E.F. Deveney/ BSC Physics Griff, Chpt 7 Electrodynamics

Note Title 3/22/2005

Here we are

Fields | **Sources**

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\varepsilon_0} \quad +$$

$$\vec{\nabla}\times\vec{E} = 0 \quad +$$

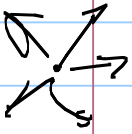
$$\vec{\nabla}\cdot\vec{B} = 0 \quad +$$

$$\vec{\nabla}\times\vec{B} = \mu\vec{J}_{free} \quad +$$

static sources of

dynamic sources?

* Note: skipping 1st couple of sections from Griff to get to Electrodyn



$\vec{E}$ & $\vec{B}$

or

$\vec{E}$ & $\vec{B}$

↳ static $\frac{\partial\rho}{\partial t} = 0$

& $\frac{dI}{dt} = 0 \Rightarrow$ steady currents.

We also have relativity; what you think is $\vec{E}$ in one inertial frame looks like $\vec{B}$ in another! pg 522 Griffith's

Just as SR mixes space-time

$$x^{\mu} = \text{4-vector} = (ct, x, y, z) \qquad \left(\begin{array}{c}\text{tensor}\\ \text{rank 1}\end{array}\right)$$

* AND

⇒ momentum energy

$$p^{\mu} = \text{4-vector} = (E/c, P_x, P_y, P_z)$$

SR mixes $\vec{E}$ & $\vec{B}$ ---- not as 4-Vector

but as <u>Tensor rank 2</u>

$$F^{\mu\nu} = \text{stress Tensor} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

It should come as no big surprise that E & B are entwined as are (t, space) & (Energy, momentum)

Now since $\vec{E}$ & $\vec{B}$ coupled by S.R.
you
also expect the coupling
should be under dynamic
situations

So

$$\vec{\nabla}\cdot\vec{E} = \rho_{free} +$$

$$\vec{\nabla}\times\vec{E} = 0 +$$

$$\vec{\nabla}\cdot\vec{B} = 0 +$$

$$\vec{\nabla}\times\vec{B} = \mu\vec{J} +$$

maybe
dynamic
sources
coupling $\vec{E}$ & $\vec{B}$
together!

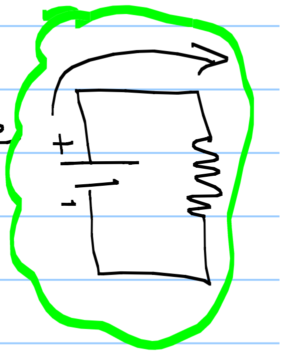
indeed this
is the case
1st born out
experimentally
then by
Maxwell!

1st Definition.

doubly misleading!

emf = electromagnetic Force

1.) emf source = ultimate source that sets up voltage and drives current



ie: emf sources = Batteries, piezo's, photo-cells, chemical

2.) emf $= \mathcal{E} = \oint \underbrace{\vec{f}_{source}} \cdot d\vec{l}$

$\vec{f}_{source} = \dfrac{\text{Force from source to move charges}}{\text{unit charge}}$

Note $\vec{E} = \dfrac{\vec{F}}{c}$

& $\int \vec{E} \cdot d\vec{l} = \dfrac{F}{c}\,\text{length} =$ Volts!

so

emf $= \oint \vec{f}_{source} \cdot d\vec{l}$ = measured in Volts!

Ex: 10V Battery or Photocell!

