Biot-Savart-Ampere: E.F. Deveney | BSC Physics   Griffs 5.2, .3 & .4

Note Title   1/23/2005

Electrostatics: q's = stationary static charges $\Rightarrow$
$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{E} &= \frac{\rho_{free}}{\varepsilon_0} \\ \vec{\nabla}\times\vec{E} &= 0 \end{aligned}\right\}\text{electro-statics}$$

We now know $\vec{F}_B = \int dq\,(\vec{v}\times\vec{B})$

So what are the equations of magneto-statics
$\left(\frac{dI}{dt} = 0\right)$
steady currents
"static" currents

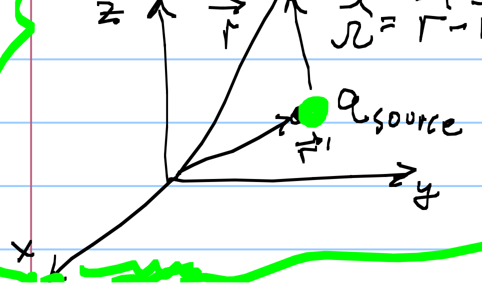
magneto-statics: $\begin{cases} \vec{\nabla}\cdot\vec{B} = ? \\ \vec{\nabla}\times\vec{B} = ? \end{cases}$

ie: obviously ideal; currents have to start & stop. Note: 60 Hz line I is good enough $\simeq$ magneto-static.

We know $\vec{F}_B$, but haven't addressed $\vec{B}$ yet

(recall, $d\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathcal{R}^2}\hat{\mathcal{R}}$ = Coulomb, by experiment)

ε_0 = permittivity of free-space $= 8.85\times10^{-12}\ \frac{C^2}{N m^2}$



$\vec{\mathcal{R}} = \vec{r} - \vec{r}'$ = pts from source to field

Now

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{\imath}}{|\vec{\imath}|^2}.$$

again by experiment & Funky ⊥ result

Called Biot-Savard Law for "Static" Currents

Therefore: $\vec{B}$ works out to be $\frac{\text{Newt}}{\text{Amp}\cdot\text{m}}$ = SI = TESLA

μ_0 = permeability of free-space $= 4\pi \times 10^{-7} \frac{N}{A^2}$

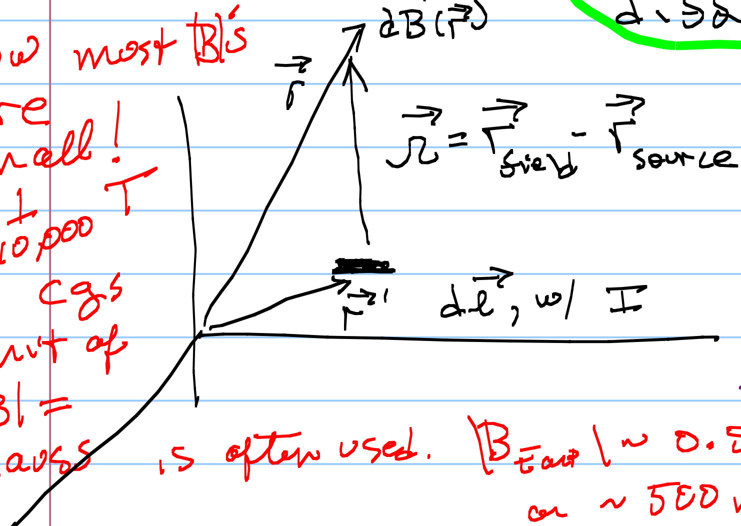
Now w/ Lot of work in Lab can $\frac{W}{m^2 \cdot s}$ $|B_0| \sim 1$ Tesla

* NOTE: see prev lecture notes & Griffs pg 522 that $\vec{B}$ = relativistic eff moving charges C

⇒ $F^{\mu\nu}$ = field tensor = correct mixing of $\vec{E}$ & B in different moving frames!

Now most $\vec{B}$'s are small! $\sim \frac{1}{10{,}000}$ T

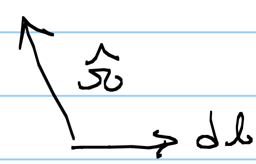
i.e. cgs unit of $|B|$ = gauss is often used. $|B_{Earth}| \sim 0.5$ g or ~ 500 mg



$\vec{\imath} = \vec{r}_{field} - \vec{r}_{source}$

In ATOMS e^- orbit creates B field the e^-, $\vec{\mu}_B$ feels of order ~ 10 T!

Note: DIR @ $B(\vec{r})$ is $d\vec{l} \times \vec{\imath}$ by RHR



So: strategy: 1.) Find $\vec{B}(\vec{r})$ brute Force! (Biot-Savard)

2.) Exploit symmetry to make Life easier
= Ampheres Law

3.) Create Vector potential, $\vec{A}$ from which
$$\vec{B}(\vec{r}) = \vec{\nabla} \times \vec{A}(\vec{r})$$

Note same as E & M!

1.) $\vec{E}(\vec{r}) = \int d\vec{E}(\vec{r})$ Brute Force

2.) Exploit symmetry: Gauss's Law

3.) Create Electric Potential
$$\vec{E}(\vec{r}) = \vec{\nabla} V(\vec{r})$$

So $\vec{B}(\vec{r})$ by Brute Force for really, by far, most significant case! USED Always! for $\vec{B}$ problems!

