we have Max's

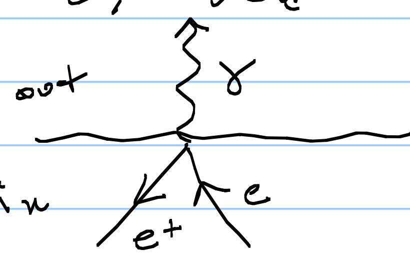
$$\vec{\nabla}\cdot\vec{E} = \rho_{free}/\varepsilon_0 + 0$$
$$\vec{\nabla}\times\vec{E} = 0 + -\frac{d\vec{B}}{dt}$$
$$\vec{\nabla}\cdot\vec{B} = 0 + 0$$
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}_{free} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

and just found $\vec{E}$ & $\vec{B}$ fields can carry
Energy
$\vec{p}$
& $\vec{L}$

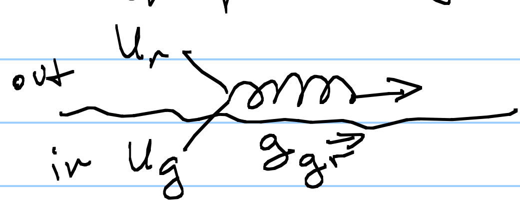
and play crucial role in

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho_{conserved}}{\partial t}$$

ie ρ_{charge} ⇒'s E&M field $(\vec{B})$
cons of E-charges

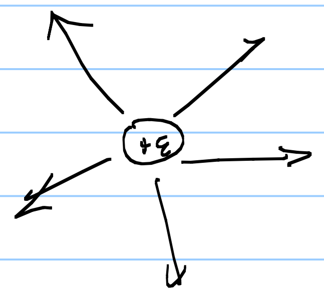


$\rho_{color\ charge}$ ⇒ gluon field



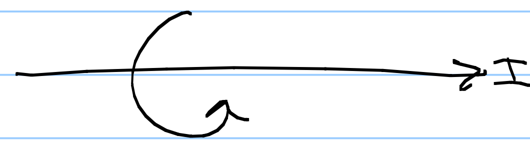
* note:
These Feynman diagrams used, but more

Maxwell's equations clearly "cover"



$\vec{E}(\vec{r})$

&



$\vec{B}(\vec{r})$

static "sources"

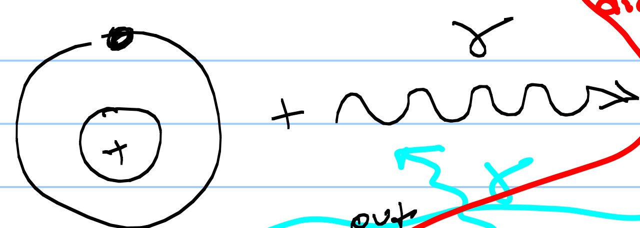
$$\frac{\rho_{free}}{\varepsilon_0} \quad \& \quad \mu_0 \vec{J}_{free}$$

But now consider the dynamic sources!

$$\frac{d\vec{B}}{dt} \quad \& \quad \frac{d\vec{E}}{dt}$$

ex:

