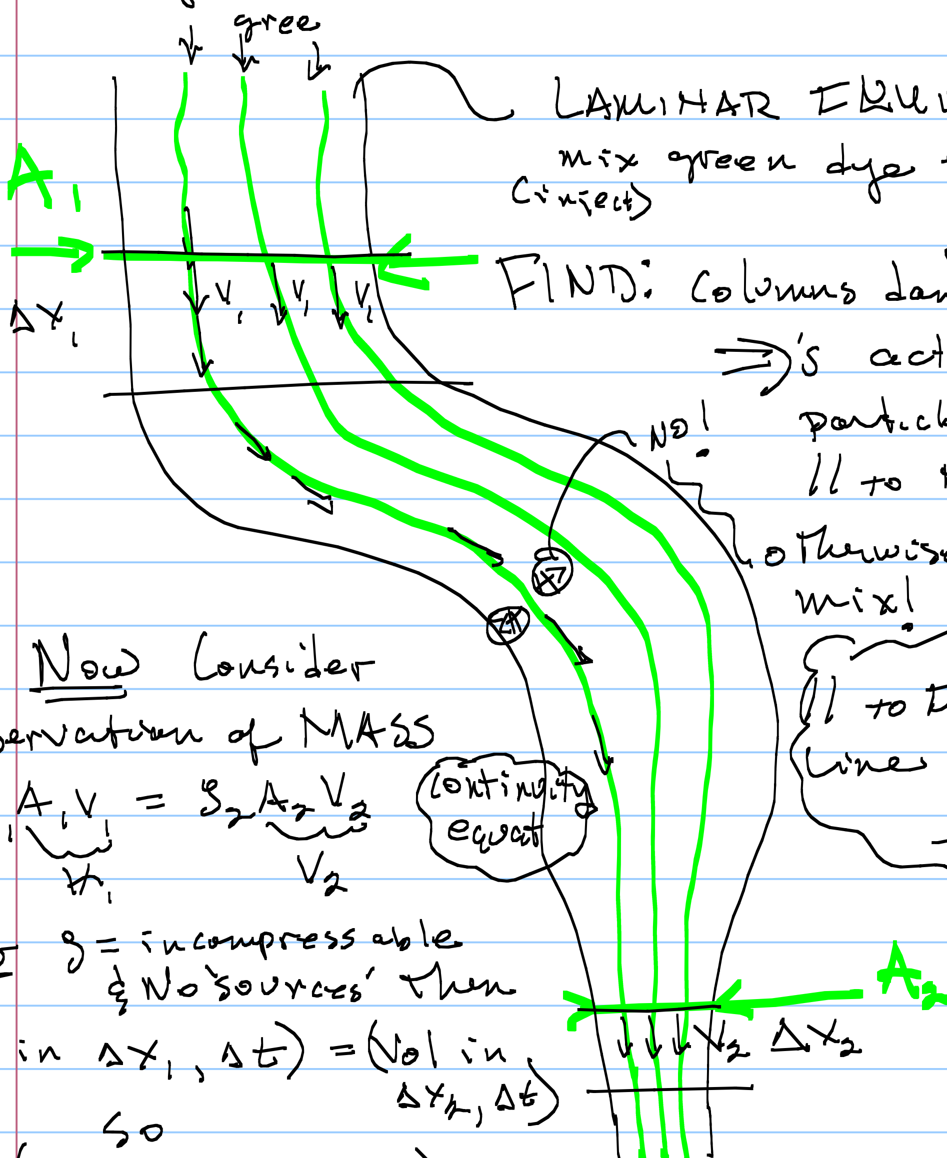
E.F. Deveney / BSC Physics — Vector Fields to Stream Lines

Note Title — 9/17/2004

Ref: Physics 2000 Huggins

Sometimes easier to visualize Vector Fields using Streamlines … FLUIDS!



LAMINAR FLUID! (not turbulent!)

mix green dye to fluid @ top! (inject)

FIND: columns don't Mix!

$\Rightarrow$'s actual velocity of particles is everywhere $\parallel$ to the FLOW

otherwise, color would mix!