Actually straight wire & circular loop of wire are only 2 cases from which you can build almost all others from

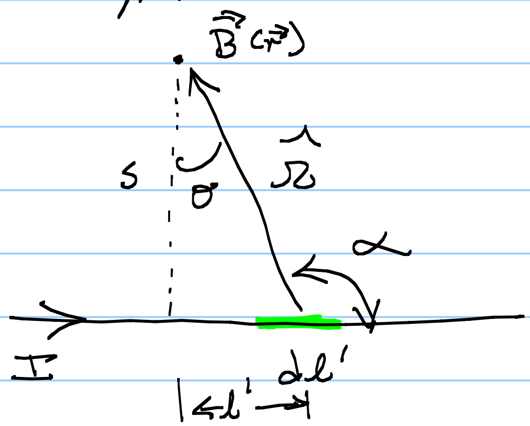
So:

Ex: 5.5 Griffs → straight wire

Ex: 5.6 Griffs loop

Are CRUCIAL

5.5 Find the $\vec{B}(\vec{r})$ a distance s from a long straight wire carrying steady current!



* SINCE $\vec{\ell} \to \infty$ use cylindrical coords

(s, ϕ, z)

& z is arbitrary so & won't depend on ϕ

re-arranging

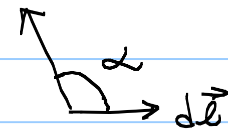
So see by Symm that $\vec{B}(\vec{r}) = \vec{B}(s)$ so get things in terms of s

OK $\qquad d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{d\vec{l}' \times \hat{r}}{|\vec{r}|^2}$

ie $\hat{\phi}$ in cylindrical coords

w/ Dir = out of page!

so $\quad d\vec{l}' \times \hat{r} = |d\vec{l}'||\hat{r}| \sin\alpha$



$= dl' \sin\alpha$

$= dl' \cos\theta \Leftarrow$ by geom

& from fig $\quad l' = s\tan\theta \Leftarrow$ Note want things in terms of s

so $\quad \frac{dl'}{d\theta} = \frac{s}{\cos^2\theta} \quad ; \quad dl' = \frac{s}{\cos^2\theta} d\theta$

Now then $\quad s = r\cos\theta$

so

$$\frac{1}{r} = \frac{s}{\cos\theta}$$

& $$\frac{1}{|r|^2} = \frac{\cos^2\theta}{s^2}$$

and

$$d\vec{B}(s) = \frac{\mu_0 I}{4\pi} \underbrace{\left(\frac{\cos^2\theta}{s^2}\right)}_{\left(\frac{1}{|r|^2}\right)} \underbrace{\left(\frac{s}{\cos^2\theta} d\theta\right)}_{dl'} \cos\theta \quad (\hat{\phi})$$

$(d\vec{l} \times \hat{r}) = dl'\cos\theta$

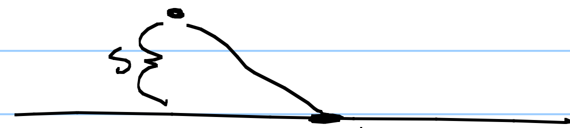
$$d\vec{B}(s) = \frac{\mu_0 I}{4\pi} \left(\frac{\cancel{\cos^2\theta}}{s^2}\right)\left(\frac{s}{\cancel{\cos^2\theta}}\right) \cos\theta \, d\theta \quad (\hat{\phi})$$

(dir = out of page!) ($\hat{\phi}$)

$$d\vec{B}(s) = \frac{\mu_0 I}{4\pi s} \cos\theta \, d\theta \uparrow\uparrow \Leftarrow \text{not Bad}$$

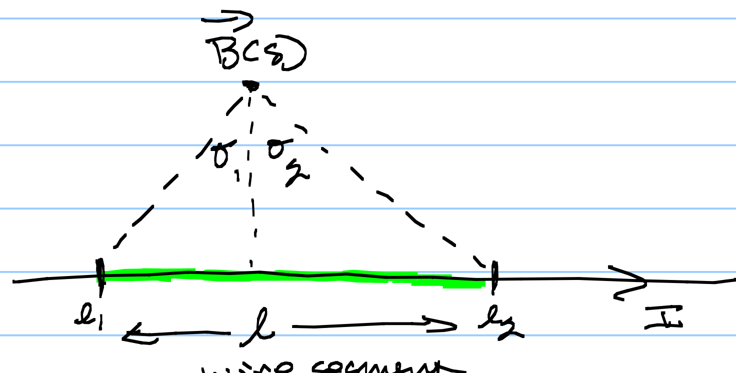
cyln

θ's entered from



$$dl' = \frac{s}{\cos^2\theta} d\theta$$

So now instead of integrating over all dl' in is transformed to integrating of θ's



$$\vec{B}(\vec{s}) = \frac{\mu_0}{4\pi} I \int_{l_1}^{l_2} \frac{d\vec{l}' \times \vec{r}}{|\vec{r}|^2} = \frac{\mu_0}{4\pi} \frac{I}{s} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta \; (\hat{\phi})$$

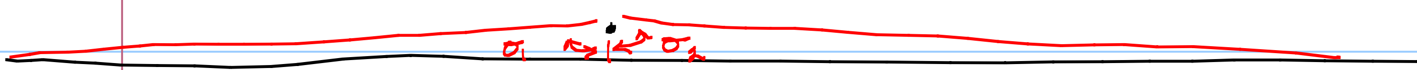
cylind

$$\vec{B}(\vec{s}) = \frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1)$$

} for Straight wire segment!

($\hat{\phi}$) cylind

For $\cong$ ∞-ly long wire



θ_1 & $\theta_2 \Rightarrow \pi/2$, $\theta_1 = -\pi/2$, $\theta_2 = +\pi/2$

& $\vec{B}(s) = \frac{\mu_0 I}{4\pi s}\left(1-(-1)\right) = \frac{\mu_0 I}{2\pi s}\hat{\phi}$ = HUGE = $\vec{B}$ field for ∞-ly long wire

NOTE: TRICK!

