It seems our job then is to find
$\vec{E}(\vec{r})$ due to some 'source' charge distributions.

Brute Force

Will find
1.) it's tough job because it's a vector
2.) Can exploit symmetry to make life a bit easier sometimes

Oops, 2b ... tricks of course

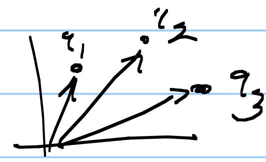
3.) Last --- Since $\vec{E}(\vec{r})$ = conservative
can write
$$\vec{E} = \vec{\nabla}\varphi$$
& then
$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla}\varphi \cdot d\vec{l} = \text{path indep}$$
$$\therefore = \varphi(\vec{b}) - \varphi(\vec{a}) \text{ only}$$

so will instead of solving for $\vec{E}$,
solve $\varphi$ = SCALAR so Easier

then get $\vec{E}$ from $-\vec{\nabla}\varphi$

# Brute Force!

I.) Discrete charge distributions

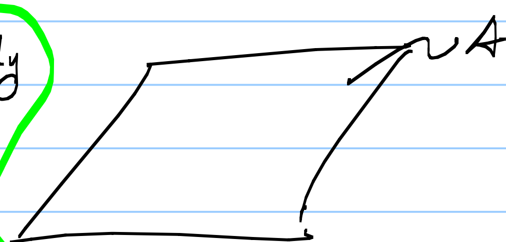



II.) Continuous Line charge distrib


; $\frac{Q_{TOT}}{L} = \lambda = \frac{\text{charge}}{\text{length}}$

in general $\lambda = \lambda(x)$
not nec const

III.) Continuous Area charge distrib




; $\frac{Q_{TOT}}{A} = \sigma = \frac{\text{charge}}{\text{area}}$

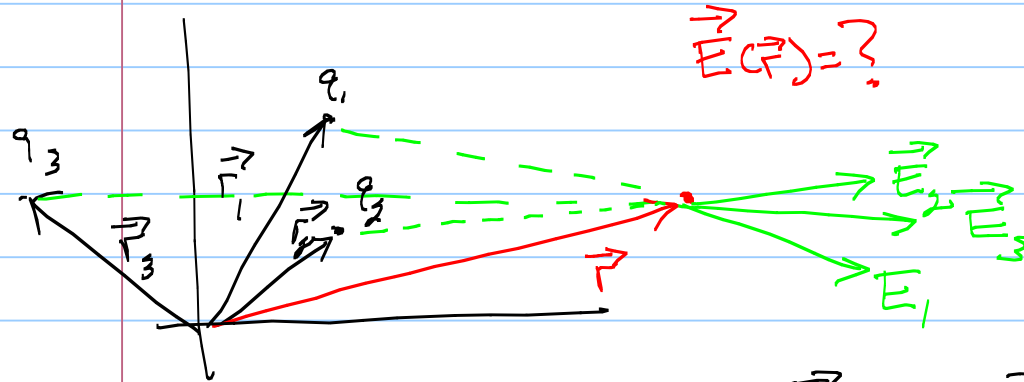
in general $\sigma \to \sigma(x,y)$
not nec const

IV.) Continuous Volum charge distrib


; $\frac{Q_{TOT}}{\forall} = \rho = \frac{\text{charge}}{\text{vol}} \to \rho(x,y,z)$
not nec const

* obviously $\lambda, \sigma, \rho$ are constant If charge is evenly distributed

# I.) discret charge distribution




Clearly: $\vec E(\vec r)$ = vector sum $\vec E_1(\vec r) + \vec E_2(\vec r) + \vec E_3(\vec r)$