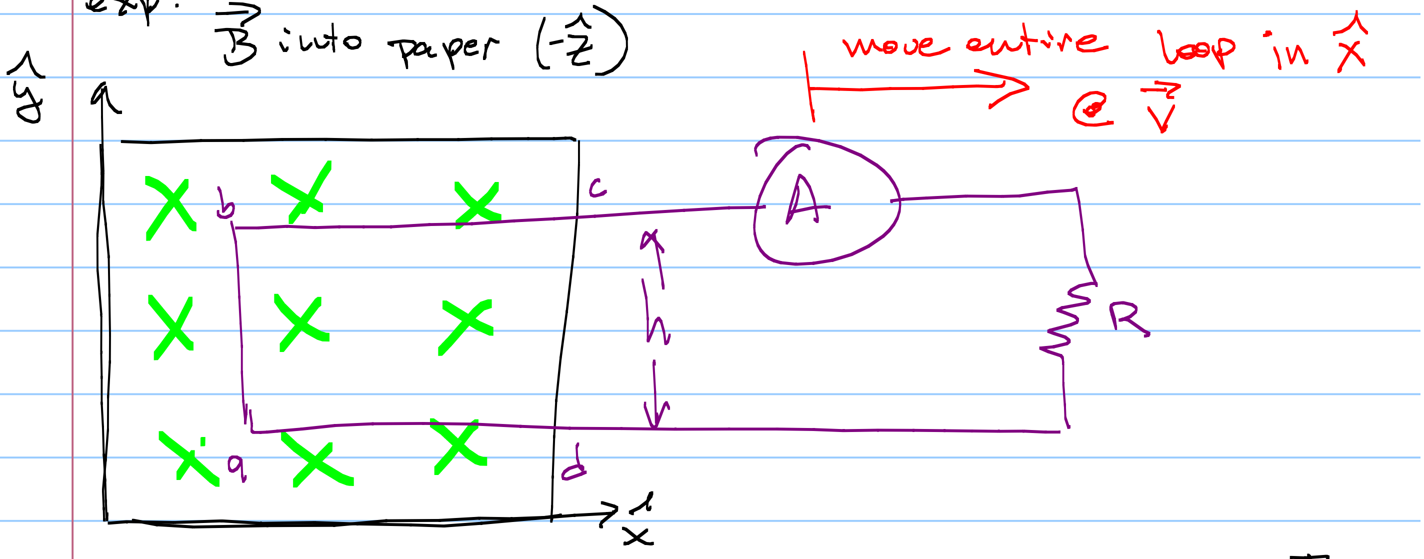
Back toward establishing dynamic connection between $\vec{E}$ & $\vec{B}$

Experimentally it was found $\begin{cases} \to \text{Motional emf} \\ \to \text{Faraday's Law of Induction} \end{cases}$

Motional emf

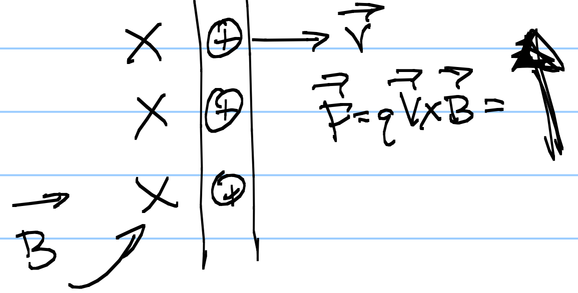
want emf or voltage "source" from just moving → check ⇒ S.R.

exp: $\vec{B}$ into paper $(-\hat{z})$



so

wire = conductor ;