$|i\rangle = 2p$ $\qquad$ $|f\rangle = 1s + \gamma$



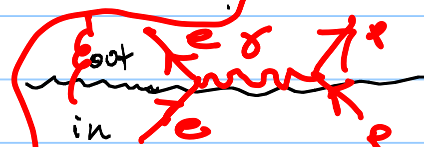
Later

These $\vec{E}$ & $\vec{B}$ are mediated by photons

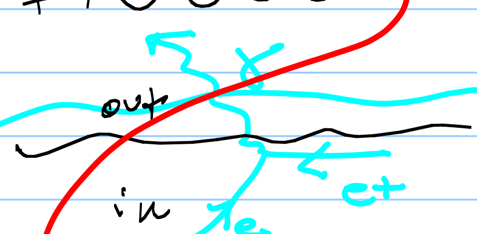
But they are

1) virtual

2.) longitudinal



= internal to Feynman diagrams



here γ's = 1) REAL

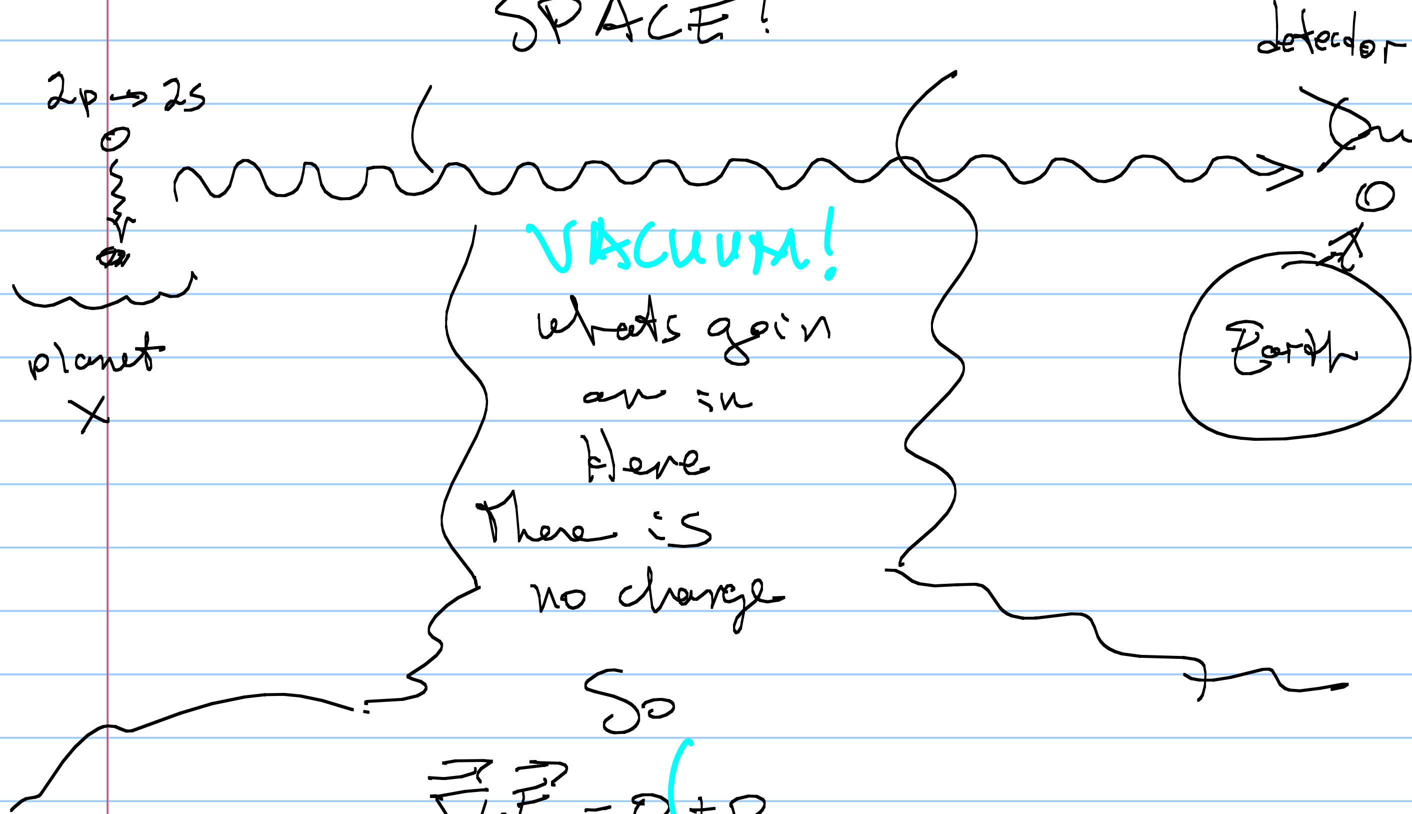
2.) transverse

& 3.) Are external lines in Feynman diagrams

That's a bit later.....

here we will concentrate on

SPACE!