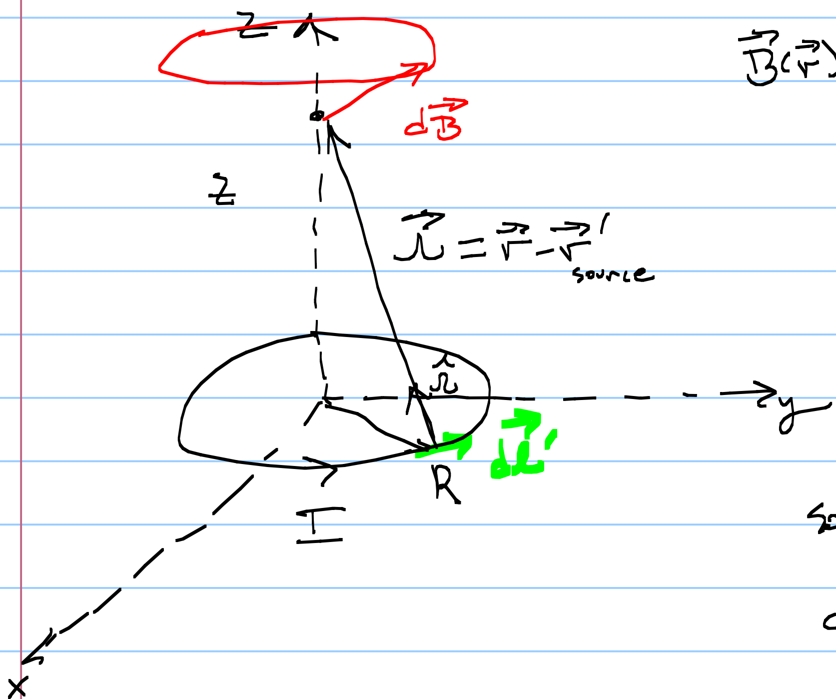
We can get the direction of $\vec{B}(s)$ from straight wires carrying I by:

RH
Put thumb in dir of (+) I and fingers roll in $\hat{\phi}$ direction

DUE to cylindrical symmetry of wire

The next most crucial case for B-S law is circular loop carrying current, I.

Ex: 5.6 Griffs: Find $\vec{B}(z)$ above circ loop



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{|\vec{\mathscr{r}}|^2}$$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}'_{source}$

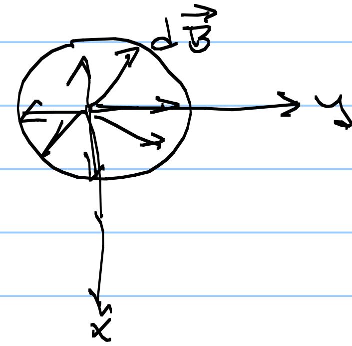
* Think of direction

Maybe Symmetry & cancelation!

so

dir $d\vec{l}' \times \hat{\mathscr{r}} = \perp$ to plane of $d\vec{l}$ & $\hat{\mathscr{r}}$ @ z

Now, as integrate around $\hat{\phi}$, azimuthal direction note that



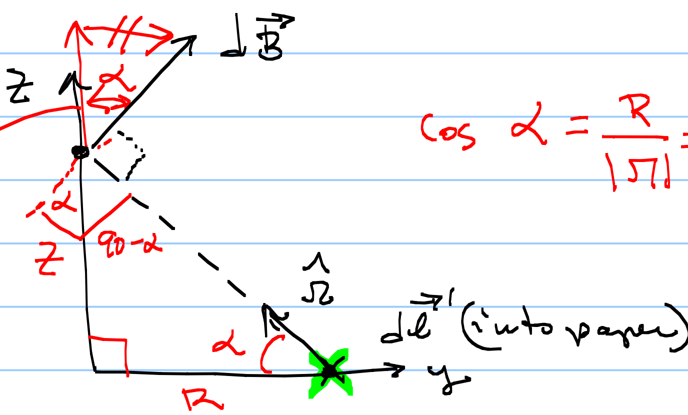
$d\vec{B}$s in x-y plane @ z Cancel exactly!

what's left is the verticle component of $d\vec{B}$

so $$\boxed{d\vec{B}(z) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{\ell}' \times \hat{\mathscr{r}}}{|\vec{\mathscr{r}}|^2}}$$

$\int [\quad]$ left w/

$$\vec{B}(z) = \underbrace{d\vec{B}(z) \text{ verticle}} + 0 \text{ horizontal}$$



$$\cos\alpha = \frac{R}{|\vec{\mathscr{r}}|} = \frac{R}{\sqrt{z^2+R^2}}$$

$d\vec{B}(z)\,\hat{z}$
$=$
$d\vec{B}\cos\alpha$

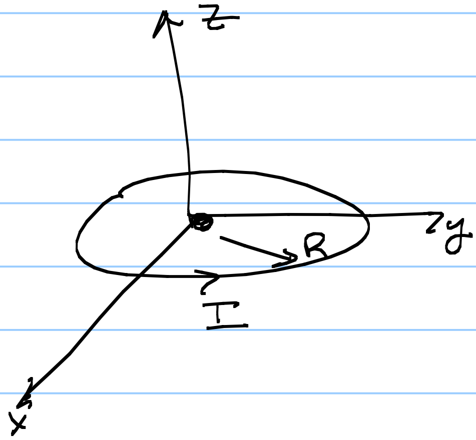
between $d\vec{\ell}'$ & $\mathscr{r}$ = 90

$$\text{so} \quad \vec{B}(z) = \int (d\vec{B})\cos\alpha = \int \frac{\mu_0 I}{4\pi} \frac{|d\ell'||\hat{\mathscr{r}}|\sin\theta}{|\vec{\mathscr{r}}|^2} \cos\alpha$$

$$= \frac{\mu_0 I}{4\pi} \int \frac{d\ell'}{(z^2+R^2)} \frac{R}{(\sqrt{z^2+R^2})} = \frac{\mu_0 I}{4\pi} \frac{R}{(z^2+R^2)^{3/2}} \int_0^{2\pi} R\,d\phi$$

$$\vec{B}(z) = \frac{\mu_0 I}{4\pi} \frac{R^2}{(\quad)^{3/2}} 2\pi = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} (\hat{z})$$

