We are just trying to solve for $\mathbf{E}$!

1) Brute Force

$$\mathbf{E}(\mathbf{r}) = \int_{\text{space}} \frac{Q(r')}{r^2} dV'$$

2) Symm. Gauss's Law

$$\oint \mathbf{E} \cdot dA = \frac{Q_{\text{inside}}}{E_0}$$

Much of the trouble, especially brute force, is integrating over the vector $\mathbf{r}$ which is not constant.

So, can we be cleverer? Yes

Electric Potential $V(E) = \text{Scalar} = \frac{\text{energy}}{\text{charge}}$

$= \text{volt}$
\[ \vec{E} \cdot \text{d}l = \text{a flux depends on the path from } \vec{a} \text{ to } \vec{b} \]

Strongly path dependent

We saw this, pg 25.

\[ \vec{\nabla} \cdot \vec{V} = \pm \vec{V} \cdot \text{d}l \]

Then

\[ \int_{\vec{a}}^{\vec{b}} (\vec{\nabla} \cdot \vec{V}) \cdot \text{d}l = T(\vec{b}) - T(\vec{a}) \]

it is 2nd Theorem of gradients just like 2nd Theorem of calc

\[ \int_{x_1}^{x_2} (\frac{dF}{dx}) \, dx = F(x_2) - F(x_1) \]

Perhaps, \[ \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot \text{d}l \] is path indep.

If so, then \[ \vec{E} \] can be expressed as gradient of some Scalar

\[ \vec{E} = -\nabla V \] (*note \( V \), not \( \vec{V} = \text{vector field} \)
and since $V = \text{Electric potential} = \text{scalar}$
it may be EASIER to solve for

$V(r) \uparrow$ Then later take $-\nabla V$ to get $E$

YES! That's idea. OK then

1st establish that

$\int_{c_a} \mathbf{E} \cdot d\mathbf{l} = \text{path independent}$

$\Rightarrow \int_{c_a} \mathbf{E} \cdot d\mathbf{l} = \text{(either $V(b) - V(a)$ or $0$)}$

will use Stokes Theorem to help

$\int_{c_a} \mathbf{E} \cdot d\mathbf{l} = \int_A \nabla \times \mathbf{E} \cdot d\mathbf{A}$

is general for all Conservative Forces: NOTE

All 4 Force ARE Conservative! friction/charge clouds, two a bit
but definitely @ smallest

but definitely @ smallest level, don't see it, just Conservative Forces.
Consider (again, this development is static) the changes in distributions:

\[ E = \frac{k \phi}{r^2} \]

\[ \vec{E} \cdot d \vec{r} = \text{pt change} \theta \text{ The on gen} \]

\[ \oint \sum E \cdot d\vec{r} \Rightarrow \text{any coord system is begun spherical is easiest} \]

\[ d\vec{E} = dr \hat{r} + r d\phi \hat{\phi} + rsin\phi d\phi \hat{\theta} \]

\[ \oint \vec{E} = E \hat{r} \]

\[ \oint_{\sigma_0} E \sigma_0 \cdot (dr \hat{r} + r d\phi \hat{\phi} + rsin\phi d\phi \hat{\theta}) \]

\[ = \oint_{\sigma_0} E \sigma_0 dr = \oint_{\sigma_0} k \frac{\phi}{r^2} dr = -k \frac{\phi}{r} \left[ \frac{1}{r_0} - \frac{1}{r} \right] \]

Now is this path dependent? 

No! doesn't depend on \( \sigma_0 \) or \( \phi \) and clearly only depends on end pts.
Clearly \( \oint \vec{E} \cdot d\vec{l} = -\kappa \cdot \left[ \frac{1}{r_0} - \frac{1}{r_a} \right] \)

\[ \oint \vec{E} \cdot d\vec{l} = 0 \]

\[ \oint \vec{E} \cdot d\vec{l} = \oint \vec{E} \times \vec{E} \cdot d\vec{l} = 0 \]

\[ \nabla \times \vec{E} (\text{pt charge}) = 0 \]

New, clearly \( d\vec{A} \times \vec{E} \neq 0 \)

