E, P & L in FIELDS

Wow! we've got Maxwell's Equations

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_s}{\varepsilon_0} + 0$$

$$\vec{\nabla}\times\vec{E} = 0 + \frac{-d\vec{B}}{dt}$$

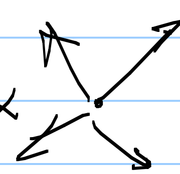
$$\vec{\nabla}\cdot\vec{B} = 0 + 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}_s + \mu_0\varepsilon_0 \frac{d\vec{E}}{dt}$$

static source | dynamic sources

of $\underbrace{\vec{\nabla}\cdot \text{ \& } \vec{\nabla}\times}$ $\vec{E}$ & $\vec{B}$

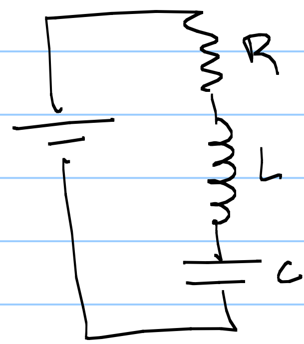
<u>Completely</u> describe Vector fields!

(recall $\vec{\nabla}\cdot \propto$  $\hat{r}$

& $\vec{\nabla}\times \propto \circlearrowleft \propto \hat{\theta}, \hat{\phi}$

so $\vec{\nabla}\cdot$ & $\vec{\nabla}\times$ span 3-D of these vector fields

Now we have the ability to look @ problems from 2 equiv reps



Work done in building current

$= \frac{1}{2} L I^2$

Energy stored in field

$= \frac{1}{\mu_0} \int |B|^2 d\tau$

use what one is best suited for problem!

Big Idea: If see a problem w/ Electricity & magnetism, if it is useful, you can cast the problem entirely in terms of fields using Max's Equations!

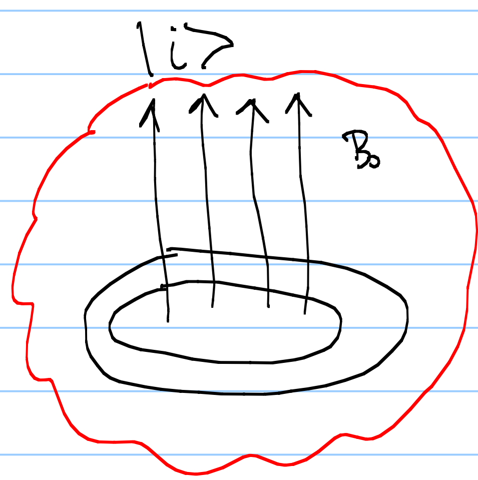
Also: if you don't think about "fields", you run into problems!

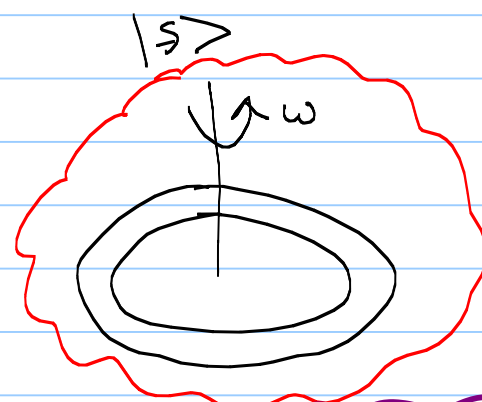
(ie Conservation laws Need "fields" to Conserve!)

So, chapt 8 Griff = properties of Fields!

Lets Begin by considering some conservation inconsistencies!