So

$$\vec{\nabla}\cdot\vec{E} = 0 + 0$$

$$\vec{\nabla}\times\vec{E} = 0 + -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla}\cdot\vec{B} = 0 + 0$$

$$\vec{\nabla}\times\vec{B} = 0 + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$$

ie
no static charges
no currents!

so
static same

Can this lead to Vacuum "supporting" Real E&M waves?

Well, $\vec{\nabla}\cdot\vec{E}=0$

$$\left.\begin{array}{l} 2\ \vec{\nabla}\times\vec{E}=-\dfrac{d\vec{B}}{dt} \\ 3\ \vec{\nabla}\cdot\vec{B}=0 \\ 4\ \vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} \end{array}\right\}$$

= "Coupled" 1^{st}-order PDE's in free space!

What to do? Uncouple them!

How? Clever!

Recall $\vec{A}\times\vec{B}\times\vec{C}=\vec{B}(\vec{A}\cdot\vec{C})+\vec{C}(\vec{A}\cdot\vec{B})$

How does that help? Do it!

$\vec{\nabla}\times 2$

$\vec{\nabla}\times 4$

$\vec{\nabla} \times 2$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

A B C

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}\psi = \frac{\partial\psi}{\partial x}\hat{i} + \frac{\partial\psi}{\partial y}\hat{j} \leftarrow \text{grad}$$

vect

Then

$$\vec{\nabla}\cdot(\vec{\nabla}\psi) = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \text{scal}$$

div. grad

$$\nabla^2\psi = \text{scalar}$$

now

$$(\vec{\nabla}\cdot\vec{\nabla})\vec{V} = \left[\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)\cdot\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right)\right]\vec{V}$$

$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\vec{V} = \nabla^2\vec{V} = \text{vector!}$$

so

$$\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

use #1

$\vec{\nabla}\cdot\vec{E} = 0$ in free space!

scal products are comutative

use #4

$$-\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$\vec{\nabla} \times 4$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

A B C

$$\vec{\nabla}(\cancel{\vec{\nabla} \cdot \vec{B}}) - \vec{B}(\vec{\nabla} \cdot \vec{\nabla}) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\underbrace{(\vec{\nabla} \times E)}_{\text{use } 2}$$

use #3 so

$$-\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{d\vec{B}}{dt}\right)$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

So, in free space (i.e. no charges
no currents)

Max's Equat's say

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\& \quad \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

recall our funny $\nabla^2 \vec{E}$ $\Rightarrow$'s

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E} \qquad \text{or}$$

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) = \left(\nabla^2 E_x = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}\right)\hat{i}$$

$$\left(\qquad \qquad \right) = \left(\nabla^2 E_y = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}\right)\hat{j}$$

$$\left(\qquad \qquad \right) = \left(\nabla^2 E_z = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial z^2}\right)\hat{k}$$

& same for $\vec{B}$

So what are $\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \dfrac{\partial^2 \vec{E}}{\partial t^2}$

& $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial^2 \vec{B}}{\partial t^2}$

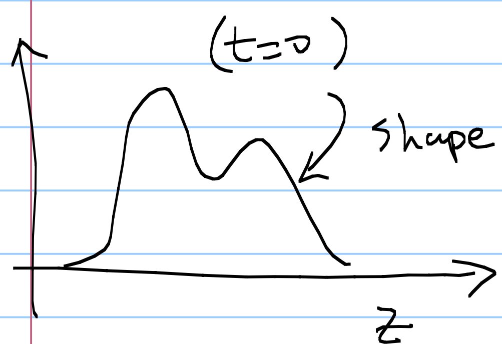
of course recognize @ traveling waves!

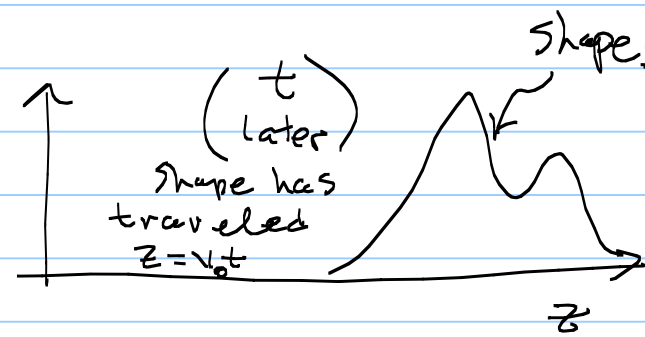
(actually 3 (each) 1-D traveling waves)

1-D Traveling wave:

Traveling wave: a disturbance (displacement, shape or function) that travels @ constant velocity w/o changing shape!

