\[ + + + + \]
\[ \theta \rightarrow \mathbf{E} = \mathbf{E}_{\text{uniform}} \]
\[ \mathbf{E}_{\text{ext}} = \mathbf{E}_{\text{uniform}} \]
\[ \mathbf{E} = \mathbf{E} \times \mathbf{n} \]
\[ \mathbf{F}_+ + \mathbf{F}_- = \mathbf{F} \]
\[ \mathbf{E} \times \mathbf{p} = \mathbf{F} \]
\[ \mathbf{F}_+ + \mathbf{F}_- = 0 \]
\[ \text{Neutral atoms stretch} \]
\[ \text{forming tiny dipoles (induced)} \]
\[ \mathbf{p} = \alpha \mathbf{E} \]
\[ \text{Polar molecules have } \mathbf{p} \text{ already} \]
\[ \text{Net by the field} \]
\[ \mathbf{E} \times \mathbf{p} = \mathbf{F} \]
\[ \mathbf{F} \times \mathbf{E} \]
\[ \mathbf{E} \times \mathbf{E} = 0 \]
\[ \text{No translation} \]
In non-uniform \( E \) field, dipoles... 

\[ \mathbf{F} = q \left( E_+ - E_- \right) = q \Delta \mathbf{E} \]

well

\[ dE_x = \frac{dE_x}{dx} \, dx \]

\[ \Delta E_x = (\nabla E_x) \cdot d \]

or

\[ \Delta \mathbf{E} = (\nabla \cdot \mathbf{A}) \mathbf{E} \]

\[ \therefore \mathbf{F}_{\text{net}} = \varepsilon (\nabla \cdot \mathbf{A}) \mathbf{E} = (\mathbf{A} \cdot \nabla) \mathbf{E} \]

So dipoles \textit{transcend}! 

But to see this, \( \nabla E_x \) must be \( P_{xy} \) not \( Q_{xy} \). 
Griss: 4.7

Energy of an ideal dipole $\vec{p}^2$ in $\vec{E}$ is

$$\vec{E} = \text{uniform} = E \hat{a}_y$$

Once at origin

Now rotate $\vec{p}$ toward $\vec{E} = 0$

Now dipole $\vec{p} \times \vec{E} = p |\vec{E}| \sin \theta = \vec{p} E \hat{a}_y$

So we have to "oppose" work by the size $\vec{H} = ?$
So \( W = U \) work by \( \vec{F} \) \( (W = \text{work done by force} \implies \text{potential energy}) \)

\[ U = \text{U stored in field} = -\int \vec{E} \cdot d\vec{r} \]

\[ = \int_{S} \rho E \sin \theta \, d\sigma + \rho E (-\cos \theta) \bigg|_{\sigma=\pi/2}^{\sigma=0} \]

\[ = -\rho E \left[ \cos(\theta) - \cos(0) \right] \]

\[ = -\rho E \left[ 0 \right] \]

\[ = -\rho E \]

U = stored in Field = - \( \rho E \cos \theta \)

( dipole + Electric field)

or \( U = -\vec{p} \cdot \vec{E} \)
Chapter 4.8

Interaction energy of two dipoles separated by a displacement $\vec{r}$

$$\vec{P}_1, \vec{P}_2$$

Since: $U = -\vec{P}_1 \cdot \vec{E}$ say $\vec{P}_1$ in field of $\vec{P}_2$

$$= -\vec{P}_1 \cdot \vec{E}_2(\vec{r})$$

Page 155-156

$$\vec{E}_2 = \vec{E}_2(r, \theta) = \frac{1}{r^3} \left( \sin \theta \hat{\theta} + \cos \theta \hat{r} \right)$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} \left[ \hat{\theta} \left( \hat{r} \cdot \vec{P}_2 \right) \cdot \frac{\hat{r}}{r^2} - \vec{P}_2 \right]$$

Page 3.10.4
\[
U = \frac{1}{8\pi\varepsilon_0} \frac{1}{r^3} \left[ \mathbf{r}_1 \cdot \mathbf{r}_2 - \frac{3}{r^2} \left( \mathbf{r}_1 \cdot \mathbf{r}_3 \right) \mathbf{r}_2 \right]
\]

So it's the \( \frac{1}{r^3} \) energy.

Dane uses \( \mathbf{r}_3 \) here.

Quantum computer!
So, in general we are talking about mostly alignment of index $\vec{p}_i$ in $\vec{E}_0$ external.