1.) pg 307 Griff ex: 7.8





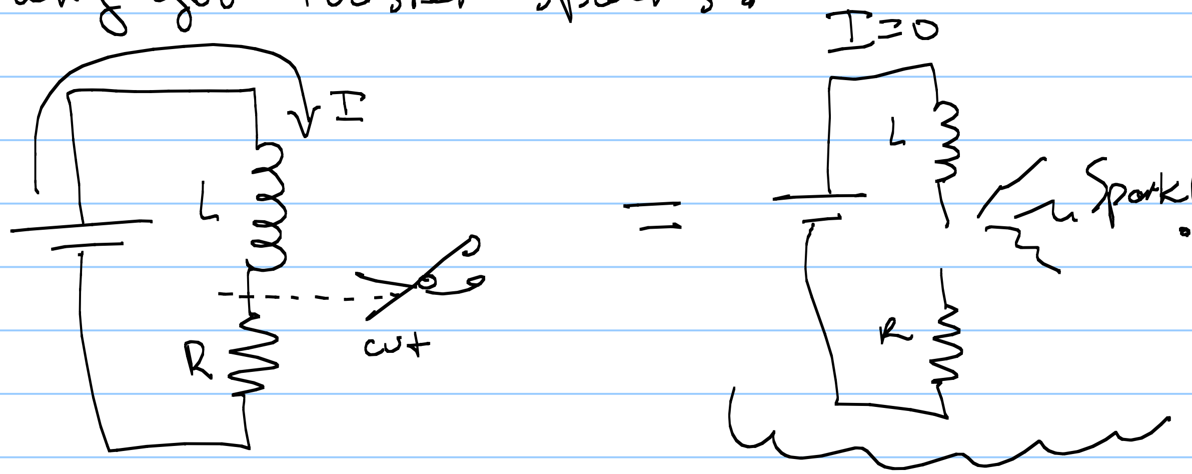
$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

$$0 = \frac{d\vec{L}}{dt}$$

So $\vec{L}$ = conserved
But here, $L_i = 0$
$L_f \neq 0$

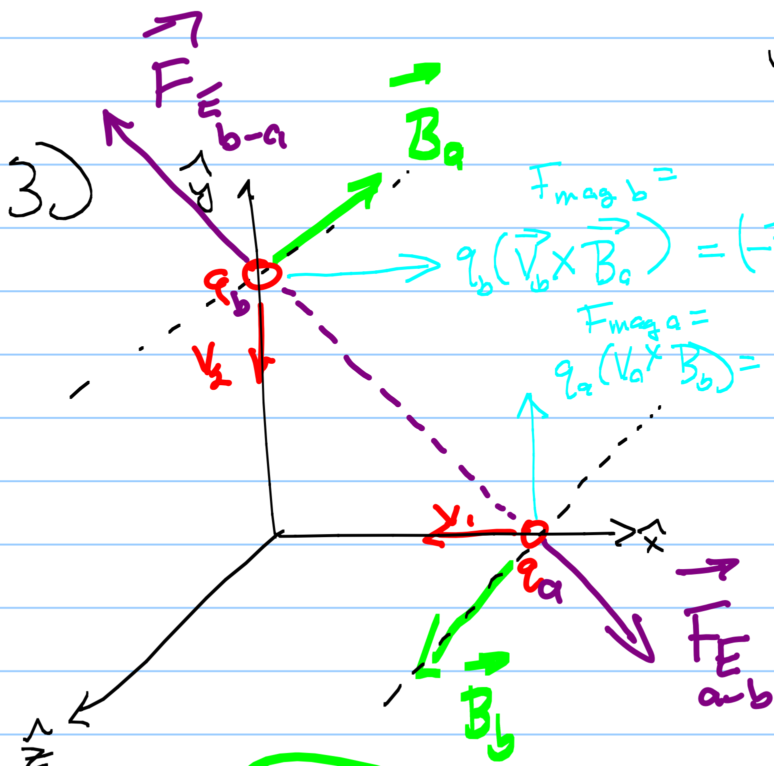
?

2) Why you Toaster Sparks?



w/ I=0, where did Energy come from?