$\vec{F}=q\vec{V}\times\vec{B}=$

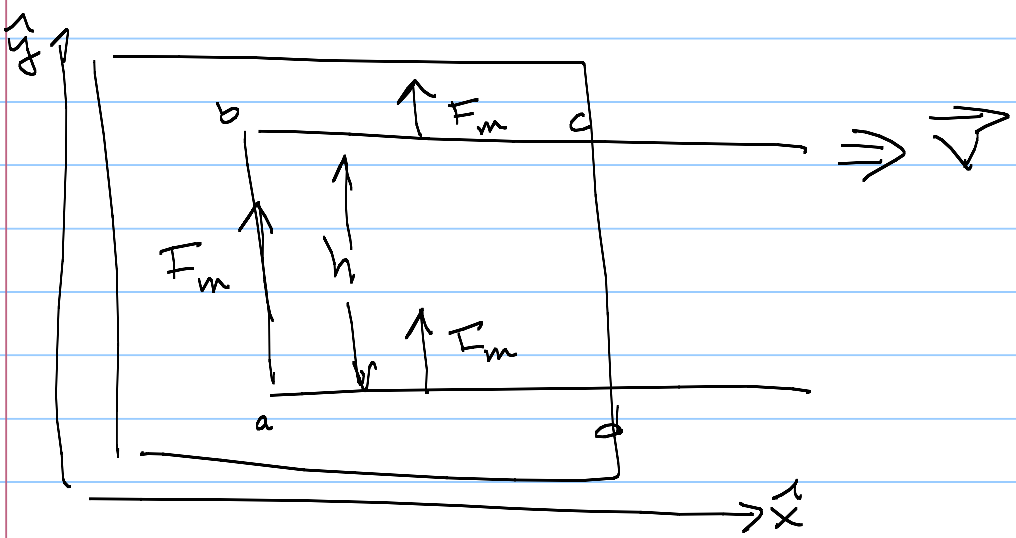
⊖-⊖-⊖-⊖ ←, I_e

=

⊕-⊕-⊕-⊕ →, $I_{\text{conventional}}$

in a sense that all the physics is the SAME so assume ⊕ charge carriers!

So you get



Now: $\text{emf} = \oint \vec{f}_{mag} \cdot d\vec{l}$

$\wr$

$\dfrac{\vec{F}_{mag}}{q}$ so

$$\text{emf} = \oint \frac{(q\vec{v}\times\vec{B})}{q} \cdot d\vec{l}$$

$$= \oint (|v||B| \sin 90)\,\hat{y} \cdot dl\,\hat{y}$$

$$= \int_a^b vB\,dl + \int_b^c \underset{\to 0}{\cancel{\uparrow \cdot \overrightarrow{dl}}} + 0 + \int_d^a \underset{\to 0}{\cancel{\uparrow \cdot \overleftarrow{dl}}}$$

* $\boxed{\text{emf} = vBh}$ motional!

$$= \frac{m}{s}\,\frac{Kg}{C\,s}\,m = Kg\,\frac{m}{s^2} \cdot m\,\frac{1}{C} = \frac{J}{C}$$

Really works!

Now there is a useful way to represent this motional emf problem.

define $\Phi_B \equiv \int \vec{B}\cdot d\vec{A}$ = Flux of $\vec{B}$

recall

$$\int_{V} \underbrace{\vec{\nabla}\cdot\vec{E}}\, dV = \int_{\text{surf of } V=A} \vec{E}\cdot d\vec{A}$$

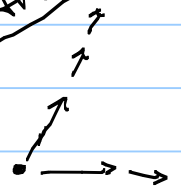
$\vec{\nabla}\cdot\vec{E} = \frac{\text{Field lines}}{\text{volume}}$

Think of $\frac{\text{Field lines}}{\text{Area}}$

* vector field lines

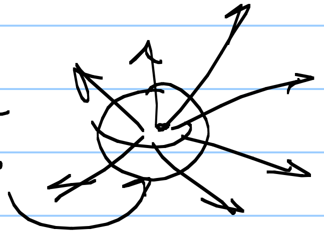
|mag| ∝ length

dir = dir



stream line

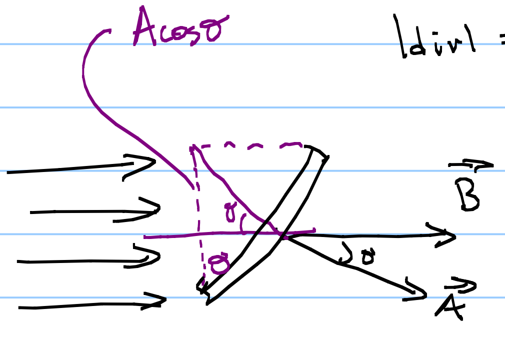
|mag| ∝ $\frac{\#\text{ lines}}{\text{Area}}$