$\vec{p}_i \parallel \rightarrow \vec{E}_0$

"Dipolarized" material talks about $\vec{P} = \frac{\varepsilon \vec{E}_0}{\varepsilon_0}$

So, $\vec{P} = \sum \vec{p}_i \Rightarrow$ total dipole

$\varepsilon \vec{P} = \frac{\vec{P}}{\varepsilon_0} \Rightarrow$ dipole density

$\varepsilon \vec{P}_i \Rightarrow$ single dipole
Our goal is to find $\mathbb{E}$ always.

Now we want to get $\mathbb{E}$ in (dielectric) why... because it happens often!

i.e.

\[ \text{Air} \rightarrow \text{Dielectric} \]

\[ \text{Vacuum} = \text{Dielectric} \]

\[ \text{Capacitor} \]

H umans
So will will start w/ finding $E^s$ yields due little 'chunks' of dielectrics.

You can do this:

\[ E^s = \frac{\partial \psi}{\partial t} \]

\[ + \text{ some materials} \]

\[ \text{"retain"} \]

\[ \text{polarization} \]

\[ \text{after } E^s_{\text{ext}} = 0 \]

For single dipole $\vec{p}$:

\[ V(\vec{r}) = \frac{k \cdot \vec{p} \cdot \vec{r}}{\vec{r}^2} \]

So

\[ V(\vec{r}) = \frac{k \cdot \vec{p} \cdot \partial(\vec{r})}{\vec{r}^2} \]
Where ultimately Gauss' pots

\[ V(\mathbf{r}) = \kappa \int_{\text{Surf}} \frac{1}{\sqrt{2}} \mathbf{p} \cdot d\mathbf{A}' - \kappa \int_{\text{Vol}} \mathbf{p} \cdot d\mathbf{A} \]

which looks like

\[
\begin{align*}
\text{Bound Charge density} & = \rho_{\text{b}} = \mathbf{E} \cdot \mathbf{n} \\
\text{Bound Charge volume} & = \mathbf{P} \cdot \mathbf{n} = \text{Bound Density}
\end{align*}
\]

Because the source see there are bound dipoles!

So

\[ V(\mathbf{r}) = \kappa \left[ \int_{\text{Polarized Zone}} \mathbf{E} \cdot d\mathbf{A}' + \int_{\text{Surf}} \frac{\rho_{\text{b}}}{\varepsilon_0} d\mathbf{A} \right] \]

looks like

\[ dV(\mathbf{r}) = \frac{\kappa \rho}{\varepsilon_0} d\mathbf{A} \quad \text{or} \quad \kappa \delta d\mathbf{A} \quad \text{for Free Charges} \]
\[ V_{\text{polarized}} = \sum \frac{ke^2}{|z_n|} \text{ for Bound } e^2 \delta \]

\[ \vec{P} \]

\[ \text{cancel in between, left w/ } \vec{\sigma} \text{ on surface} \]

\[ \text{new is surface} \]

\[ \vec{P} \cdot \vec{n} \]

\[ \vec{\sigma} \]

\[ \text{IS polarization = UNIFORM} \]

\[ \text{CLEARLY } - + - + - + \text{ cancel, } S_{\text{bound}} = 0 \]

\[ \text{IS not uniform} \]

\[ \int S \delta \, d\Omega = -S \int \vec{P} \cdot dA = -S \vec{\sigma} \cdot d\vec{A} \]
Example 4.2 pg 168

So, for example, consider a uniformly (ALREADY Polarized) fixed sphere of radius $R$.

Choose $\hat{z} \parallel \vec{n}$

Then

1) $\vec{E}_r = \vec{n} \cdot \hat{r} = |\vec{n}| \cos \theta$

2) $\vec{E}_\theta = 0$ (uniform)

Find

Looks like a bulky dipole

$E(r > R)$

$E(r < R)$

$E_{\text{inside}}$
So:

The effect of polarization is to produce accumulations of bound charge, 

\[ \mathbf{\mathcal{E}}_b = -\nabla \cdot \mathbf{\mathcal{D}} \]  \text{ within dielectric and} 

\[ \mathbf{\mathcal{D}}_b = \mathbf{\mathcal{P}} \cdot \mathbf{n} \]  \text{ on the surface.}

The field due to polarization of the medium is just the field of the bound charge.
With the idea of $T_6$ & $S_{bond}$ changes of dielectric, polarized, material.
It is now time to solve for $E$ inside a Dielectric.
Consider:

