E.F. Deveney / BSC Physics PH438 Electric Potential Intro

Note Title 11/1/2004

We are just trying to solve for $\vec{E}$!

* For static charge Distributions

1.) Brute Force $\vec{E}(\vec{r}) = \int_{\text{space}} \frac{\rho(r')\,d\tau'}{|\vec{\mathscr{r}}|^2}\,\hat{\mathscr{r}}$

2.) Symm, Gauss's Law

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

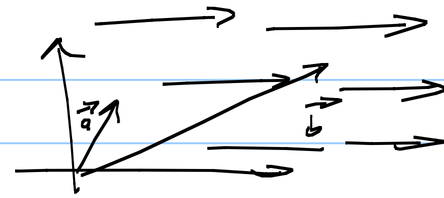
Much of the trouble, especially brute force, is integrating over the vector $\vec{\mathscr{r}}$, which isn't constant.

So, Can we be Clever-er? yes

Electric-Potential $V(\vec{r})$ = Scalar = $\frac{\text{energy}}{\text{charge}}$

= volt

$\vec{V}$ = vector field



How to get there...

we know

$$\int_{\vec{a}}^{\vec{b}} \vec{V} \cdot d\vec{l}$$

= HUGELY depends on the path from $\vec{a}$ to $\vec{b}$

strongly path dependant

we saw this, pg 25.

However, if $\vec{V} = \pm \vec{\nabla} T$

Then

$$\int_{\vec{a}}^{\vec{b}} (\vec{\nabla} T) \cdot d\vec{l} = T(\vec{b}) - T(\vec{a})$$

I.E. $\neq$ Path Depend

it is fund theorem of gradients just like fund theorem of calc

$$\int_{x_1}^{x_f} \left(\frac{df}{dx}\right) dx = f(x_f) - f(x_1)$$

Perhaps, $\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$ is path indep.

if so, then $\vec{E}$ can be expressed as gradient of some scalar

$\vec{E} = -\nabla V$ (*note V, not $\vec{V}$ = vector field)

and since V = Electric potential = scalar
it may be EASIER to solver for
$V(r)$ & then late take $-\vec{\nabla}V$ to get $\vec{E}$

YES! That's idea. OK then

1st establish that

$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$ = path indepont

1.) either $= V(\vec{b}) - V(\vec{a})$

or 2.) $\oint \vec{E} \cdot d\vec{l} = 0$

ie $\vec{E}$ = conservative

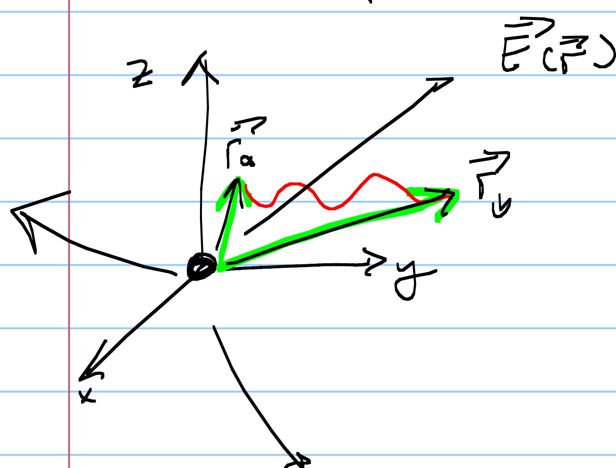
& discussion is general for all conservative forces: NOTE All 4 force ARE conservative / Friction clouds this a bit but definitely @ smallest level, don't see it, just conservative forces.

will use stokes theorem to help

$$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} = \int_A (\vec{\nabla} \times \vec{E}) \cdot d\vec{A}$$

consider (again, this development is Sop) static pt charge distributions ...

$$\vec{E} = K \frac{q}{r^2} \hat{r} \quad = \text{pt charge @ the origin}$$



consider

$$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} \Rightarrow$$ any coord system is legit, spherical is easiest

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\& \quad \vec{E} = E(r)\,\hat{r}$$

$$\int_{\vec{a}}^{\vec{b}} E(r)\,\hat{r} \cdot \left(dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}\right)$$

$$= \int_{\vec{a}}^{\vec{b}} E(r)\,dr = \underbrace{\int_{\vec{a}}^{\vec{b}} K \frac{q}{r^2}\,dr = -K\frac{q}{r}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]}_{\text{now is this path dependent?}}$$

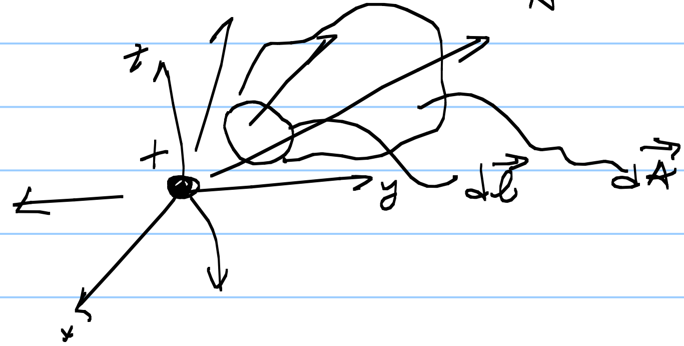
NO! doesn't depend on θ, or ϕ

and clearly only depends on end pts

Clearly $\int_{\vec{a}}^{\vec{b}} \vec{E}(\text{point charge}) \cdot d\vec{l} = -\frac{kq}{r}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$

$$\oint \vec{E}(\text{pt charge}) \cdot d\vec{l} = 0$$

$$\therefore \oint \vec{E}(\text{pt charge}) \cdot d\vec{l} = \int_A (\vec{\nabla} \times \vec{E}(\text{pt charge})) \cdot d\vec{A} = 0$$