3)



$F_{mag\,b} =$

$q_b(\vec V_b \times \vec B_a) = (-\hat x)$

$F_{mag\,a} =$

$q_a(V_a \times B_b) = (+\hat y)$

Note: $\vec E$ & $\vec B$ not static, but indeed directions ok Gauss

so

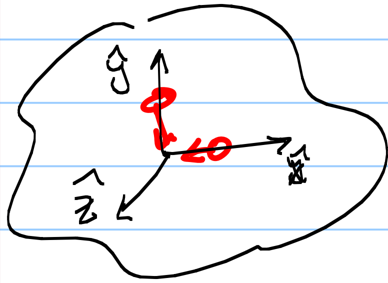
$|F_{m_b}| = |F_{m_a}|$

but not opposite direction!

In violation of Newton's 3rd Law

Now might not seem too big a deal BUT conservation of linear moment

Very much depends on cancelation of internal Forces



$$\sum \vec{F} = m\ddot{x} = \frac{d}{dt}(\vec{p})$$

$$\underbrace{0}_{\text{is all internal}} = \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{p} = \text{constant!}$$

conserved.

How do reconcile these "violations" to

Conservation of angular momentum, $\vec{L}$

" " energy, E

" " linear momentum, $\vec{p}$

The Answer of course is ---.....

The Field "Carries" $\vec{L}$, E & $\vec{p}$ to Conserve!

Here is Basic idea again.....

Continuity Equation (general)

$$\vec{\nabla}\cdot\vec{J} = -\underbrace{\frac{\partial}{\partial t}(\text{conserved stuff})}$$

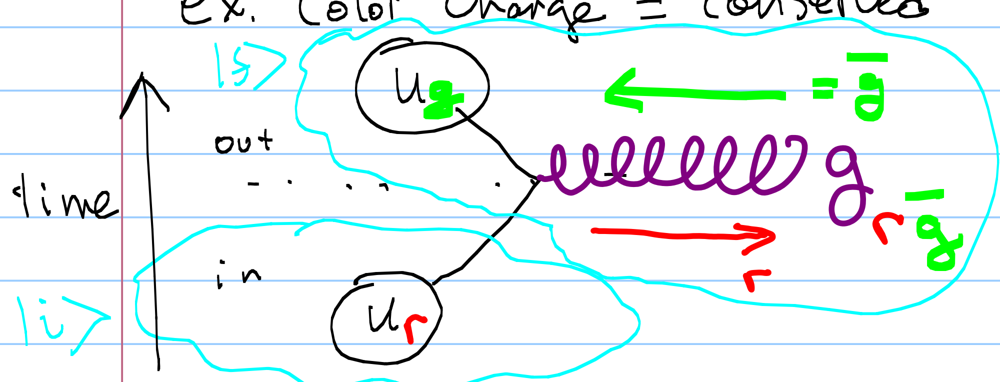
if you have stuff that is conserved & it "disappears" (ie $\frac{\partial}{\partial t} \neq 0$)

$\Rightarrow$'s a Field is "sourced"

so Creates a field Current & this carries the conserved quantities!

= Strong nuclear Force

ex: Color Charge = conserved so if quark color changes, A Field current must be created that conserves color



$|i\rangle$ = red

gluon color field

$|f\rangle = g + \bar{g} + r = r$

So in order to conserve

$\vec{L}$, E & $\vec{p}$ you need Fields

or

Since $\vec{L}$, E & $\vec{p}$ MUST be

conserved based on symmetry arguments (ie no perfect

symm	cons
$x_0 \Rightarrow$	$\vec{p}$
$\theta_0 \Rightarrow$	$\vec{L}$
$t_0 \Rightarrow$	E

you MUST HAVE FIELDS $\therefore$ HAVE FORCES!