Special metallic bonds, now

dielectrics = more

Common = Covalent

Vacuum

Helium

Air

Water

Silicon

Methanol

ICE

and so on

So HUGE!

\[ E_{\text{due to } +e} = \text{"Free-charge"} \]

Note \[ E_{\text{due to "Bound" charges induced}} \] REDUCE \[ E_{\text{inside}} \]
So

Conductor  \[ \begin{array}{c}
\text{dielectric} \\
\text{All } \vec{E}_{\text{inside}} = 0 \\
\end{array} \]

So

\[ \text{dielectrics = imperfect conductor!} \]

Don't quite make

\[ \text{\textbf{outside}} \quad \text{\textbf{Zero}} \]

But they do Diminish the Sfields

set up by Free charges or External Sfields.
In a dielectric:

\[ \nabla \cdot \vec{E} = \rho \text{ free charges} / \varepsilon_0 \]

or

\[ \varepsilon_0 \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

But \( \vec{E} \) is what induces atoms or molecules to polarize.

\( \varepsilon \) to create bound charge density (i.e., due to the dipoles).

Where

\[ \vec{P}_{\text{bound}} = -\nabla \cdot \vec{P} \]

2: So

\[ \varepsilon_0 \nabla \cdot \vec{E} = \rho_{\text{free}} - \rho_{\text{bound}} = \rho_{\text{free}} - \nabla \cdot \vec{P} \]

How cause call plus opposite free change E-field.

\[ \varepsilon_0 \nabla \cdot \vec{E}_{\text{total}} \text{ inside} \]
so \( \varepsilon_0 \vec{E}_{\text{tot}} = \vec{P}_{\text{inside}} \)

\[ \nabla \cdot (\varepsilon_0 \vec{E}_{\text{tot}} + \vec{P}) = \rho_{\text{free}} \]

\[ \text{Now } \text{Polarization} = \frac{\# \text{dipoles}}{\text{Volume}} \]

For "linear" dielectrics \( \vec{P} \) is simply related to the \( \vec{E} \) field that creates it.

\[ \vec{P} = \varepsilon \vec{E} \]

ie Linear \( \vec{P} \) is \( \parallel \vec{E} \)

Very often, there are materials we will work with

Convenient to write as

\[ \vec{P} = \varepsilon_0 \chi \vec{E} \]

\[ \text{Electric Susceptibility} \]

\[ \varepsilon_0 \text{ permittivity of free space!} \]
So back to problem

\[ \nabla \cdot (\varepsilon_0 \vec{E}_{\text{inside}} + \vec{E}) = \sigma \text{ s/v/m} \]

\[ \nabla \cdot (\varepsilon_0 \vec{E}_{\text{Tot}} + \varepsilon_0 \varepsilon_r \vec{E}_{\text{inside}}) = \sigma \text{ s/v/m} \]

\[ \nabla \cdot \left[ \varepsilon_0 (1 + \varepsilon_r) \vec{E}_{\text{Tot}} \right] = \sigma \text{ s/v/m} \]

Now two Eqns work at once:

1. \[ \nabla \cdot \varepsilon \vec{E}_{\text{Tot}} \text{ inside} = \sigma \text{ s/v/m} \]

\( \varepsilon = \) permittivity of material

2. \[ \nabla \cdot \varepsilon_0 \varepsilon_r \vec{E}_{\text{Tot}} \text{ inside} = \sigma \text{ s/v/m} \]

\( \varepsilon_r = \) relative permittivity

\( \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \)

Displacement vector:

\[ \nabla \cdot D = \sigma \text{ s/v/m} \]

But then:

\[ \nabla \cdot \vec{E}_{\text{Tot}} \text{ inside} = \frac{1}{\varepsilon_0} \frac{\sigma \text{ s/v/m}}{\varepsilon} \]

Dielectric constant of material.
So you have

\[ \vec{E}_{\text{tot}} = \vec{E}_{\text{free}} \]

inside

Dielectrics

1) Solve \( \nabla \cdot \vec{D} = \sigma_{\text{free}} \), Solve \( \vec{D} \)

Then \( \vec{E} = \frac{\vec{D}}{\varepsilon} \)

2) Solve \( \nabla \cdot \vec{E}_{\text{tot}} = \frac{1}{\mu} \sigma_{\text{free}} \)

\( \frac{1}{\mu} = \varepsilon + \varepsilon_0 \)

Both are equivalent & look almost like

\[ \nabla \cdot \vec{E} = \frac{\sigma_{\text{free}}}{\varepsilon_0} \]