So





So is

$$\boxed{\text{shape} \equiv f(\underset{\uparrow \text{ posit}}{z}, \ \& \text{ time})}$$

& travelling @ $V_0 (\hat{z})$

$\Longrightarrow$ is mathematically IF you have

$\underbrace{f(z,t)}_{\text{shape @ z \& @ some t}}$ and it $= \underbrace{f(z - V_0 t, 0)}_{\text{shape @ orig}}$ then

z is moving $\rightarrow$ @ $V_0 (\hat{z})$

and $t=0$

$f(z,t)$ is really just a shape or disturbance (constant) traveling $\rightarrow$

so

<u>if</u> $f(z,t) = f(\underbrace{z-vt}_{\text{pos}}, \underset{\uparrow \text{ time}}{0}) = f(z-vt)$

Then you have a traveling wave!

$f(z-vt)$ = shape or disturbance in z direction traveling $\longrightarrow$ @ v

clearly

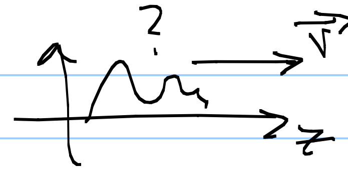
$f(z,t) = f(z+vt)$ = shape in z traveling $\leftarrow$ @ v

So, clearly not all functions are traveling waves!

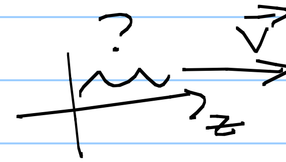
look for
$$\left.\begin{aligned} f(z,t) &= f(z-vt) \\ f(y,t) &= f(y-vt) \end{aligned}\right\} \text{ got traveling waves!}$$

ex!

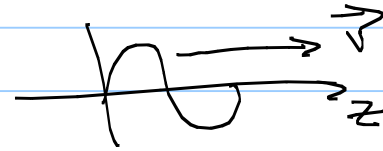
$$S_1(z,t) = A e^{-b(z-vt)^2} =$$



$$S_2(z,t) = \frac{A}{b(z-vt)^{1.5}+1} =$$



$$S_3(z,t) = A \sin\left[b(z-vt)\right] =$$



But

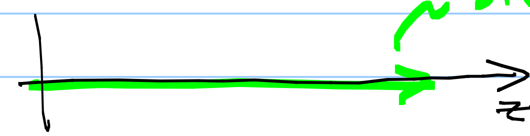
$$S_4(z,t) = A e^{-b(z^2-vt)}$$ X not

$$S_5(z,t) = A \sin(bz)\cos(bvt)^3$$ X not!

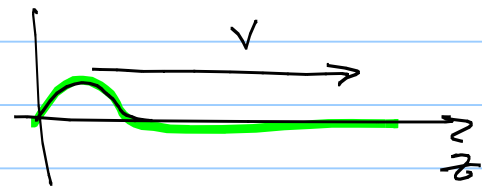
A great example is a wave traveling
on a rope
or in air

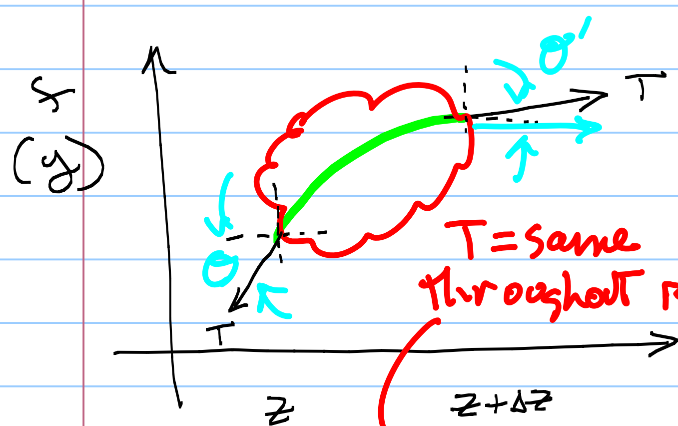
rope: why does stretched string support traveling waves?