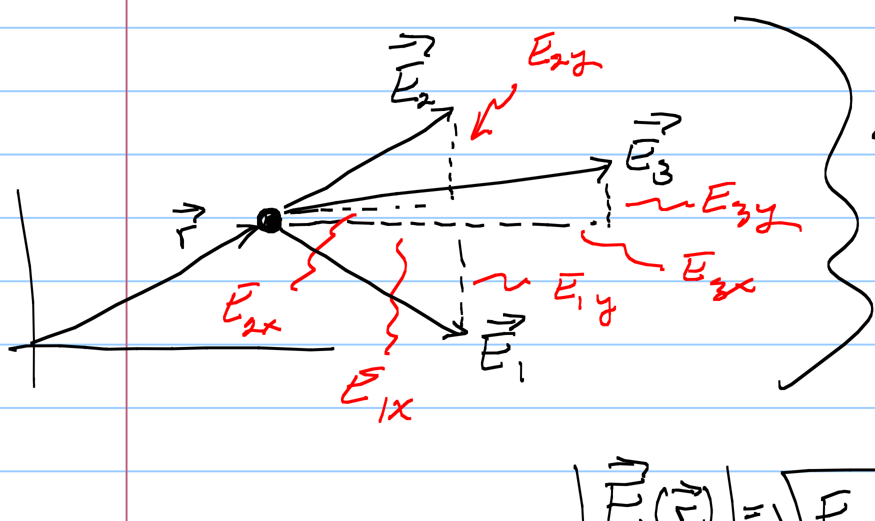
or

$$\vec E(\vec r) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|\vec{\mathfrak{r}}_i|^2} \hat{\mathfrak{r}}_i$$

where $\vec{\mathfrak{r}}_i \equiv \vec r - \vec r_i$ source

ie SUPERPOSITION

Not an algebraic sum so gets messy.



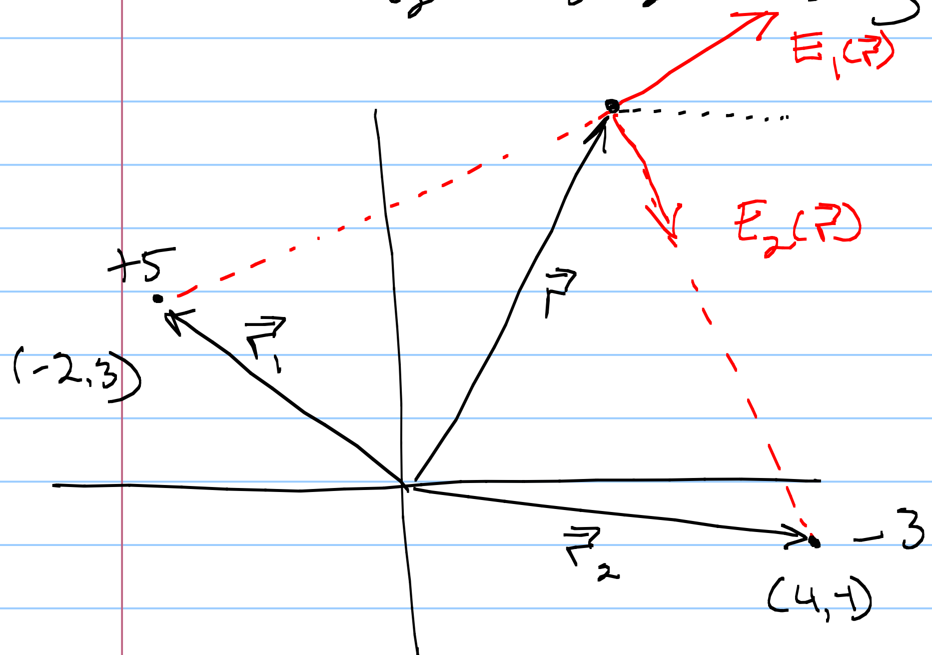

so $\vec E(\vec r) = (E_{1x} + E_{2x} + E_{3x})\hat i + (E_{1y} + E_{2y} + E_{3y})\hat j$

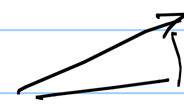
&

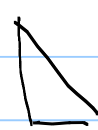
$|\vec E(\vec r)| = \sqrt{E_{TOT_x}^2 + E_{TOT_y}^2}$ w/ dir $\theta = \tan^{-1} \frac{E_{TOT_y}}{E_{TOT_x}}$

