Now we've got Maxwell's Equations:

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} + \frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

\[ \nabla \cdot \mathbf{E} \] and \( \nabla \times \mathbf{B} \) of \( \nabla \cdot \mathbf{E} \), \( \nabla \times \mathbf{E} \), and \( \nabla \times \mathbf{B} \)

Completely describe Vector Fields!

(recall \( \nabla \cdot \mathbf{A} = \nabla \times \mathbf{B} \)

\( \nabla \times \mathbf{A} \) span 3-D of those vector fields.)
Now we have the ability to look at problems from 2 equivalent reps.

\[
\begin{align*}
R & \\
L & \\
C & \\
\end{align*}
\]

Work done in building current:

\[\frac{1}{2}LI^2\]

Energy stored in field:

\[\frac{1}{2} \int_0^L \frac{1}{2}B^2 dy\]

Use what you are best suited for problems!

Big Idea: I see a problem with electricity & magnetism if it is useful, you can cast the problem entirely in terms of fields using those equations!
Also: if you don't think about "Fields," you run into problems!

*Conservation Laws*

Need "Fields" to conserve!

So, chapter 8: Gross = properties of Fields!

Let's begin by considering some conservation inconsistencies.

\[ \text{pg 307 Gross ex: 7.8} \]

\[ i \rightarrow \downarrow \uparrow \uparrow \uparrow \text{B} \]

\[ i \rightarrow \text{L} \text{conserved} \]

But here, \( i \rightarrow \text{D} \) \( i \neq 0 \)
2) Why does a Toaster Spark?

\[ \text{Diagram:} \]

\[ I = 0 \]

\[ \text{Diagram:} \]

\[ \omega / I = 0 \]

Where did the energy come from?

Note: \( \vec{E} \) & \( \vec{B} \) are not static, but indeed directions on.

So, \[ |F_{\text{m}}| = |F_{\text{m}}| \]

but not opposite direction.

In violation of Newton's 3rd Law.

Now might not seem too big a deal, but conservation of linear momentum.
Very much depends on cancelation of internal forces

\[ \sum \mathbf{F} = \mathbf{0} = \frac{d}{dt} (\mathbf{p}) \]

\[ \mathbf{P} = \text{const} \]

"is all internal"

How do reconcile these "violations" to conservation of angular momentum, \( \mathbf{L} \)

energy, \( E \)

linear momentum, \( \mathbf{P} \)

The answer of course is ---.....
The Field "Carries" $E$, $\mathbf{B}$ & $\rho$ To Conservation!

Here is basic idea again......

Continuity Equation (general)

$$ \nabla \cdot \mathbf{J} = -\frac{\partial \rho_{\text{conserved stuff}}}{\partial t} $$

If you have stuff that is conserved & it "disappears" (i.e. $\rho_{\text{in}} \neq 0$)

$\Rightarrow$'s a Field is "Sourced"

So creates a Field Current & this carries the conserved quantities!

Ex: Color charge = conserved, so is quark color changes

A field current must be created that conserves color

$1s^+ = \text{red}$, green, color field

$1s' = g + g + r = \text{r}$
So in order to conserve $E$, $\mathcal{P}$ you need SUSY.

Since $\mathcal{P}$, $\mathcal{E}$ $\mathcal{P}$ MUST be conserved based on symmetry arguments (ie no par SQED $X_0 \Rightarrow \bar{\varphi}$ $\varphi_0 \Rightarrow \bar{\varphi}$ $0 \Rightarrow \bar{\varphi} \Rightarrow E$)

you MUST HAVE FIELDS $\Rightarrow$ HAVE FORCES! (ie Standard Model)
OK. $E \cdot B$ yields "corre. $E, E' \frac{E'}{E}$

and combine to conserve $E, E'$

---

**Final results on $E \times B$ fields from Chpt 8**

Energy $E \times B = U_{\text{EM}} = \frac{1}{2} (E_0 E^2 + \frac{1}{4} B^2) = \frac{\text{energy}}{\text{volume}}$

Momentum: $\vec{P} = \varepsilon_0 (\vec{E} \times \vec{B}) = \frac{\text{momentum}}{\text{volume}}$

Angular momentum $= \vec{P} \times \vec{E}_{\text{EM}} = \varepsilon_0 [\vec{P} \times (\vec{E} \times \vec{B})]$

Carried by FIELDS

Great results $\frac{E}{E}$ will yield only. Example $\Rightarrow$
Why need \( \Psi \), \( E \neq 0 \) of fields?

Excited atom interacting with light:

\[ \hat{H}_E = E\Psi + E \Psi = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial \phi^2} - \frac{e^2}{r} \]

Solve \( \frac{d}{dt} \Psi(t, \phi) = \hat{H}_E \Psi(t, \phi) \)

\[ \Psi_{\Psi_0} = e^{-i \hat{H}_E t} R(n) \frac{\psi_{nm}(\phi)}{\sqrt{\hbar}} \]

Now there is Stationary!