So, for example,



$$\vec{B}(z=0) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+0^2)^{3/2}} (\hat{z})$$

$$= \frac{\mu_0 I}{2} \frac{R^2}{R^3}$$

$$\boxed{\vec{B}(z=0) = \frac{\mu_0 I}{2R} \hat{z}}$$

= mag field @ center of current carrying loop!

Note: TRICK still works! RH thumb in dir of I & fingers roll in dir of $\vec{B}$

H.W. Gr.ss 5.8 & 5.9b

So Law of Biot-Savard

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I} \times \hat{\mathscr{r}}}{|\mathscr{r}|^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{|\mathscr{r}|^2}$$

= Line of charge, λ, moving @ $\vec{v}$

so $\vec{I} = \lambda \vec{v}$

Likewise surface charge σ @ $\vec{v}$, $\vec{K} = \sigma \vec{v} = \frac{dI}{dl_\perp}$

Then

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}') \times \hat{\mathscr{r}}}{|\mathscr{r}|^2}\, da'$$

and

volume charge ρ @ $\vec{v}$, $\vec{J} = \rho \vec{v} = \frac{dI}{da_\perp}$

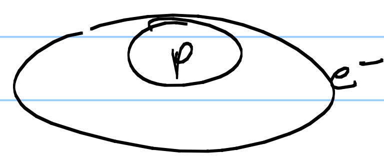
&

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}') \times \hat{\mathscr{r}}}{|\mathscr{r}|^2}\, d\tau'$$

Note: Tempted, then from $\vec{I}$, $\vec{K}$ & $\vec{J}$ to write

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{q\vec{v}\times\hat{\mathscr{r}}}{|\mathscr{r}|^2}$$

for 1 charge moving in a circular path like



But, Griffiths emphasizes, is wrong because 1 q ≠ steady current

e–e–e–e–e

& B-S only hold for steady or static currents.

* **However:** It is $\cong$ correct for non relativistic charges!

so $V_{e^-} \cong 10^6$ m/s or 1% of c, so not Bad!

w/ $\vec{B}$ given by Biot-Savard

Now our goal is have an idea of what "Sources" $\vec{B}$

Just as w/ $\vec{E}$, we get $\left\{ \begin{array}{l} \vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\varepsilon_0} \\ \vec{\nabla}\times\vec{E} = \underbrace{0}_{} \\ \text{sources (static) for } \vec{\nabla} \text{ \& } \vec{\nabla}\times E \end{array} \right.$

Look for

$$\left\{ \begin{array}{l} \vec{\nabla}\cdot\vec{B} = ? \\ \text{\& } \vec{\nabla}\times\vec{B} = \underbrace{?}_{} \\ \text{Static sources of } \vec{\nabla} \text{ \& } \vec{\nabla}\times \vec{B}\text{s} \end{array} \right.$$

right from Biot-Savard

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times\hat{\mathscr{r}}}{|\mathscr{r}|^2}\,d\tau'$$

WORK thru Griffs pg 222 - 224

results $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{|\mathscr{r}|^2}\, d\tau'$

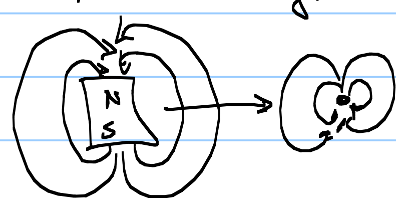
1.) $\boxed{\vec{\nabla}\cdot\vec{B}(\vec{r}) = 0}$

↳ ie $0 = \oint_S \vec{B}\cdot d\vec{A} = \int_{Vol} \vec{\nabla}\cdot\vec{B}\, d\tau' = S_{B_{free}}$

Also: no net flux out of closed surface!

∴ No free, charge-like, source of $\vec{B}$ that diverges.

No Magnetic monopoles!



unlike Electric monopoles = charges

2.) $\vec{\nabla}\times\vec{B} = ?$

$$= \frac{\mu_0}{4\pi}\int \vec{\nabla}\times\left(\vec{J}\times\frac{\hat{\mathscr{r}}}{|\mathscr{r}|^2}\, d\tau'\right)$$

where

$$\vec{\nabla} \times \left(J \times \frac{\hat{\imath}}{|\imath|^2} \right) = \underbrace{J \left(\vec{\nabla} \cdot \frac{\hat{\imath}}{|\imath|^2} \right)}_{\text{we encountered before!}} - \underbrace{\left(\vec{J} \cdot \vec{\nabla} \right) \frac{\hat{\imath}}{|\imath|^2}}_{\to 0}$$

in $\vec{E}$-field
$\vec{E} \propto Kq \left(\frac{\hat{\imath}}{|\imath|^2} \right)$

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{|r|^2} \right) = \text{undefined @ } r = 0$$

so

$$\int_{V} \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) dV' = \oint_{\text{Area}} \frac{\hat{r}}{r^2} \cdot d\vec{A} = 4\pi$$

independ of area surrounding V Even is let V around $\frac{\hat{r}}{r^2} \to 0$

HUGE!

so found $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right)$ acted like $4\pi \delta(\vec{r})$

Dirac delta function

"source" of diverging $\left(\frac{\hat{r}}{r^2} \right)$

$$\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = \underbrace{4\pi \delta(\vec{r})}_{\text{source of}}$$

Continuing.....