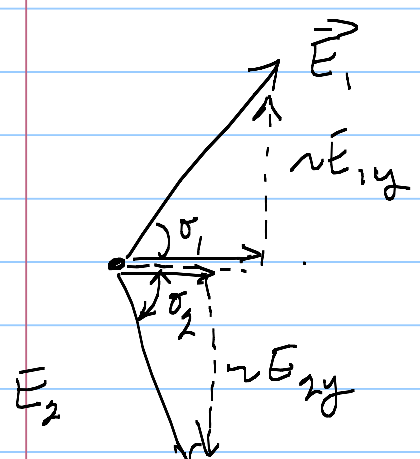
yikes! Messy

H.W.: $q_1 = +5C$, $\vec{r}_1 = (-2,3)$
$q_2 = -3C$, $\vec{r}_2 = (4,-1)$ } Find $E(\vec{r})$
$\vec{r} = (2,6)$



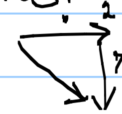

$\vec{\mathfrak{r}}_1 = (2,6) - (-2,3) = (4,3) =$  , $|\vec{\mathfrak{r}}_1| = \sqrt{25}$

$\vec{\mathfrak{r}}_2 = (2,6) - (4,-1) = (-2,7) =$  , $|\vec{\mathfrak{r}}_2| = \sqrt{53}$




$$\vec{E}(\vec{r}) = k\left[\frac{q_1}{|\mathfrak{r}_1|^2}\hat{\mathfrak{r}}_1 + \frac{q_2}{|\mathfrak{r}_2|^2}\hat{\mathfrak{r}}_2\right]$$

$$= k\left[\frac{5q}{25}\left(\frac{4}{5}\hat{x} + \frac{3}{5}\hat{y}\right) + \frac{-3q}{53}\left(\frac{-2}{\sqrt{53}}\hat{x} + \frac{7}{\sqrt{53}}\hat{y}\right)\right]$$

Note $-3q$ means attractive!
so points $(+2,-7)$ = 

$$= kq\left[\left(\frac{4}{25}\hat{x} + \frac{3}{25}\hat{y}\right) + \left(\frac{6}{53\sqrt{53}}\hat{x} + \frac{-21}{53\sqrt{53}}\hat{y}\right)\right]$$

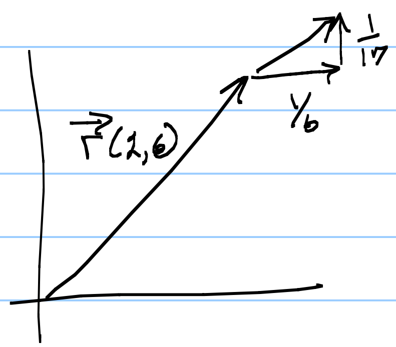
$$53\sqrt{53} \cong 7 * 50 = 350$$

$$= kq\left[\left(\frac{4}{25} + \frac{6}{350}\right)\hat{x} + \left(\frac{3}{25} - \frac{21}{350}\right)\hat{y}\right]$$

$$\sim kq\left[\frac{56+6}{350}, \frac{42-21}{350}\right]$$

$$\sim kq\left[\frac{1}{6}\hat{x}, \frac{1}{17}\hat{y}\right]$$

So

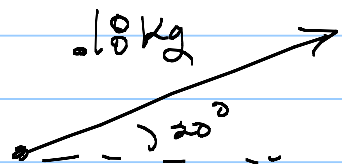



$$\left|\vec{E}(\vec{r})\right| = kq\sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{17}\right)^2} = .18\,kq$$