|dir| = dir tangent to stream lines

& $\vec{B}\cdot\vec{A}$

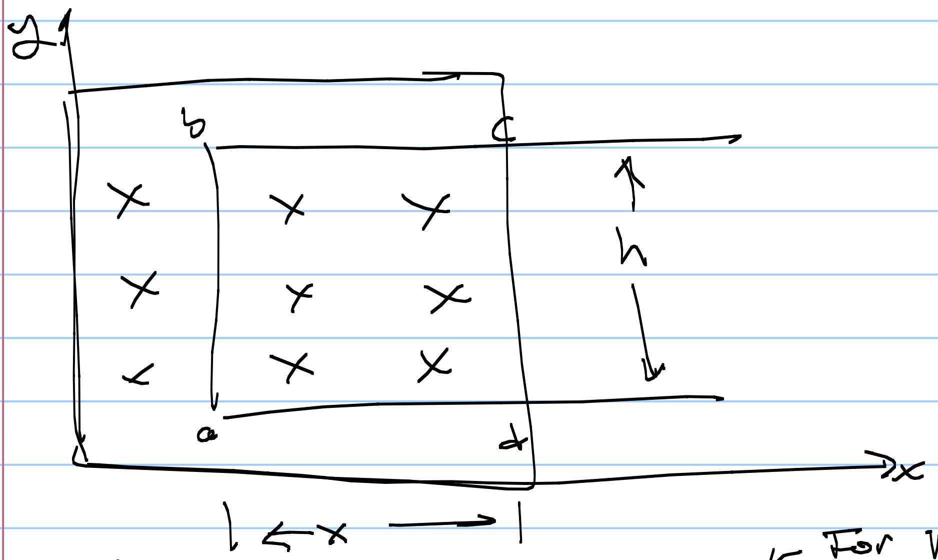
$|B||A|\cos\theta$



$\vec{B}\cdot\vec{A} = BA_\perp$ = # Field lines out of $\vec{A}$

so $\Phi_B = \int \vec{B}\cdot d\vec{A}$ = # field lines of $\vec{B}$ out of $\vec{A}$

Now for our motional emf prob



$$|\Phi_B| = \int B_0(-\hat{z})\cdot dA(+\hat{z}) \leftarrow \text{For now}$$

$$= B\int dA$$

$$|\Phi_B| = Bxh$$

Then consider

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(Bxh) = Bh\frac{dx}{dt} = Bh|\vec{v}|$$

Let's worry about signs now

$$\frac{d\Phi}{dt} \text{ for this case } = \underset{\substack{\uparrow \\ \text{cause Flux decreases.}}}{-} Bhv$$

So....

since emf = Bhv

we

can say $\boxed{\text{emf} = -\frac{d\Phi}{dt}}$

Convention!

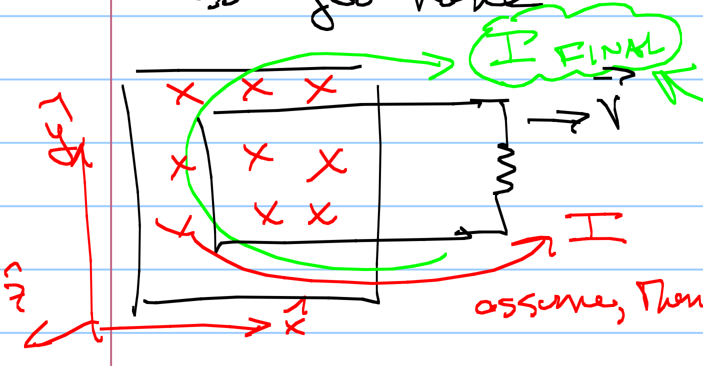
assume (+) charge carriers, use rhr

(is need (−) current we know ⊖-⊖ ⊖-⊖ $\xrightarrow{\vec{v}}$
= ⊕.⊕ ⊕-⊕ ←)

Then

Fingers define(+) dir & $\hat{A}$ = Thumb!

so you have



means choose I dir wrong so

assume, Then $\hat{A} = +\hat{z}$

$$\text{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[B(-\hat{z})\cdot(A)(+\hat{z})\right]$$

$$= +\frac{d}{dt}[BA] = B\underbrace{\frac{dA}{dt}}_{\text{since }\downarrow}$$

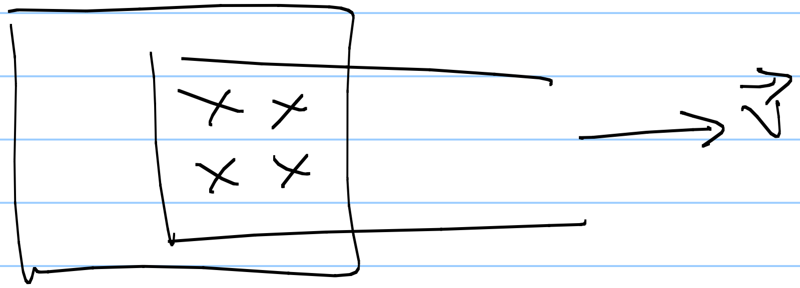
$$= \textcircled{-}B \times v$$

🚩 HW
7.7
7.11 ⇒
EMF AC
generator

Later we will learn Lenz's Law.....