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}')\, 4\pi\, \delta^3(\vec{r}-\vec{r}')\, d\tau' \quad = \mu_0 J(\vec{r})$$

picks out

$\vec{J}(\vec{r}')$ @ $\vec{r}$

YEAH

* recall

$\vec{J} = \rho\vec{v} = \frac{C}{m^3}\,\frac{m}{s}$

$\vec{J} = \frac{\vec{I}}{m^2} = \frac{I}{A_\perp}$

So, we are done w/ STATIC Maxwell's Equations!

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_s}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

Static Source's for div & curling E & B fields

So, that's Huge step, but we are still mired w/ problem of $\vec{B}$ in general & How to Solve it!

Just got $\vec{B}$ = Biot-Savard ⇒ Brute Force

Now

Ellegant symmetry ⇒ Ampheres Law's Like Gauss's Law for $|\vec{E}|$

Found

$$\underbrace{\vec{\nabla}\cdot\vec{B}}_{\text{fields}} = \underbrace{0}_{\text{sources}}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Leftarrow$$

static source of curling $\vec{B}$ = (ie magneto Steady I's)

Ampheres Law ⇒ Different-ial form

Play w/ it

$$\int_A \left[\vec{\nabla}\times\vec{B} = \mu_0\vec{J}\right] d\vec{A}$$

$$\int_A (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \int \mu_0\vec{J}\cdot d\vec{A}$$

now use Stoke's Theorem.

$$\underbrace{\int_{A} (\vec{\nabla}\times\vec{B})\cdot d\vec{A}}_{\text{Area}} = \underbrace{\oint \vec{B}\cdot d\vec{l}}_{\text{Line integral on Bounding of } A} = \mu_0 \underbrace{\int \vec{J}\cdot d\vec{A}}_{\text{Flux of } \vec{J} \text{ thru that } A}$$

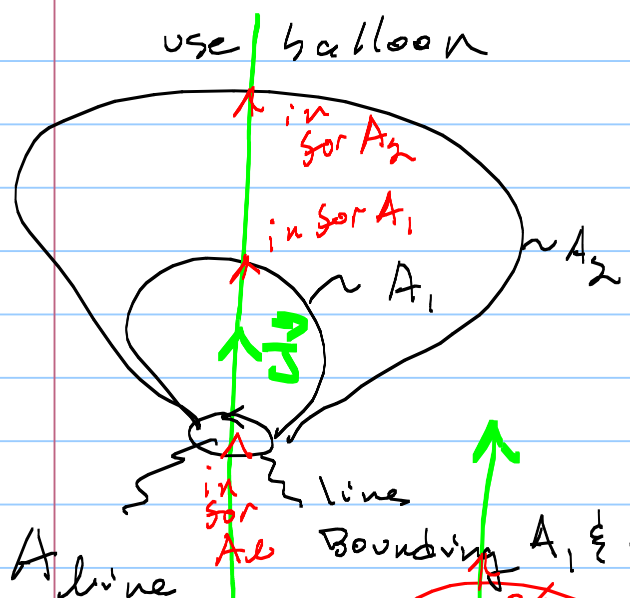
* Note Area NOT closed so have boundary

Note: not a net flux ing out But thru!

So : A = area is arbitrary!
use balloon

Further, in Gauss's Law $\int \vec{\nabla}\cdot\vec{V}\, d\tau = \oint_A \vec{V}\cdot d\vec{A}$ Area = closed

Note: A is not a closed surface or l = Boundary would = 0

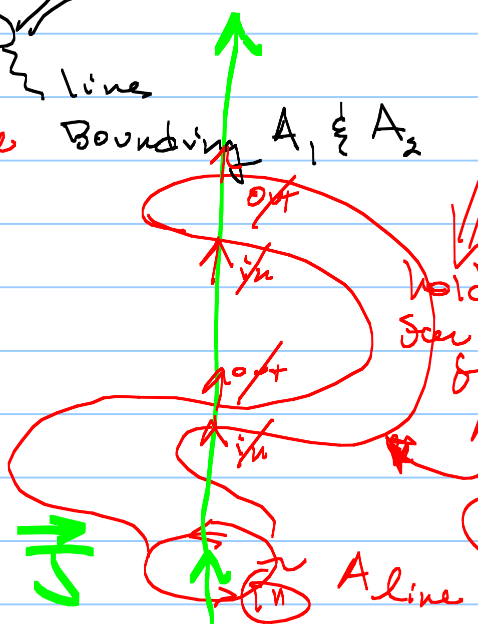


A_{line} = area of line integral

Net Flux of $\vec{J}$ passing thru

A_1 = same for A_2

= Same as Area bound by the A_l



holds for even funkier A_{sunky}

= 1 J in

So A_{sunky} & A_l have 1 J in!

So $$\oint_{\substack{\text{closed}\\ \text{Line}}} \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{A}$$

$$= \mu_0 \int \underbrace{\left(\frac{\vec{I}}{dA_\perp}\right)}\cdot d\vec{A}$$

$$=$$

$dA_\perp$ is // to dA

$\therefore$ this is really just

$$\oint_{\substack{\text{closed}\\ \text{Line}}} \vec{B}\cdot d\vec{l} = \mu I_{\text{thru } A}$$

or

enclosed by

Ampheres Law Integral form

Always TRUE

But not always use Sol$_{\text{n}}$!