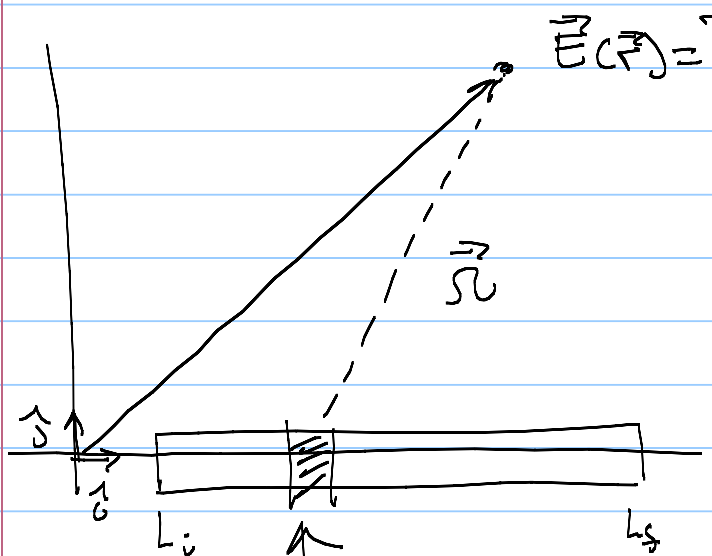
dir

$$\theta = \tan^{-1}\frac{\frac{1}{17}}{\frac{1}{6}} = \tan^{-1}\frac{1}{3} \sim 20^{\circ}$$

20

$\vec{E}(\vec{r}) =$ 

II.) Continuous ($\infty$-ly thin) Line distribution




start for continuous distributions!

$$\vec{E}(\vec{r}) = \int \underbrace{d\vec{E}(\vec{r})}$$

infinitesimal $d\vec{E}(\vec{r})$ due to infinitesimal $dq$

here $dq = \lambda dr$

so $$dE(\vec{r}) = \frac{K\,dq}{|\vec{\mathscr{R}}|^2}\vec{\mathscr{R}} = \frac{K\lambda dr}{|\vec{r} - \vec{r}_{source}|^2}\widehat{(\vec{r} - \vec{r}_{source})}$$

Now notice $\vec{r}$ = field position = Constant! not integrating over it!

Then get tired of writing $\vec{r}_{source}$

So say $\left.\begin{array}{l}\vec{r} = \text{field position still} \\ \vec{r}\,' = \text{source position}\end{array}\right\}$ now, what are you integrating over?

$$d\vec{E}(\vec{r}) = \frac{k\lambda\, dr}{|\vec{r}-\vec{r}\,'|^2}\,\widehat{(\vec{r}-\vec{r}\,')}$$

Clearly, need to integrate over $r'$

so

$$d\vec{E}(\vec{r}) = \frac{k\lambda\, dr'}{|\vec{r}-\vec{r}\,'|^2}\,\widehat{(\vec{r}-\vec{r}\,')}$$

Dummy variable idea again

Then $$\vec{E}(\vec{r}) = \int d\vec{E}(\vec{r}) = \int_{L_i}^{L_f} \frac{k\lambda\, dr'}{|\vec{r}-\vec{r}\,'|^2}\,\widehat{(\vec{r}-\vec{r}\,')}$$


$A(x) = \int_0^x x^2\,dx$ yuck!

$A(x) = \int_0^x x'^2\,dx' = \frac{1}{3}x'^3\Big|_0^x = \frac{x^3}{3}$

idea: x = variable BUT want to integrate a 'dummy' variable 'up to' what variable x is

* in general,

$\widehat{(\vec{r}-\vec{r}\,')}$ = unit vector

& it is NOT constant

∴ you CAN'T take it out of the integral

* note $|\vec{r}-\vec{r}\,'|^2$ = scalar so not a problem!

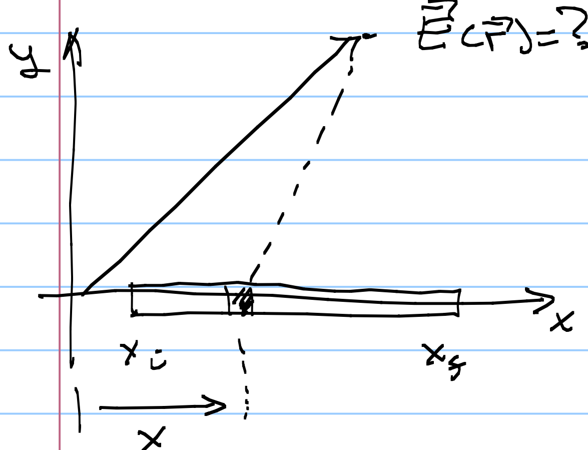
So $\int(\text{stuff})\,\hat{x} = \hat{x}\int(\text{stuff})$ cause $\hat{x}$ = const

but $\int(\text{stuff})\,\hat{\mathfrak{r}} = \int(\text{stuff})\,\hat{\mathfrak{r}}$

So for continuous charge distributions,

1.) Build the infinitesimal from $dq$
2.) Build the whole!