Pluck a string



Notice: pulse travels
medium (string)
↕ but not ⟷





T = same throughout rope

otherwise small pieces of rope would be in ⟵⟶, ⟶⟵ oscillations themselves

$\Sigma \vec{F} = m\vec{a}$

$\Sigma F_y = m\ddot{y}$

$\Sigma F_z = m\ddot{z}$ = 0 — medium doesn't ⟷ so

$T\cos\theta' - T\cos\theta = 0$

$\theta' \neq \theta$ but small
so $\cos\theta' \approx \cos\theta$

$+\uparrow \quad \sum F_y = m\ddot{y}$

$$T\sin\theta' - T\sin\theta = \mu\,\Delta z\,\frac{d^2y}{dt^2}$$

where $\mu = \frac{\text{mass}}{\text{length}}$

& Δz = segment of rope

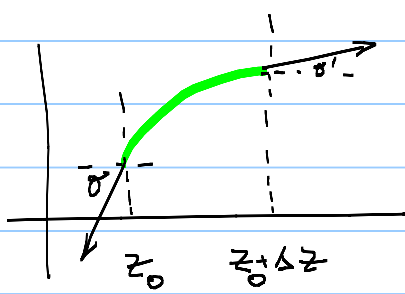
$T(\sin\theta' - \sin\theta)$

again $\theta' \neq \theta$

trick $\tan\theta = \frac{\sin\theta}{\cos\theta}$

for small θ $\quad \cos\theta \cong 1 \quad$ so $\quad \tan\theta \cong \sin\theta$

$$T(\tan\theta' - \tan\theta) = \mu\,\Delta z\,\frac{d^2y}{dt^2}$$



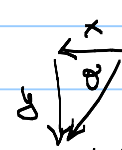
so

ah..... want $S(z_0+\Delta z)$ about z_0

i.e.

Taylor series $\Rightarrow S(z_0+\Delta z) \cong S(z_0) + \left.\frac{dS}{dz}\right|_{z_0}\Delta z$

$S(z_0) = \tan\theta$



$\tan\theta = \frac{y}{x}$ = instant slope @ z_0

$$S(z_0) = \tan\theta = \left.\frac{dS}{dz}\right|_{z_0}$$

so $$f(z_0+\Delta z) \simeq f(z_0) + \left.\frac{df}{dz}\right|_{z_0} \Delta z$$

now

$$f(z_0) = \boxed{\tan\theta = \left.\frac{df}{dz}\right|_{z_0}}$$

So

$$f(z_0+\Delta z) = \tan\theta' = \left.\frac{df}{dz}\right|_{z_0} + \frac{d}{dz}\left(\left.\frac{df}{dz}\right|_{z_0}\right)\Delta z$$

$$+\uparrow \sum F_y = m\ddot{y}$$

so

$$T(\tan\theta' - \tan\theta) = \mu\,\Delta z\,\frac{d^2f}{dt^2}$$

$$T\left(\cancel{\frac{df}{dz}} + \frac{d^2f}{dz^2}\Delta z - \cancel{\frac{df}{dz}}\right)\Bigg|_{z_0} = \mu\,\Delta z\,\frac{d^2f}{dt^2}$$

$$T\,\frac{d^2f}{dz^2}\cancel{\Delta z} = \mu\,\cancel{\Delta z}\,\frac{d^2f}{dt^2}$$

or

$$\frac{d^2f}{dz^2} = \frac{\mu}{T}\frac{d^2f}{dt^2}$$

recall

$$\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

Now what?

we, pluck a rope & get

$$\frac{\partial^2 \xi}{\partial z^2} = \frac{\mu}{T}\frac{\partial^2 \xi}{\partial t^2} = ?$$