$\parallel$ to Flow $\Rightarrow$ Tangent Lines

Now Consider Conservation of MASS

$$\rho_1 \underbrace{A_1 v_1}_{H_1} = \rho_2 \underbrace{A_2 v_2}_{V_2}$$

Continuity equat

IF ρ = incompressable & No 'sources' then

(Vol in $\Delta x_1, \Delta t$) = (Vol in $\Delta x_2, \Delta t$)

so

$$\frac{1}{\Delta t}\left(A_1 \Delta x_1 = A_2 \Delta x_2\right)$$

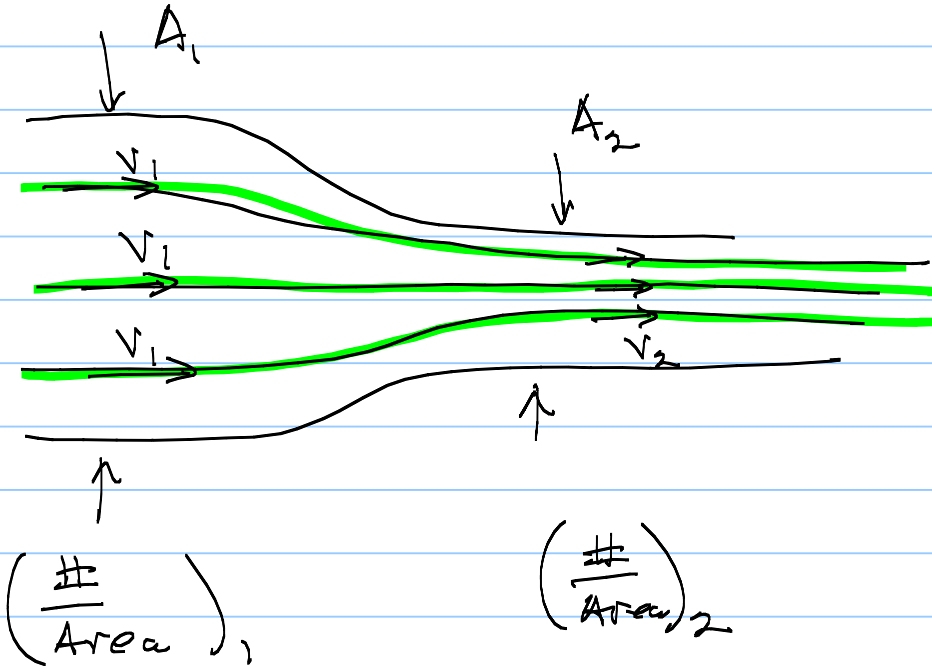
$$A_1 |V_1| = A_2 |V_2| \quad \text{so} \quad |V_2| = \left|\frac{A_1}{A_2}\right| V_1$$

Note

1.) continuity equat can also be seen as momentum FLUX

2.) $\frac{3\text{ Lines}}{A_1} < \frac{3\text{ Lines}}{A_2}$

So



So VECTOR FIELD LINES! $(\vec{F}(x,y))$

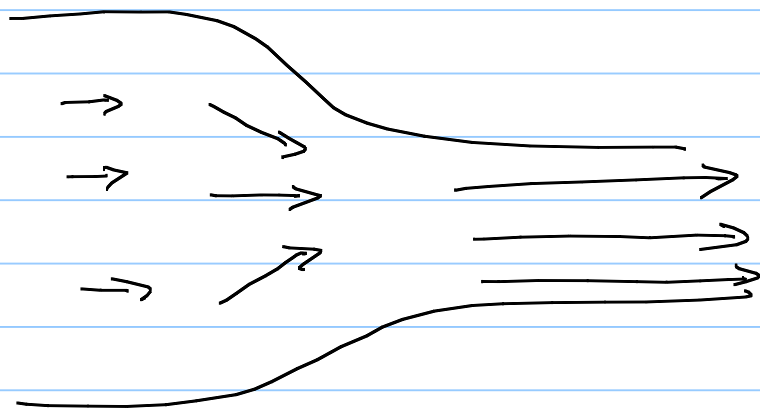
Instead of $I.$ $\begin{cases} |\text{arrows}| \propto |\vec{F}(x,y)| \quad (\text{mag}) \\ \text{arrow dir} \propto \hat{F}(x,y) \quad (\text{dir}) \end{cases}$

use

STREALINES where....