So specifical




$$d\vec{E}(\vec{r}) = K \frac{dq}{|\vec{\mathscr{r}}|^2} \hat{\mathscr{r}}$$

$$dq = \lambda\, dx$$

$$\vec{\mathscr{r}} = (x, y) - (0, x)$$
$$= (x - x)\hat{i} + y\hat{j}$$

Wait: const x-posit of $\vec{r}$ variable x

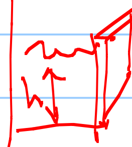
use dummy integration variable $x'$

$$d\vec{E}(\vec{r}) = \frac{K\lambda dx'}{\left((x-x')^2 + y^2\right)} \frac{(x-x')\hat{i} + (y)\hat{j}}{\sqrt{(x-x')^2 + y^2}}$$

clearly $\hat{\mathscr{r}} \neq$ const.
so can't remove from integral

$$d\vec{E}(\vec{r}) = \frac{K\lambda\, dx' \left((x-x')\hat{i} + y\hat{j}\right)}{\left[(x-x')^2 + y^2\right]^{3/2}}$$

my 1st being able to see this: Craig Richardson, I got surface area of sphere. then Engineering.




$P(h) = P_0 + \rho g x$

$dF = P\,dA$
$= P(x)\,dy\,dx$
$= P(x)\,dy\,dx'$

$F = \int dF = \int_0^h \int_0^L (P_0 + \rho g x')\,dy\,dx'$
$= \int (P_0 + \rho g x')\,dx' L$
$= P_0 h L + L\rho g \int_0^x x'\,dx'$

so

$$\vec{E}(\vec{r}) = \int d\vec{E}(\vec{r}) = \int_{x_i}^{x_f} \frac{K\lambda\left[(x-x')\hat{i} + y\hat{j}\right] dx'}{\left[(x-x')^2 + y^2\right]^{3/2}}$$

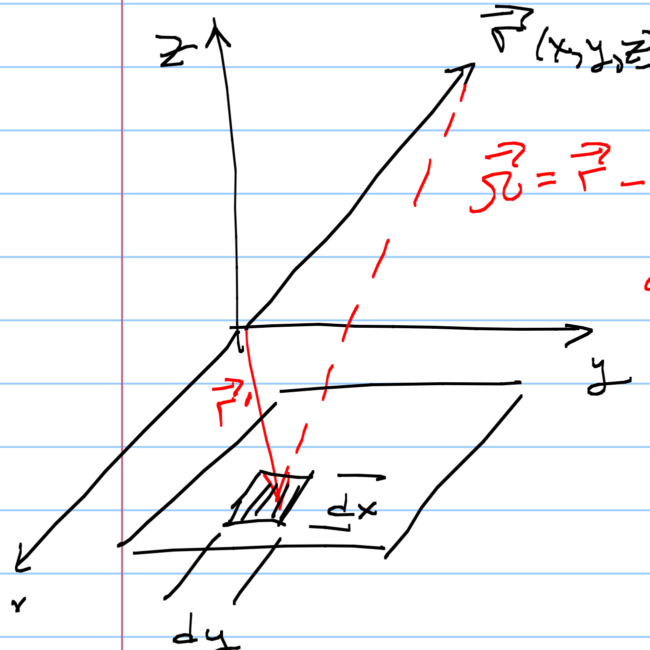
where $x$ & $y$ are obviously constant!

$\therefore$ get

$$E(\vec{r}) = \left[\int_{x_i}^{x_f} \frac{K\lambda(x-x')\,dx'}{[\quad]^{3/2}}\right]\hat{i} + \left[\int_{x_i}^{x_f} \frac{K\lambda y\,dx'}{[\quad]^{3/2}}\right]\hat{j}$$

clearly here, I have taken the $\hat{i}$ & $\hat{j}$ out of the integral's because they are constant, ie $\neq f(x')$

# III.) Continuous Area Distribution




$\vec{\mathscr{r}} = \vec r - \underbrace{\vec r'}_{\text{note source} \equiv \vec r'} = (x-x')\hat i + (y-y')\hat j + (z-0)\hat k$