Just as $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ Gauss's law

$$|E| \int dA = \frac{q_{in}}{\varepsilon_0} \Leftarrow \text{when symmetry}$$

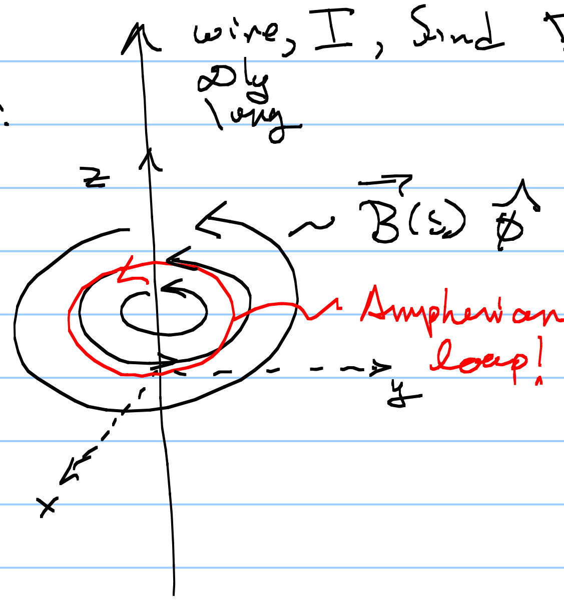
Now $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

when symm

$$|B| \int d\vec{l} = \mu_0 I_{enclosed} \Leftarrow \text{Useful.}$$

EXAMPLE:

Say: wire, I, find B. ∞ly long



well ⇒ symmetry

$B \neq f(z)$

$\therefore \underbrace{B \neq f(\phi)}$

ie cylindrical

From trick of finding dir of $\vec{B}$ outside long wire carrying I

RH Thumb in dir of +I

Fingers roll in $\hat{\phi}$

so $\oint_{loop} \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

choose an "Ampherian Loop" — just like Gaussian Surface!

that is useful!

$\oint_{Loop} \vec{B}\cdot d\vec{l}$

$|\vec{B}(s)| = $ constant on loop

& dir $= \hat{\phi}$

$\oint |B(s)|\, \hat{\phi}\cdot s\, d\phi\, \hat{\phi}$

$B(s) \int_0^{2\pi} s\, d\phi$

$B(s)\, s\, 2\pi = \mu_0 I_{enclosed}$

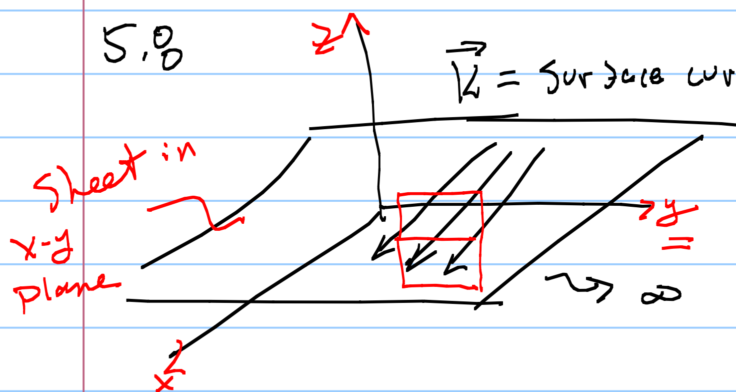
so

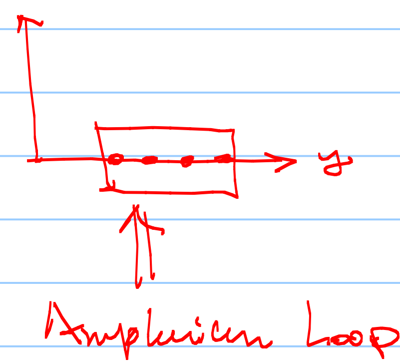
$$\boxed{\vec{B}(s) = \frac{1}{2\pi s} \mu_0 I_{enclosed}\ \hat{\phi}}$$

Work thru 2 examples in Griffiths:

5.8

$\vec{K}$ = surface current density $= \frac{I}{d\ell_\perp} = |K|\hat{x}$

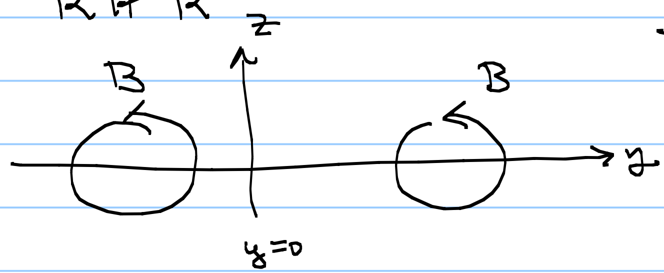




so choose

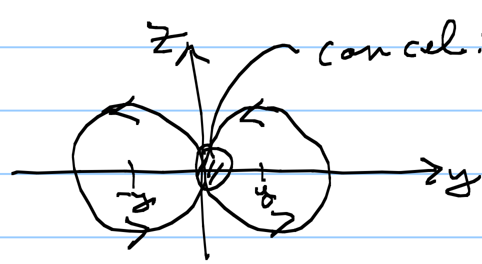
Think hard for symmetry 1st

by RHR



so no $\vec{B}$ in $\hat{x}$ direction

Now

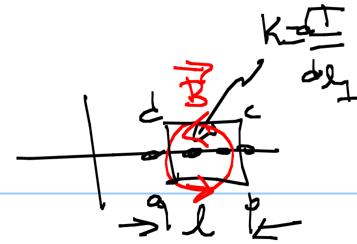


cancel: argue due to symm & superposition

ALL $\vec{B}$ in $\hat{z}$ will cancel!