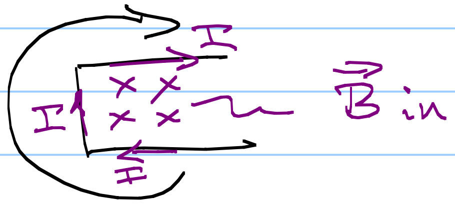
"The induced current flows in a direction that the flux it produces tends to cancel the change in flux"

is



Since this reduces the flux thru

current is induced by motional emf



NATURE abhors a change of flux
so works to keep it!

So tied motion to $\vec{E}$ & $\vec{B}$ $\left(\frac{dx}{dt}\right)$

now other dynamics $\left(\frac{d}{dt}\right)$

Emf by Faradays law of Induction

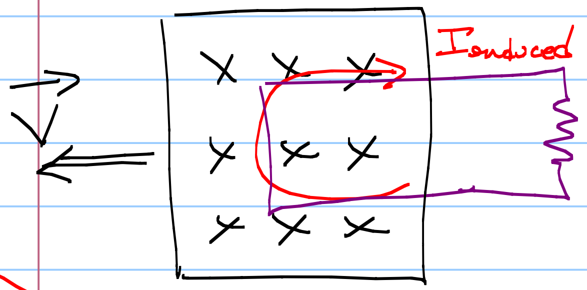
1831 Michael Faraday on 3 experiments

(& same thing, induced current happened in all 3)

1.) motional emf (not Faraday induct officially)

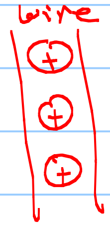
Then

2.)



Here the magnet moves

so

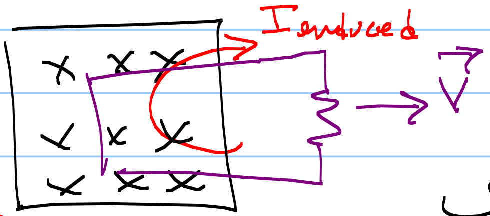


not moving!

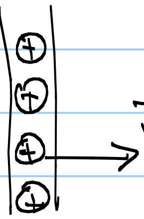
So, $\vec{F}_{Mag} = q\vec{V}\times\vec{B} = 0$

So this motion is NOT the emf (source)

recall #1



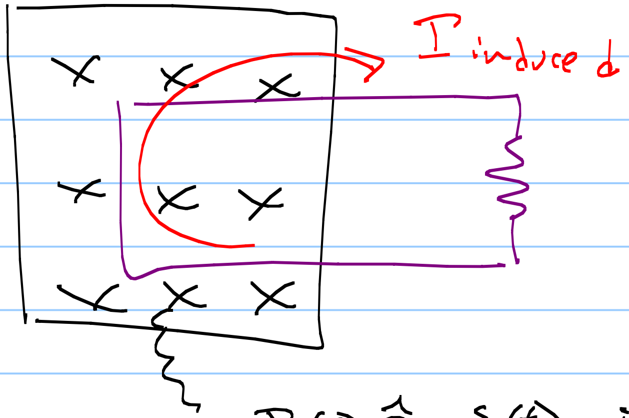
here



$\vec{V}$ in $\vec{B}$ so

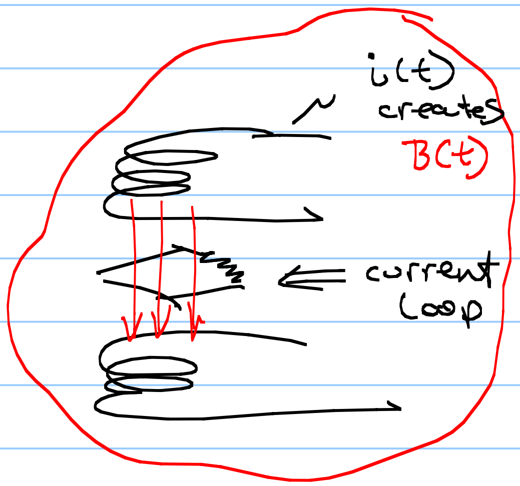
$\vec{F}_{Mag} = q\vec{V}\times\vec{B}$

Case 3



so here

$B(z)\,\hat{z} = f(t)$ ie



Faraday Found.