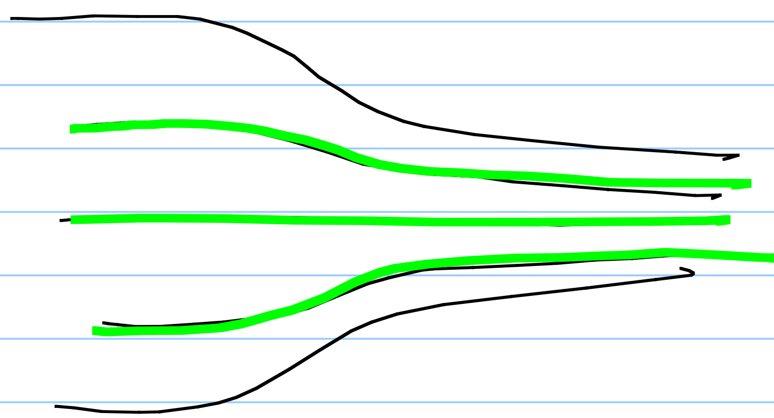
$\vec{F}(x,y)$ $\begin{cases} \#/\text{Area of lines} \propto |\vec{F}(x,y)| \quad (\text{mag}) \\ \text{Tangents to lines} \propto \hat{F}(x,y) \quad (\text{dir}) \end{cases}$

Instead of



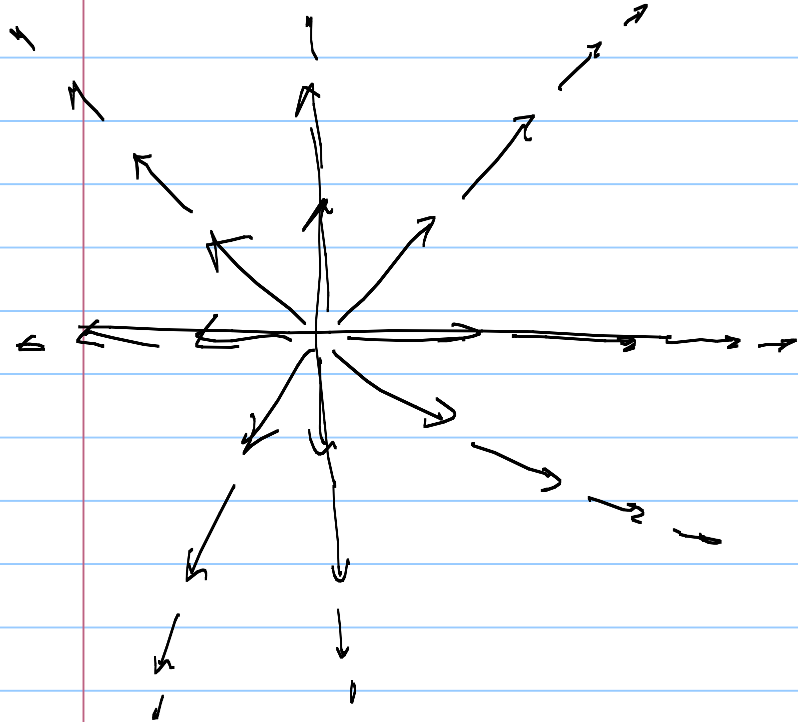
Vector
Field
Lines

use stream lines
to visualize
vector fields



Vector
Stream
Lines!

Try $\vec{E}(\vec{r}) \propto \frac{1}{r^2}\hat{r}$



* Note: Problem @ $\vec{F}(0) \propto \frac{1}{0^2}$?

problem!