ie standard Model

OK! $\vec{E}$ & $\vec{B}$ fields "carry" $\vec{L}$, $\vec{p}$ & E

and Combine to conserve $\vec{L}$, $\vec{p}$ & E

Final results on $\vec{E}$ & $\vec{B}$ fields from Chpt 8

Energy E&B $= u_{EM} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) = \frac{\text{energy}}{\text{volume}}$

momentum: $\vec{p}_{E\&M} = \varepsilon_0 (\vec{E} \times \vec{B}) = \frac{\text{momentum}}{\text{volume}}$

angular momentum $= \vec{r} \times \vec{p}_{E\&M} = \varepsilon_0 \left[\vec{r} \times (\vec{E} \times \vec{B})\right]$

Carried by FIELDS

Great results & will need ∞-ly. Example $\Longrightarrow$

why need $\vec{P}$, $\vec{L}$ & E of fields!

ex: atom interacting with Light

$$\hat{H}_0 = E_K + E_P = \frac{\hat{P}^2}{2M} - \frac{Ze^2}{r}$$

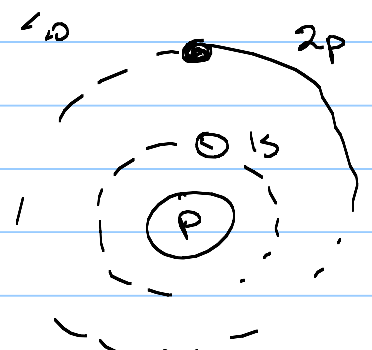
Solve $i\hbar \frac{\partial \Psi(t,\vec{r})}{\partial t} = \hat{H}_0 \Psi(t,\vec{r})$

& get $\Psi_{tot} = e^{-i\frac{E_n t}{\hbar}} R_n(r)\, Y_\ell^m(\theta,\phi)$

now these = Stationary!

They don't change in time

See of Course Townsend Chapt 14 GREAT



stays Forever!

Now is couple to $\Rightarrow$

self prop $\vec{E}$ & $\vec{B}$ field

$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$

$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \frac{d\vec{E}}{dt}$

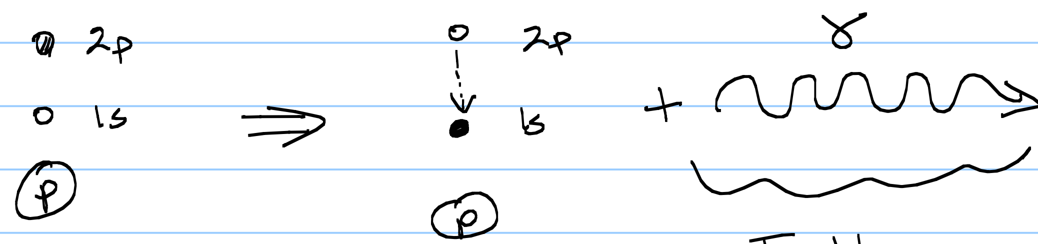
$\sim \int [\epsilon_0|\vec{E}|^2 + \frac{1}{\mu_0}|\vec{B}|^2] d\gamma$

$= \gamma$ Transvers real photon

need $\hat{H} = \hat{H}_{0\,atom} + \hat{H}_{E\&M}$

Now atom can decay cause it is coupled to γ field

So



Field
carries
Energy, momentum
& angular moment

Now

$$\begin{matrix} E_i & & E_f \\ \vec{p}_i & = & \vec{p}_f \\ \vec{L}_i & & \vec{L}_f \end{matrix}$$