So left w/ $\vec{B}$ in $\hat{y}$ direction

So now $\displaystyle\oint_{\text{closed}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$



$$\left.\begin{aligned} &\int_a^b B_{\rightarrow}\cdot dl_{\rightarrow} + \underbrace{\int_b^c 0\cdot d\vec{l}}_{\to 0} \\ &+\int_c^d B_{\leftarrow}\cdot dl_{\leftarrow} + \underbrace{\int_d^a 0}_{\to 0} \end{aligned}\right\} = \mu_0 K l$$

$= \mu_0 K l$

$$2B(y)\cancel{l} = \mu_0 K \cancel{l}$$

$$B(y) = \frac{\mu_0 K}{2}$$

So $$\vec{B}(y) = \begin{cases} -\dfrac{\mu_0}{2} K\ \hat{y} & \text{for } y > 0 \\ +\dfrac{\mu_0}{2} K\ \hat{y} & \text{for } y < 0 \end{cases}$$

* KEY is $B(y) \neq f(y)$

ie

it is uniform

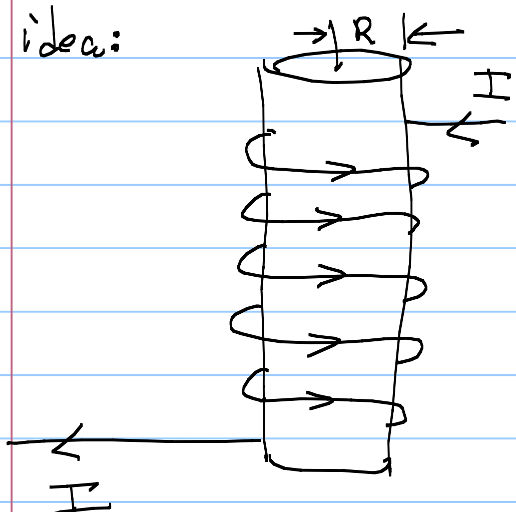
just as $|\vec{E}|$ above

an ∞ sheet of charge

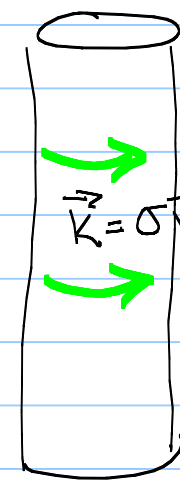
Griff Example 5.9 : $\vec{B}$ fields outside long solenoid

HUGE problem, use in physics alot $\Rightarrow$ Aharonov-Bohm effect pg 399

idea:



closely spaced, packed $\approx$



$\vec{K} = \sigma \vec{v} = \frac{dI}{dl_\perp}$

* is worry about $I \uparrow$, imagine sheet of alum foil or wires wrapped $\uparrow$ & $\downarrow$ to elim vertical current.

* see Griff

OK:

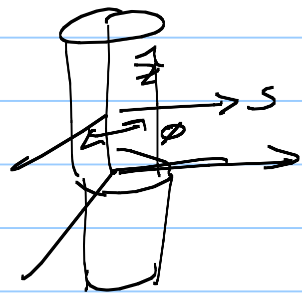
to get anywhere you again have to stop & think about symmetry.

So

Start: what is the direction of $\vec{B}$?

Think

Cylindrical coords:



$\vec{B}$ radial?

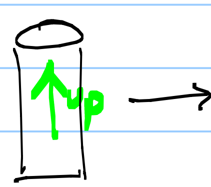
No way: why ......?

assume $\vec{B} = +\hat{s}$ $\longrightarrow$

Now, is reverse direction of I, then would $\vec{B} = -\hat{s}$

But reversing I = flipping solenoid upside down

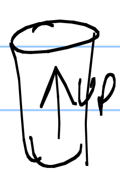
so is



$\longrightarrow$ $\vec{B}$ $+\hat{s}$

$\longrightarrow$ $\vec{B}$ $\underbrace{+\hat{s}}$ should still be up!

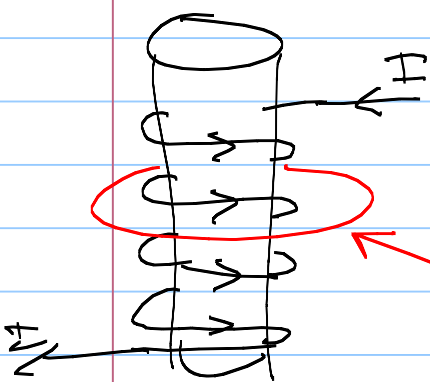
But since this is the same as



w/ currents reversed where $\longleftarrow$ $\vec{B}$ $(-\hat{s})$ = contradiction

so $|B(s)| = 0$

Solenoid:



How about $B(s)\,\hat{\phi}$ ie circumference?

