams

e+ = spinor = "carries spin"

vertex: must → have properties to "pass" spin + other props.



propagator = Greens funct soln to $(i\not\partial_\mu - m)\phi$ = unit impulse

e⁻ = spinor