Again: start by building $d\vec E(\vec r)$ from $dq$

$$dq = \sigma(x',y')\,dA$$
$$= \sigma(x',y')\,dx'dy'$$

Then $d\vec E(\vec r) = K\,\dfrac{\sigma(x',y')\,dx'dy'}{|\mathscr{r}|^2}\,\hat{\mathscr{r}}\,(x',y',z)$

$|\mathscr{r}|^2 = \vec{\mathscr{r}}\cdot\vec{\mathscr{r}}$

Clearly $\hat{\mathscr{r}}$ not constant

so

$$\vec E = \int d\vec E(\vec r)$$

$$= K\int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{\sigma(x',y')\left\{(x-x')\hat i + (y-y')\hat j + z\hat k\right\}}{\left[(x-x')^2 + (y-y')^2 + z^2\right]^{3/2}}\,dx'dy'$$

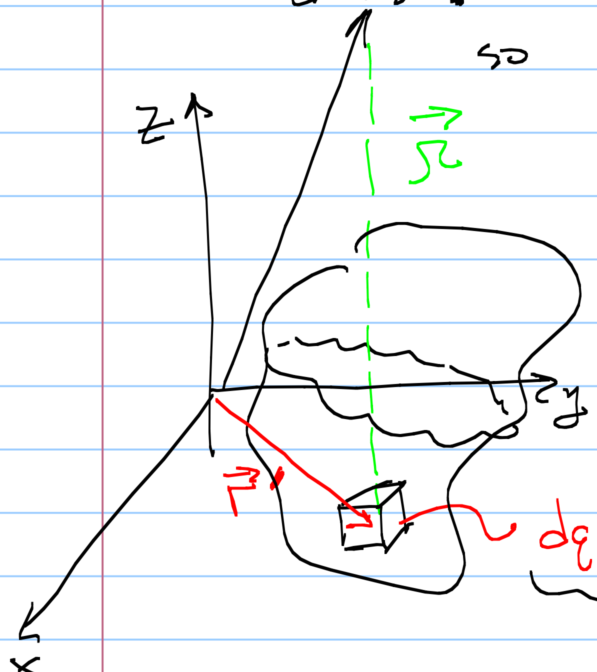
$\vec E = E_x\hat i + E_y\hat j + E_z\hat k$ = 3 - double integral

## IV.) Continuous Volume distributions

strategy: get $d\vec{E}(\vec{r})$ from $dq$

$\vec{E}(\vec{r}) = ?$ then $\vec{E}(\vec{r}) = \int d\vec{E}(\vec{r})$

so




$$\underbrace{dq = \rho(x',y',z')\,dx'dy'dz'}_{\text{then}}$$

$$d\vec{E}(\vec{r}) = \frac{k\,dq}{|\vec{\mathscr{r}}|^2}\,\hat{\mathscr{r}}$$

$\hat{\mathscr{r}}$ clearly not constant!

$$= \frac{k\,\rho(x',y',z')\,dx'dy'dz'}{|\vec{\mathscr{r}}|^2}\,\hat{\mathscr{r}} \quad \text{and}$$

$$\vec{E}(\vec{r}) = \int d\vec{E}(\vec{r}) = \int_{x_i}^{x_f}\int_{y_i}^{y_f}\int_{z_i}^{z_f} \frac{k\,\rho(x',y',z')\left\{(x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}\right\}dx'dy'dz'}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$= E_x\hat{i} + E_y\hat{j} + E_z\hat{k} \;=\; 3 \text{ messy triple integrals!}$$

Conclude: ALL 4-cases

$$dE \propto \frac{K\,dq}{|\mathfrak{r}|^2}\,\hat{\mathfrak{r}}$$

pain Because Not constant, a variable vector, ∴ can't take out of any integrals!

Will have to be

Clever

& Rely on
1.) Symmetries
2.) tricks possibly

to assist doing problems like this

Recall: Future: Give up on solving $\vec{E}$ vector directly!

Solve for scalar Electric potential, $V(\vec{r})$

Then $\vec{E}(\vec{r}) = -\vec{\nabla} V$