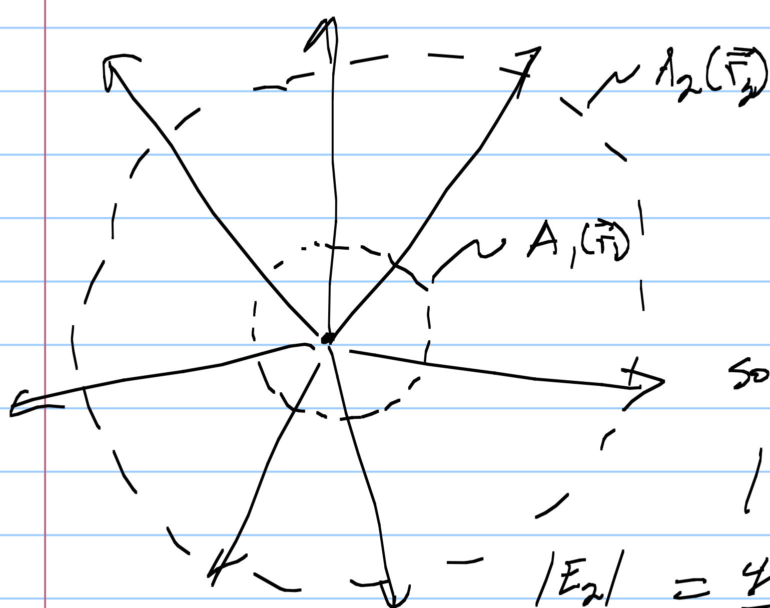
see Griff pg 46!

Vector field Lines

problem gets back to E&M, classical, only $\cong$ to QED where ∞-ities are renormalized. until then

Things are smoothed out w/ math not real phys

Now w/ Stream Lines



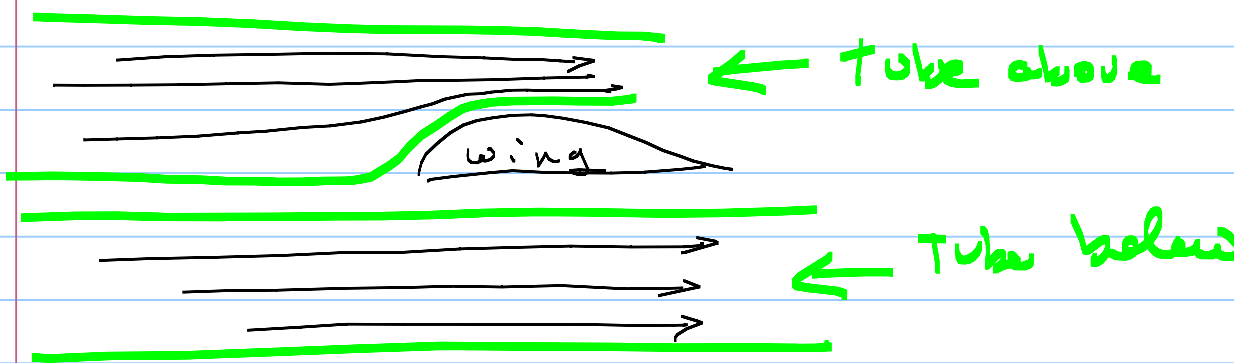
Here dir = Tangent vect = SAME as above

But mag

$$\text{So } \frac{|\vec{E}(\vec{r}_2)|}{|\vec{E}(\vec{r}_1)|} = \frac{\frac{\#\text{ Lines}}{A_2}}{\frac{\#\text{ Lines}}{A_1}} = \frac{\#\ A_1}{\#\ A_2}$$

$$\frac{|E_2|}{|E_1|} = \frac{4\pi r_1^2}{4\pi r_2^2}$$ say $|r_1| = 1$ Then $|E_2| = \frac{1}{r^2}|E_1|$

Another example: Air flow



Obviously here from stream lines above wing $|\vec{V}_{above}| > |\vec{V}_{below}|$