But also \( d\vec{A} \times \vec{E} \neq 0 \) AND Since \( A = \text{arbitrary} \) Anything and everything

Then \( \nabla \times \vec{E} (\text{pt charge}) = 0 \)

\[ \oint x \, dx = 3 \oint x^2 \, dx = 1 \]

But clearly integrands are not equal only way

\[ \oint \text{integrand} \, d\vec{x} = \oint \text{integrand} \, d\vec{x} \]

Dave Spiegel
Anchorage College
1968
\[
\begin{align*}
\text{so} \quad \int_0^b E \cdot dl & \neq \text{path dep} \\
\text{and} \quad \nabla \times \mathbf{E} & = 0
\end{align*}
\]

Change!

Clearly a "Bigger" at change distribution
\( \mathbf{E} \) field is built by vector superposition

\( \mathbf{E}_{\text{tot}} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots \)

Then \( \nabla \times \mathbf{E}_{\text{tot}} = \nabla \times \mathbf{E}_1 + \cdots \) \( \Rightarrow 0 \)

Thus

\[
\int_0^b \mathbf{E} \cdot dl = \text{path indep} \\
\text{For} \quad \forall \mathbf{E}_{\text{static}} \quad \text{Charge distributions}
\]

Thus is Key
Because \( \mathbf{E} = -\nabla \psi \)
it says that $E$ (static charge distributions) are IRROTATIONAL!

ie 
non-zero curls
$\neq 0$ Curls $
\Rightarrow$

$\nabla \times \mathbf{E} = 0$

Could
never ever be constructed by static charge distributions

$\nabla \times E_{\text{static charge}} = 0$
Ok in Business for static charge distribution.

Just got

\[ \oint_{\Delta} \mathbf{E} \cdot d\mathbf{S} = \Phi(\text{path}) \]  

we know this is only true when vector field

\[ \sum_{\Delta} (\mathbf{E} \cdot d\mathbf{S}) \]

But let see here

\[ \oint_{\Delta} (\mathbf{E}) \cdot d\mathbf{S} \]

looks alot like definition of work

\[ \int_{\Delta} (\mathbf{E}) \cdot d\mathbf{S} \]

so lets define a funct

\[ \frac{\text{Work}}{\text{change}} = \mathbf{V} \]

since the path from \( \mathbf{a} \) to \( \mathbf{b} \) doesn't matter

\[ \text{work} \iff \text{is between from } \mathbf{a} \text{ to } \mathbf{b} \text{ we'd like to associate a potential at a pt instead of between to points} \]
So we all agree on some \( \mathbb{E} = \mathbb{E}_0 \) then

\[
V(\mathbb{P}) = - \int_c^{\mathbb{P}} \mathbb{E} \cdot d\mathbb{L} = \text{Electric potential}
\]

\[
E_0 = \text{Energy}\quad \text{Coulomb} = \text{Volt},
\]

So

\[
V(c) - V(c) = - \int_c^{c} \mathbb{E} \cdot d\mathbb{L} + \int_c^{\mathbb{P}} \mathbb{E} \cdot d\mathbb{L}
\]

\[
= \int_c^{\mathbb{P}} \mathbb{E} \cdot d\mathbb{L} - \int_c^{c} \mathbb{E} \cdot d\mathbb{L}
\]

\[
V(c) - V(c) = - \int_c^{c} \mathbb{E} \cdot d\mathbb{L}
\]

ie path indep

But more formally

\[
\int_c^{\mathbb{P}} \nabla V \cdot d\mathbb{L} = - \int_c^{\mathbb{P}} \mathbb{E} \cdot d\mathbb{L}
\]

For both sides \( \mathbb{E} + 0 \mathbb{E} \)

is completely path independent
again This is key

\[ \int_0^1 f(x) \, dx = 2 \int_0^1 g(x) \, dx \]

But integrands \( \neq 0 \)

to be = how to be
true for all limits

We just showed

\[ \int_0^k \nabla V \cdot dl = - \int_0^k \nabla \cdot E \cdot dl \quad \text{for all limits } \]

\( E = - \nabla V \)