Case 1, $\underbrace{\text{emf}}_{\text{voltage source}}$ due to $\vec{F}_{mag} = q\vec{v}\times\vec{B}$ = motional emf

$$\text{emf} = -\frac{d\Phi}{dt} \quad -\vec{B}\cdot\left(\frac{d\vec{A}}{dt}\right)$$

Case 2 & 3 emf also $= -\frac{d\Phi}{dt}$

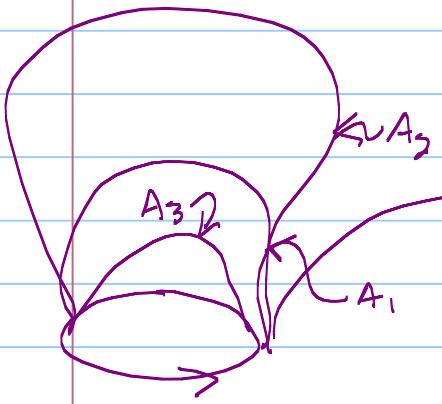
but here

$$\text{emf} = \left(-\frac{d\vec{B}}{dt}\right)\cdot\vec{A}$$

induction emf

$$\text{now} \quad \text{emf} = \oint \frac{F_{source}}{q} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$= \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$



$$\underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{use stokes Theorem}} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

so, this contour is not specific to $\vec{A}$, $\vec{A}$ is arbitrary

Thus

$$\boxed{\int_{Area} (\vec{\nabla} \times \vec{E}) \cdot d\vec{A} = -\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}}$$

this equality is true for all Boundaries! ∴ can equate the integrands

an so

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

or

$$\vec{\nabla} \times \vec{E} = \underset{\substack{\uparrow \\ \text{static} \\ \text{source}}}{0} + \underset{\substack{\uparrow \\ \text{dynamic} \\ \text{source of } E}}{-\frac{\partial \vec{B}}{\partial t}}$$

A time varying $\vec{B}$ produces an $\vec{E}$!

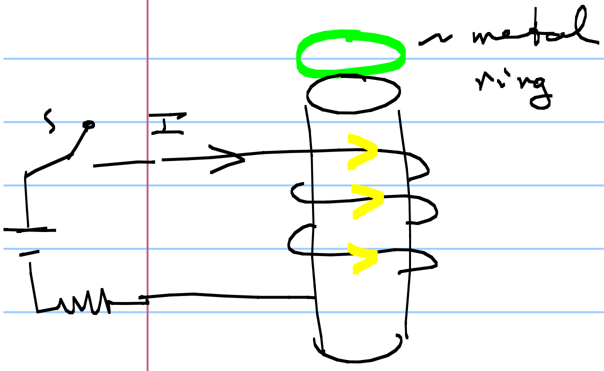
Again Lenz's law help's us keep track of induced flow...

flow is in direction to create $\vec{B}$ flux to cancel the change in $\vec{B}$

or restore

Do Ex: 7.6 = jumping ring

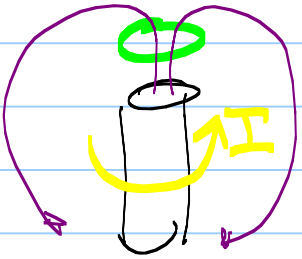
Jumping ring:



w/ switch off:

$\vec{B}=0 \therefore \Phi_B = 0$

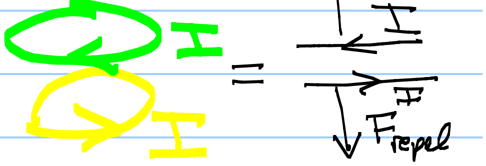
Throw switch



$\text{emf} = -\frac{d\Phi}{dt}$

I induced to counter change in flux

but now



$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

opposes!

so read $\frac{\partial \vec{B}}{\partial t}$ "sources" curling $\vec{E}$

in such a way to oppose the change

(−)