∴, w/ BERNOULLI = cons of Energy)

$$\rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1 = \text{constant}$$

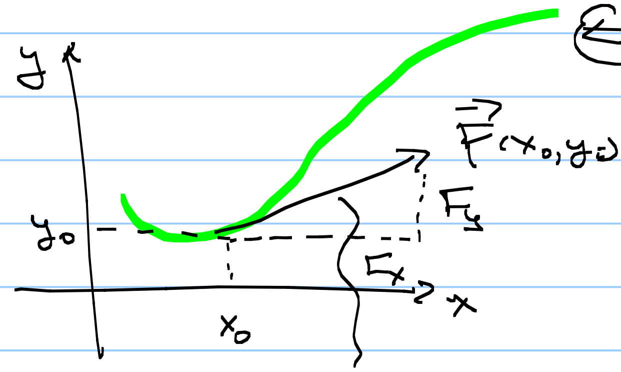
that $P_{above} < P_1$

∴

$$F_{NET} = \Uparrow = \text{lift}$$

Stream Lines FROM Field Lines:

A curve $y=y(x)$ = stream line of $\vec{F}(x,y)$ (ie vector field) is @ every pt (x_0,y_0) on the curve, $\vec{F}(x_0,y_0)$ is tangent!



$\vec{F}(x_0,y_0)$ = vector field line

stream line looking to get $y(x)$

Note: $\left.\frac{dy}{dx}\right|_{x,y} = \frac{F_y(x,y)}{F_x(x,y)}$

slope of streamline $= \frac{dy}{dx}$

So

$$\frac{dy}{dx} = \frac{F_y}{F_x}$$

ex: $\vec{F}(x,y) = \frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2+y^2}}$ $\quad \frac{dy}{dx} = \frac{F_y}{F_x} = \frac{+x/\cancel{\sqrt{}}}{-y/\cancel{\sqrt{}}} = -\frac{x}{y}$

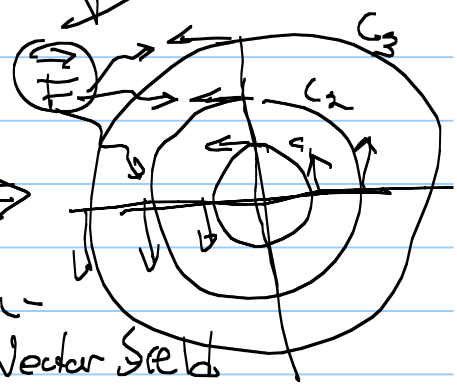
$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 + C = \frac{1}{2}x^2 + C$$

$$\boxed{x^2+y^2=C}$$

These are set of stream-line curves for vector field



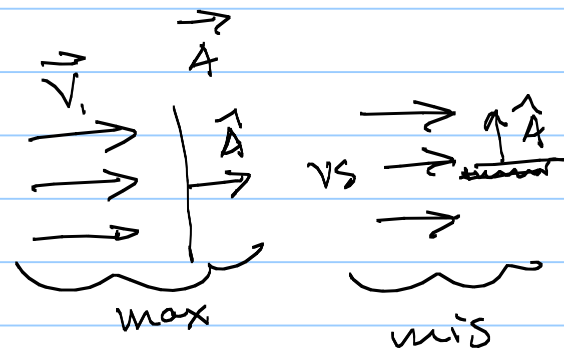
FLUX Foreshadowing ⇒ from phys 2000

If you have a continuity equation it ⇒'s things don't bunch up! They move along

$$\underbrace{\vec{A}_1 \vec{V} = \vec{A}_2 \vec{V}_2}_{\text{in general}}$$

$$\vec{A}_1 \cdot \vec{V} = \vec{A}_2 \cdot \vec{V}_2$$

Call Flux!



If no "sources" Vector Flux is conserved!

$\vec{J}$, say,

$$\therefore \quad \left.\begin{aligned} \vec{A}_1 \cdot \vec{E}_1 &= \vec{A}_2 \cdot \vec{E}_2 \\ \text{or} \quad |E_2| &= \frac{A_1}{A_2} E_1 \end{aligned}\right\} \text{Gauss-Law idea}$$