atom = atom + field

So $\vec{E}$ & $\vec{M}$ fields

$$u_{E\&M} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

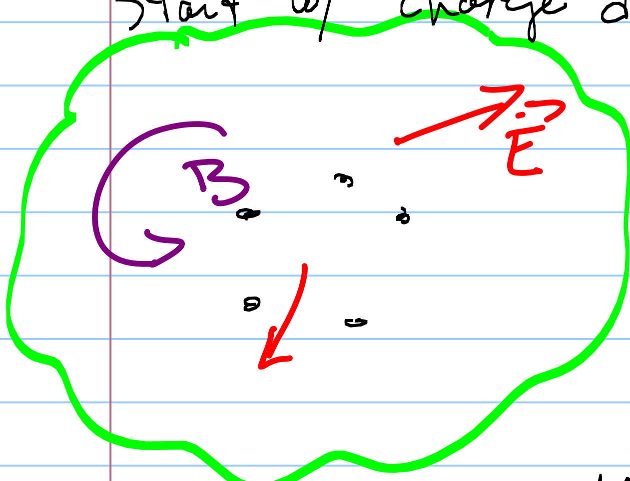
$$\vec{p}_{E\&M} = \epsilon_0(\vec{E} \times \vec{B}) \qquad = \mu_0 \epsilon_0 \vec{S}$$

$$\vec{l}_{E\&M} = \epsilon_0[\vec{r} \times (\vec{E} \times \vec{B})] \qquad = \mu_0 \epsilon_0 [\vec{r} \times \vec{S}]$$

Introduced $\vec{S}$ & $\vec{S}$ = Poynting Vector & plays Big role........

Poynting Vector:

Start w/ static charge distribution & $\vec{E}$'s & $\vec{B}$'s



Now let charges move abit.....

?= how much work is done by fields on these charges

well $dw = \vec{F} \cdot d\vec{l}$

$$dw = \vec{F} \cdot d\vec{l} = q(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{v}\,dt$$

$$dw = q\vec{E} \cdot \vec{v}\,dt$$

* again $\vec{B}$'s do not work!

since

$dq = \rho\,d\tau$

& $\rho\vec{v} = \vec{J}$

then

$$\frac{dw}{dt} = \vec{E} \cdot q\vec{v}\,dt$$

$$= \vec{E} \cdot \rho\,d\tau\,\vec{v}$$

$$\frac{dw}{dt} = \vec{E} \cdot \vec{J}\,d\tau$$

or ?

$$\frac{dw}{dt} = \int_{\tau} (\vec{E} \cdot \vec{J})\,d\tau$$

This is cool, but $\vec{J}$.... Huh, represent in terms of pure fields ....

use Max's equat's

Lots of steps.... Some magic   pg 347

get ....

$$\frac{dW}{dt} = \frac{-d}{dt}\int_{V} \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right) d\tau - \frac{1}{\mu_0}\oint_{surf} (\vec{E}\times\vec{B})\cdot d\vec{a}$$

So .... $\frac{dw}{dt}$ = power

So   RHS = power supplied by FIELD's!

we know $\frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) = \frac{\text{Energy}}{\text{Volume}} = u_{EGM}$

Introduce $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ must $= \frac{\text{Energy}}{(\text{time})(\text{Area})} = \frac{\text{Power}}{\text{Area}}$

= poynting Vector!