Now, clearly $d\vec{l}$ & $\vec{E} \neq 0$
But also

$d\vec{A}$ & $\vec{E} \neq 0$ AND Since

idea

$\int_0^1 x\,dx = 3\int_0^1 x^2\,dx = 1$

But clearly integrands are not $=$

only way

$A =$ arbitrary – Anything and everything)

Then $\vec{\nabla} \times \vec{E}(\text{pt charge}) = 0$

$\int (\text{integrand } 1)\,dx = \int (\text{integrand } 2)\,dx$ is is you consider All limits

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So $\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} \neq$ path dep
and
$\vec{\nabla} \times \vec{E} = 0$
for pt charge!

Cleary a "Bigger" pt charge distribution $\vec{E}$ field is built by vector superposition

$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2 + \cdots$$

Then $\vec{\nabla} \times \vec{E}_{TOT} = \underbrace{\vec{\nabla} \times \vec{E}_1}_{=0} + \cdots = 0$

Thus $\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} =$ path indep
&
$\vec{\nabla} \times \vec{E} = 0$
For ALL Static Charge distributions

This is Key Because

& of course then $\vec{E} = -\nabla V$

it says that $\vec{E}$ (static charge distributions) are

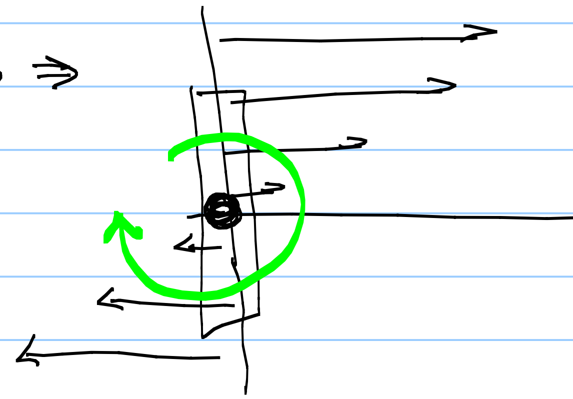
IRROTATIONAL!

ie

nonzero curls

$\neq 0$ Curls $\Rightarrow$

$\vec{V} = y\hat{x}$



Could

Never ever be constructed by static charge distributions

$$\vec{\nabla} \times \vec{E}(\text{static-charge dist}) = 0$$

OK in Business for static charge distributions

Just got

$\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} \neq f(\text{path})$ $\therefore$ we know this is only true when vector field

$\int_{\vec{a}}^{\vec{b}} (\vec{\nabla} T) \cdot d\vec{l}$

But let see here

$\int_{\vec{a}}^{\vec{b}} \left(\frac{\vec{F}}{q} \right) \cdot d\vec{l}$ looks alot like definition of WORK

$\vec{E}$

so lets define a funct

$\frac{\text{work}}{\text{charge}} = V$

& since the path from $\vec{a}$ to $\vec{b}$ doesn't matter

& work $\Rightarrow$ is between, from, $\vec{a}$ to $\vec{b}$, we'd like to associate
a potential at a pt instead of
between to points

So we all agree on some $\vec{c} = \vec{O}$rigin
then

V for any $\vec{r}$

$$V(\vec{r}) = -\int_{\vec{O}}^{r} \vec{E} \cdot d\vec{l}$$

= Electric potential

$$\frac{E}{q}\, dr = \frac{\text{Energy}}{\text{Coulomb}}$$

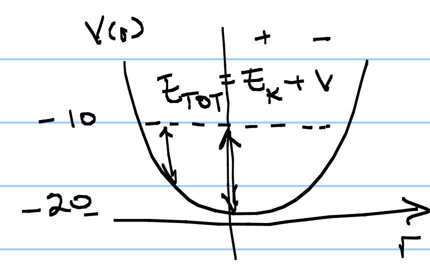
= Volt.

minus: convention
For BOUND system! $E_{TOT} = (-)$

so

$$V(\vec{b}) - V(\vec{a}) = -\int_{O}^{\vec{b}} \vec{E} \cdot d\vec{l} + \int_{O}^{\vec{a}} \vec{E} \cdot d\vec{l}$$

$$-\int_{O}^{\vec{b}} \cdot d\vec{l} - \int_{\vec{a}}^{O} \vec{E} \cdot dl$$



$E_K = E_{TOT} - V$

$-10 - (-20) +$

$$V(\vec{b}) - V(\vec{c}) = -\int_{\vec{c}}^{\vec{b}} \vec{E} \cdot d\vec{l}$$

ie path indep

But more formally

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} V \cdot d\vec{l} = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$$

$\vec{a}$ to $\vec{b}$

For both sides, $\vec{a}$ to $\vec{b}$ is completely path independ again This is Key

$$\int_0^1 dx = 2\int_0^1 x\,dx$$

But integrands $\neq$

To be $=$, has to be True for all limits

We just showed

$$\int_{\vec{0}}^{\vec{b}} \vec{\nabla} V \cdot d\vec{l} = -\int_{\vec{0}}^{\vec{b}} \vec{E} \cdot d\vec{l}$$ for all limits & paths

$$\vec{E} = -\vec{\nabla} V$$