$\mu = \frac{\text{mass}}{L}$

$T = \text{Force}$

$$= \frac{\text{kg}/\text{m}}{\text{kg m}/\text{s}^2} = \frac{\text{s}^2}{\text{m}^2}$$

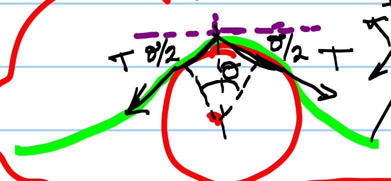
so note $\sqrt{\frac{1}{\mu/T}} = \sqrt{\frac{T}{\mu}} = \frac{\text{m}}{\text{s}} =$

$=$ velocity!

so

$$\frac{\partial^2 \xi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}$$

Also



$$\Sigma F_{rad} = -\frac{mv^2}{r}$$

$$-2T\sin\theta/2 = \mu \Delta s \frac{v^2}{r}$$

$\cong$

$$\cancel{2}T\theta/\cancel{2} = \mu \Delta s \frac{v^2}{r}$$

$$T\cancel{\theta} = \mu \cancel{R}\cancel{\theta}\, v^2/\cancel{R}$$

$$v = \sqrt{\frac{T}{\mu}}$$

Now recall $\xi(z - vt)$ = traveling wave traveling @ v ⟶

Is $\xi(z-vt)$ = ALL travling waves ALSO is soln to

we identify traveling waves w/ plucked strings & $\vec{E}$ & $\vec{B}$ fields in

FREE SPACE!

So check if $f(z-vt)$ a soln to $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$?

how would you know?

Substitute & see if → works!

So

assume: $f(z,t) = g(z-vt)$ ---- traveling wave

$g(z,v,t) \cdot \underbrace{u(z-vt)}$

g any other — special combo of z, v, t

g can be anything!

check

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

Note diff w/ respect to that special funct form

$u(z-vt)$

$$\frac{\partial f}{\partial z} = \frac{dg}{du}\frac{\partial u}{\partial z} = \frac{dg}{du}(1)$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{dg}{du}\right) = \frac{d}{du}\left(\frac{dg}{du}\right)\frac{du}{dz} = \frac{d^2 g}{du^2}(1) \quad (1)$$

Then

$$\frac{\partial f}{\partial t} = \frac{dg}{du}\frac{\partial u}{\partial t} = \left(\frac{dg}{du}\right)(-v)$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{d}{du}\left(-v\frac{dg}{du}\right)\frac{du}{dt} = +v^2\frac{d^2 g}{du^2} \quad (2)$$

1.) $\frac{d^2 g}{du^2} = \frac{\partial^2 f}{\partial z^2}$

2.) $\frac{d^2 g}{du^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

or

indeed

$$\frac{\partial^2 s}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

where rope

$$\frac{\partial^2 s}{\partial z^2} = \frac{1}{\sqrt{\frac{T}{\mu}}}\frac{\partial^2 s}{\partial t^2} \quad ; \quad v = \sqrt{T/\mu}$$

Then $\vec{E}$ & $\vec{B}$

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \underbrace{\frac{\partial^2 E_x}{\partial y^2}}_{\to 0} + \underbrace{\frac{\partial^2 E_x}{\partial z^2}}_{\to 0} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}\right)\hat{i}$$

$$\left(\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}\right)\hat{i}$$

$$v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

μ_0 = permeability $= 4\pi \times 10^{-7}\, N/A^2$

ε_0 = permitivity = $8.85 \times 10^{-12}\, C^2/Nm^2$

$$v = \sqrt{\frac{1}{(4\pi)(8.85)\times 10^{-19}\,\frac{\cancel{N}}{\left(\frac{\cancel{C}}{S}\right)^2}\,\frac{C^2}{\cancel{N}\,m^2}}} \qquad \frac{1}{\frac{1}{\frac{m^2}{s^2}}}$$

$$= \sqrt{\frac{1}{100 \times 10^{-19}}\,\frac{m^2}{s^2}} = \sqrt{10 \times 10^{16}\, m^2/s^2}$$

$$\simeq 3 \times 10^{8}\, m/s$$