E.F.Deveney, PH 460 Quantum II
BSC/Physics, Spring 07, Exam I – closed book take home

Rules:

- When ready, it is just you versus the exam.
- This is a take-home, closed everything (texts, homeworks, notes… everything) exam. Please take no more than 2 continuous hours - no breaks in between).
- Please make this as neat as possible and do not use two sides of the same piece of paper. Please put all problems in order. **Show all work and I will deduct points if good sketches with well-defined variables are not used. You must show all unit work.**
- You are allowed to use hints from the web site. **However,** please let me know in your test when you are using a hint. The idea is that ideally you should not need the hint so I will deduct a tiny bit when you do.

Strategy:
There are 5 problems (all of different values so please keep an eye out). Some may be harder than others so be wise with your time - As always, if you get stuck on one, quickly construct aluminum framed pyramid and try the yoga position while burning incense. If this does not work, move on to the next problem.
There is a 5 point bonus question.
Good luck, Prof. Deveney (Ed)

Formula:
Special relativity:

$$E_{tot} = \gamma m_0 c^2 = \sqrt{p^2c^2 + m_0^2c^4}\ ,\ \vec{p} = \gamma_u m_0 \vec{u}\ ;\quad E_{kinetic} = (\gamma - 1)m_0c^2\ ,\ \gamma_u = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

Waves:

$E_{tot} = \frac{hc}{\lambda} = \hbar\omega$, $p = \frac{h}{\lambda} = \hbar k$, $\omega = \frac{2\pi}{T} = 2\pi f$, $k = \frac{2\pi}{\lambda}$, Plane wave = $Ae^{i(kx-\omega t)}$

Charged particles in electric potential fields:

$$E_{potential} = qV\ ,\ Volt = V \equiv \frac{J}{C}$$

Modern Physics:

DeBroglie: $\lambda = \frac{h}{p}$, Heisenburg; $\Delta x \Delta p \geq \frac{\hbar}{2}$ and $\Delta E \Delta t \geq \frac{\hbar}{2}$

Constants:

$h = 6.62\times10^{-34}\ J\cdot\sec$; $m_{e^-} = 9.11\times10^{-31}\ kg$, $m_p = 1836 * m_{e^-}$, $q_{e^-} = -1.6\times10^{-19}\ C$

Calculus and trig: $\int u dv = uv - \int v du$; $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

Problem #1 (10 points):
Write Schrödinger's time dependent and time independent wave equations, the Klien-Gordon (K-G) equation and Dirac equation.

- For each specify to what extent spin (if spin, identify what type of spin eg, 0 or ½) and relativity are incorporated into each.
- HINT: This is standard hint for everyone – the K-G equation was 1st attempt at relativistic equation and started by using $(E_{tot})^2$. Additional standard hint: go back to the general idea that we used to 'build' wave equations working from the general plane wave solution, expression for p, and energy of a wave (see formula on prev. pg.).
- Next, reduce the K-G equation for the case of a massless particle. Comment on what this final form is.

Problem #2 (Four not so-long, 5 point problems):
#2a: In a complete CAV (complex abstract vector space) one can expand a general state ket as follows:

$$\left|\Psi\right\rangle = \underline{1}\left|\Psi\right\rangle \qquad \text{where } \underline{1} \equiv \sum_{all\,'base'\,kets} \left|'base'\,ket\right\rangle\left\langle 'base'\,ket\right| .$$

For a 2-dimensional spin space, say of up and down spin base kets, show what the expansion of the general ket looks like and describe what each term means. Include a brief discussion of the connection to the regular 'vector space' analogy of this expansion.

#2b: In an up/down spin ½ basis, a general ket is found to be:

$$\left|\Psi\right\rangle = 3\left|\uparrow\right\rangle + 4i\left|\downarrow\right\rangle .$$

- What is the significance of this 'superposition' solution and in particular, the appearance of the complex 'i'?
- Construct the 'bra' space representation of the general ket.
- Is the general ket normalized?
- If not, normalize it.

#2c: Lowering and raising operations were defined as follows (and respectively):

$$\hat{S}_-\left|s,m_s\right\rangle = \hbar\sqrt{s(s+1) - m_s(m_s - 1)}\left|s,m_s - 1\right\rangle$$
$$\hat{S}_+\left|s,m_s\right\rangle = \hbar\sqrt{s(s+1) - m_s(m_s + 1)}\left|s,m_s + 1\right\rangle$$

Please apply both operators to each spin state of a spin ½ particle, i.e. $\left|\frac{1}{2},\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2},\frac{-1}{2}\right\rangle$ and briefly discuss results.

#2d: At time =0, a $\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|z\right\rangle + \left|-z\right\rangle\right)$, which we recognize as 'x' state ket in a +/- 'z' basis, evolves in some later time to

$$\left|\Psi(t_{later})\right\rangle = \begin{pmatrix} \dfrac{e^{\frac{i\omega 2\pi}{2\omega}}}{\sqrt{2}} \\ \dfrac{e^{-i\pi}}{\sqrt{2}} \end{pmatrix}.$$

Please factor this result (factor implies pull out common amplitudes and phases).
What does your final factored answer for the general ket at this later time mean and why is it helpful to do this factoring?

Problem #3 (10 points, might take some time): Give the Pauli Spin Matrices in a +/- z basis: $\underline{\vec{\sigma}} = (\sigma_x, \sigma_y, \sigma_z) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

And spin operators $\underline{\hat{\vec{S}}} = \frac{\hbar}{2}\underline{\vec{\sigma}}$.

Please find the eigen-values and vectors for $\hat{S}_x$ all in the +/- z basis. General Hint: this will 1st require setting up the eigen-equation, $\hat{S}_x\left|\Psi_x\right\rangle = c\left|\Psi_x\right\rangle$, then solving for the eigenvalues and vectors.

Problem #4 (10 points, this should be quick):

A Stern-Gerlach device in the X-Y plane and mounted at an angle theta with respect to the x-axis prepares a spin ½ particle in a general state ket in the +/- z basis given by:

$$\left|\Psi_\sigma\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle + e^{i\sigma}\left|\downarrow\right\rangle\right).$$

If this particle is then directed through a Stern-Gerlach device mounted purely in the x-direction, what is the probability of detecting particles in the $\left|-x\right\rangle = \frac{1}{\sqrt{2}}\left(\left|z\right\rangle - \left|-z\right\rangle\right)$ state? Please show a sketch as best you can.

Problem #5 (15 points, might take a while):

Consider a sin ½ particle initially prepared in a $\left|\Psi(t=0)\right\rangle = \left|\uparrow\right\rangle$ state (defined in a +/- z basis) in a homogeneous external magnetic field in the z direction given by $B_0\hat{z}$.
Given that the energy of a magnetic dipole in an external field is:
$E = \vec{\mu}\cdot\vec{B}$ where the magnetic dipole due to a general angular momentum, $\vec{L}$, is given by

$$\vec{\mu}_{\vec{L}} = \frac{gq}{2m}\vec{L},$$

Please compute $\left|\Psi(t)\right\rangle$. Hint: this will require solving the time dependent Schrödinger equation just as we did in class.

BONUS (5 points):
Please compute the precession frequencies (called Larmor frequencies) for both an electron and proton in an external homogeneous magenetic field of order 1 Tesla. Use $g_e \cong 2$ and $g_p \cong 5$ (these are what I have been calling the gyromagnetic spin factors but they are actually called the Lande' factors).

RULES: no calucators!