They don't change in time.

\[ \text{Stays Forever!} \]

Now is couple to

\[ \hat{H} = \hat{H}_E + \hat{H}_E^{\text{atom}} \]

Now atom can decay cause it is coupled to \( \Psi \) field.
So

\[ \begin{align*}
\psi_1 & \Rightarrow \psi_2 \\
0 & \Rightarrow 0 \\
\Phi & \Rightarrow \Phi
\end{align*} \]

Field

carries
Energy, momentum
& angular moment

Now

\begin{align*}
E_i & = E_0 \\
\vec{P}_i & = \vec{P}_0 \\
\vec{L}_i & = \vec{L}_0
\end{align*}

atom = a low + field
\[ S_0 \in \mathbb{R} \text{ fields} \]

\[ \mu_0 E_0 = \frac{1}{2} (\varepsilon_0 E^2 + \mu_0 B^2) \]

\[ \mu_0 B_0 = \varepsilon_0 (E \times B) = \mu_0 \varepsilon_0 \vec{B} \]

\[ \mu_0 H_0 = \varepsilon_0 \left( \mathcal{E} \times \mathcal{B} \right) = \mu_0 \varepsilon_0 \left( \mathcal{E} \times \mathcal{B} \right) \]

Introduced \( \vec{S} \in \mathbb{R} \): Poynting vector \& plays big role.....

Poynting vector:

\[ \text{Static} \]

\[ \text{Start with charge distribution} \in \mathbb{E} \text{'s} \in \mathbb{B} \text{'s} \]

\[ \text{Now let changes move about} \]

\[ ?= \text{how much work is done by fields on these changes} \]

\[ \text{well } dw = \mathcal{F} \text{d} \ell \]
\[ \frac{dw}{dt} = \mathbf{E} \cdot \mathbf{v} \cdot \mathbf{v} \cdot dt \]

Since
\[
\mathbf{E} = \Phi \mathbf{E} \quad \text{and} \quad \mathbf{v} = \Phi \mathbf{v}
\]

Then
\[
\frac{dw}{dt} = \mathbf{E} \cdot \Phi \mathbf{v} \cdot \mathbf{v} \cdot dt
\]

\[
\frac{dw}{dt} = \mathbf{E} \cdot \Phi \mathbf{v} \cdot dt
\]

or?

\[
\frac{dw}{dt} = \int (\mathbf{E} \cdot \Phi \mathbf{v}) \, dp
\]

This is cool, but \( \int \) … huh

represent in terms of
pure yields …

use Moses' equations
Lots of steps... some magic, yet...

\[ \frac{d\omega}{dt} = \frac{1}{2} \left( 3 \varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \] 

So...

\[ \frac{d\omega}{dt} \text{ = power} \]

So RHS = power supplied by FLED's!

we know \[ \frac{1}{2} (\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) = \frac{\text{Energy}}{\text{Volume}} = \frac{\text{Work}}{\text{Volume}} \]

Introduce \[ \vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \text{ must = Energy} \] (time x Area) = \[ \frac{\text{Power}}{\text{Area}} = \text{Poynting Vector!} \]

So

\[ \frac{d\omega}{dt} = -\frac{\text{dWork}}{\text{dt}} - \int \varepsilon_0 \vec{E} \cdot \vec{dA} \]
Now what does this mean?

\[ \frac{dw}{dt} = \frac{d}{dt} \left( \text{energy} \right) = \oint \mathbf{E} \cdot \mathbf{n} \, dA \]

Here's the deal:

- Power that leaves the surface, i.e., in Maxwell's waves: \( \mathbf{E} \times \mathbf{H} \)
- Power that enters the volume, i.e., for changes in energy density of field to make things like current & voltage = power

\[ \epsilon \frac{d\mathbf{E}}{dt} = \frac{d}{dt} \left( \text{energy} \right) \]

Stay within volume.
So, in chpt 9 an EM waves,

Starting pts one

1) \( \psi_{EM} = \frac{1}{2} (\varepsilon_0 E^2 + \mu_0 B^2) = \frac{\text{energy}}{\text{vol}} \)

2) \( S = \frac{1}{6} (E \times B) = \frac{\text{energy}}{(\text{Area} \times \text{time})} = \frac{\text{power}}{\text{Area}} \)
H.W.:

Example problems:

Ex: 7.8

Ex: 8.1 using $S$

Ex: 8.3 $P_{EM} \rightarrow$ hidden momentum

Ex: 8.4 $l_{EM} \rightarrow$ no hidden momentum

where did \( L \) come from