No

why

$$\oint \vec{B}\cdot d\vec{l} = B_\phi 2\pi s = \mu_0 I_{enclosed}$$

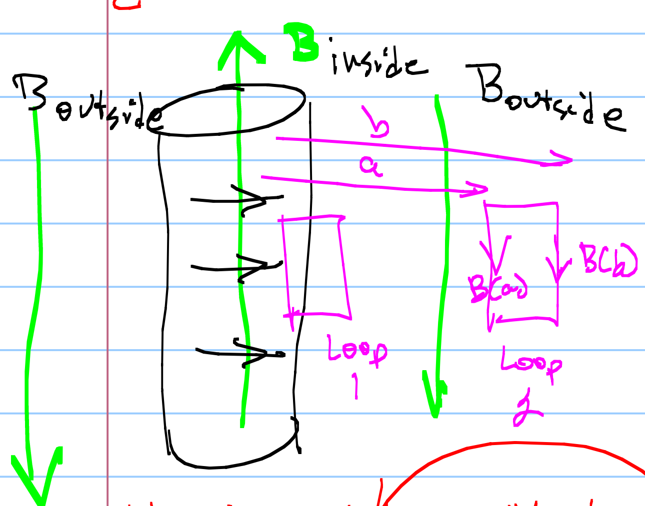
But no I flows ↑ through Ampherian Loop!

$$\therefore B(s)\,\hat{\phi} = 0$$

So $B\,\hat{s} = B\,\hat{\phi} = 0 \;\therefore$

{ $\vec{B}$ for ∞ly long solenoid, closely packed wires)

{ runs $\hat{z}$ ie axially!



By RHR & assume $B(s \to \infty) = 0$ outside

Now Loop 2

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 J_{enclose}$$

$(B(a) - B(b))\,l = 0$ or $B(a) = B(b)$?

$\therefore$ $B_{outside} = 0$ As Gauss SAYS

ASTONISHING!

can get same result Brute-Force so it is RIGHT!

Also key to A-B is that $\vec{A}$ enters the $\mathcal{L}$, not $\vec{B}$ itself (while $\vec{B} = 0$ outside, $\vec{A}$ does not!) * in Abrohanov-Bohr potential

$\mathcal{L}$ for q in $\vec{E}$ $\Rightarrow$ $\mathcal{L}_E = \frac{1}{2}m\dot{x}^2 - q\varphi$

get $\Sigma\vec{F} = m\vec{a}$

$q\vec{E} = m\vec{a}$

Then $\mathcal{L}$ for q in $\vec{E}$ & $\vec{B}$ $\Rightarrow$ $\mathcal{L}_{E\&B} = \frac{1}{2}m\dot{x}^2 - q\varphi + \frac{q}{c}\vec{A}\cdot\dot{\vec{x}}$

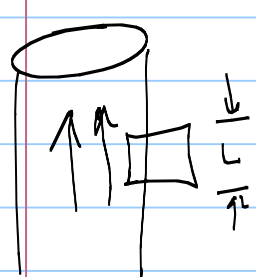
get $\Sigma\vec{F} = m\vec{a}$

$q\vec{E} + q\vec{v}\times\vec{B} = ma$

Back to Solenoid

$\vec{B}(s) = 0$ outside HUGE!

For loop 2, half in & half out



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

where $n = \frac{\#\text{wires}}{\text{length}}$

$$\int_{in} B\uparrow\cdot dl\uparrow + \cancel{\int B\uparrow\cdot dl\rightarrow} + \int_{outside} 0\cdot dl\uparrow + \cancel{\int B\uparrow\cdot dl\leftarrow} = \mu_0 n I L$$

$\rightarrow 0$

$$B_{in}\cancel{L} = \mu_0 n I \cancel{L}$$

$$\vec{B}_{solenoid} = \begin{cases} \mu_0 n I \ (\hat{z}) & \text{inside} \\ 0 & \text{outside} \end{cases} \Leftarrow \text{HUGE!}$$

ie

 B_0 uniform $\Longleftrightarrow$  E_0 uniform

solenoid

$\vec{B} = \mu_0 n I$ vary

Capacitor

$|E| = \dfrac{Q}{A\epsilon_0}$ ~ control?

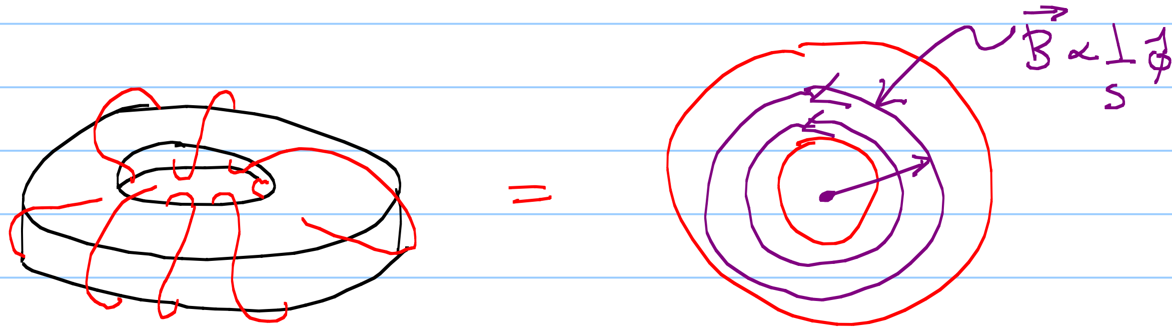
USEFUL in Lab to make uniform $\vec{E}$ & $\vec{B}$ fields!

So: Ampère's Law! much easier than Brute Force but still tricky!

* Always true: But not always useful (i.e. some w/ Gauss) prototypes you must know!

1) ∞ straight line ex: 5.7
2) ∞ planes ex: 5.8
3) ∞ solenoid ex: 5.9
4) ∞ Toroid ex: 5.10

Ex: 5.10 Torroid = Donut



Result:

$$B(\vec{r}) = \begin{cases} \frac{\mu_0 N I}{2\pi s}\,\hat{\phi} & \text{inside} \\ 0 & \text{outside} \end{cases}$$

KEY For Plasma containment (Fusion)
& Magneto hydrodynamics

charged-fluid dynamics!
Sun stuff
& plasmas - again fusion!