So

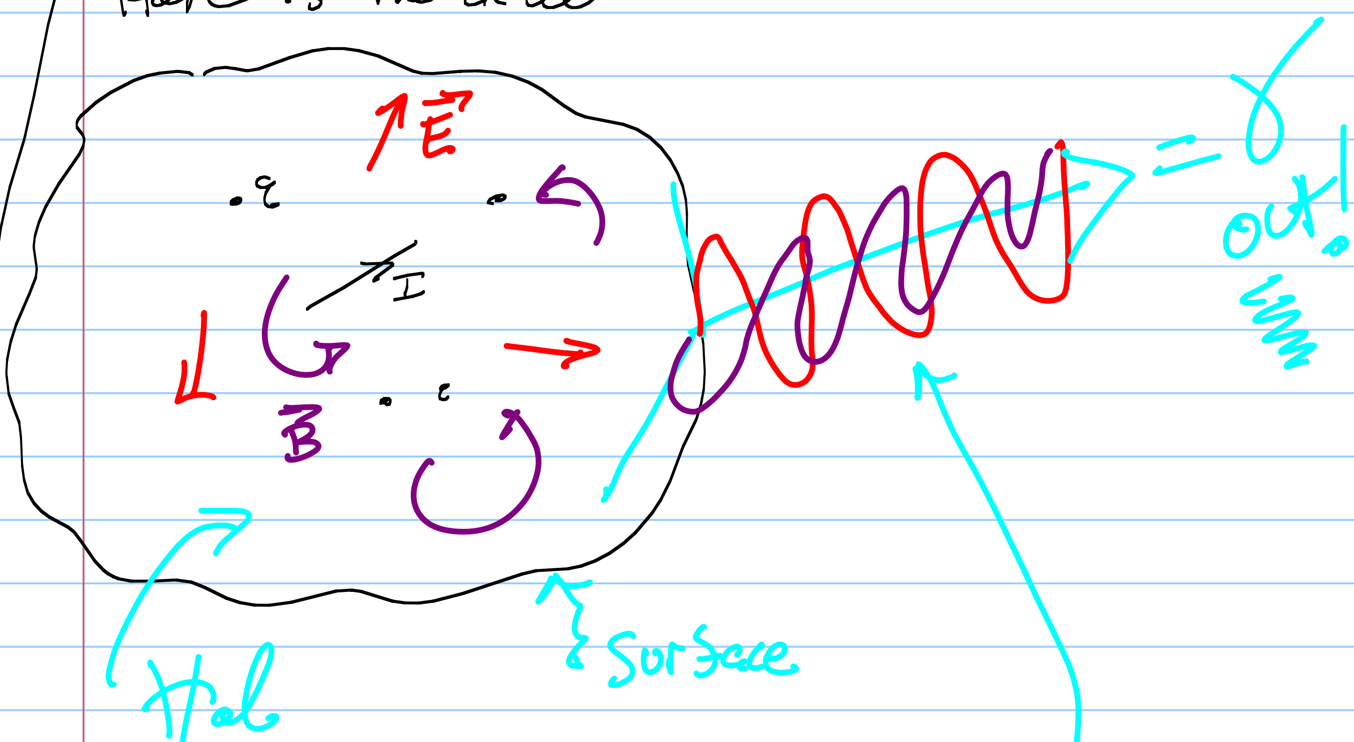
$$\boxed{\frac{dw}{dt} = \frac{-d\,u_{EGM}}{dt} - \oint \vec{S}\cdot d\vec{a}}$$

Now what does this

$$\boxed{\frac{dW}{dt} = -\frac{dU_{E\&M}}{dt} - \oint \vec{S}\cdot d\vec{a}}$$

mean?

Here is the deal



= Changes of energy density of field to make things like current & voltage = power stay w/in volume $\quad emf = -\frac{d\Phi_B}{dt}$

\+ Power that leaves the surface ie $\vec{E}$ & $\vec{M}$ waves = photons!

So, in chpt 9 on $\vec{E}$ & $\vec{M}$ waves,

Starting pts are

1.) $u_{E\&M} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu} B^2\right) = \frac{\text{energy}}{\text{Vol}}$

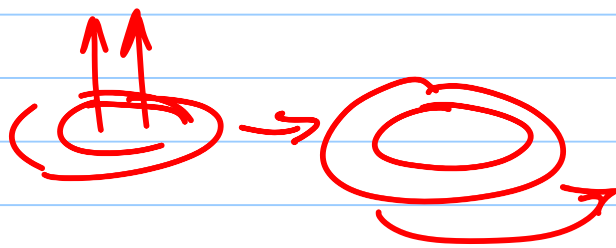
& 2.) $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) = \frac{\text{energy}}{(\text{Area})(\text{time})} = \frac{\text{power}}{\text{Area}}$

H.W.:

Griffiss

Example problems:

Ex: 7.8



where did $\vec{L}$ come from

Ex: 8.1 using $\vec{S}$

Ex: 8.3 $\vec{p}_{E\&M}$ → hidden momentum

Ex: 8.4 $\vec{l}_{E\&M}$ → no hidden momentum