This was always assumed in

\& They are!

in a Dielectric as we discussed

\( \varepsilon_{\text{inside}} \) is simple reduced, which is born out by both \( \varepsilon \) & 2
So $ \nabla \cdot \vec{D} = \sigma_f$

Then $\vec{E} = \frac{\vec{D}}{\varepsilon}$

or $\nabla \cdot \vec{E} = \frac{1}{\varepsilon_r} \frac{\sigma_r}{\varepsilon_0}$

or $\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_r} \frac{\sigma_r}{\varepsilon_0}$

Gauss Law Rem

$\vec{E}_{\text{tot}} = \text{Total } \vec{E} \text{ field inside dielectric in due to }$

1. Some charges that make an $\vec{E}$

2. Then that $\vec{E}$ induces $\vec{p}$'s (dipoles) which oppose $\vec{E}$

3. Add $\vec{E}$ is the total $\vec{E} \in \text{WLD}$

Note we never get lost in the particulars of the bound charges induced on the dielectric except for the fact it is all conveniently swept under rug w/ $\vec{E} = \varepsilon_0 \vec{E}$ --- i.e. linear dielectric.

So all the "mess" or "complication" of dielectric is stowed in constants

1. $\vec{E}_{\text{tot}} = \frac{\vec{D}}{\varepsilon} = \text{permittivity}$

2. $\nabla \cdot \vec{E}_{\text{tot}} = \frac{\sigma_{\text{free}}}{\varepsilon_0}$
So:

\[ \vec{E}(r) = \frac{q}{4\pi \varepsilon_0 r^2} \]

by div theorem

\[ \nabla \cdot \vec{D} = \frac{\varepsilon_r \varepsilon_0}{\varepsilon_0} \]

Now IMMERS

That problem in WATER = DIELECTRIC

By 2 methods

In either CASE!

\[ \vec{E}_{\text{dil}} \text{ is reduced by } \frac{1}{\varepsilon_r} \]

dielectric const
So, I usually do it this way

\[ \nabla \cdot \mathbf{D} = \frac{1}{\varepsilon_r} \varepsilon_0 \nabla \cdot \mathbf{E} \quad \text{or Gauss's Law} \quad \oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_r} \nabla \cdot \varepsilon_0 \mathbf{D} \]

where \( \varepsilon_r \) = dielectric constant = 'How much \( \mathbf{E} \) is reduced in a dielectric.'

So: Table 4.2 (C. A. Compton, 'Handbook of Chemistry and Physics')

<table>
<thead>
<tr>
<th>Material</th>
<th>dielectric constant, ( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0000</td>
</tr>
<tr>
<td>Helium</td>
<td>1.000065</td>
</tr>
<tr>
<td>Neon</td>
<td>1.00013</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1.00025</td>
</tr>
<tr>
<td>A. argon</td>
<td>1.00052</td>
</tr>
<tr>
<td>A. air (dry)</td>
<td>1.00054</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.00055</td>
</tr>
<tr>
<td>Water vapor (100°C)</td>
<td>1.00587</td>
</tr>
<tr>
<td>Benzene</td>
<td>2.38</td>
</tr>
<tr>
<td>Diamond</td>
<td>5.7</td>
</tr>
<tr>
<td>Salt</td>
<td>5.9</td>
</tr>
<tr>
<td>Silicium</td>
<td>11.8</td>
</tr>
<tr>
<td>Methanol</td>
<td>33.0</td>
</tr>
<tr>
<td>Water</td>
<td>80.1</td>
</tr>
<tr>
<td>Ice</td>
<td>99</td>
</tr>
</tbody>
</table>
\[ \nabla \cdot \vec{E} = \frac{1}{\varepsilon_r} \frac{\sigma_{\text{free}}}{\varepsilon_0} \]

\[ \oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_r} \frac{\sigma_{\text{tot}}}{\varepsilon_0} \]

Inside dielectric: Gauss #1 inside

\[ E(z) \ 2\pi S \ \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_r} \frac{\sigma_{\text{free}}}{\varepsilon_0} \]

\[ \vec{E}(z) = \frac{1}{\varepsilon_r} \left( \frac{\sigma_0}{2\pi\varepsilon_0} \hat{z} \right) = \frac{1}{\varepsilon_r} \ \text{b/o dielectric} \]

\[ \text{outside dielectric:} \]

\[ \vec{E}(z) = \frac{1}{\varepsilon_r} \left( \frac{\sigma_0}{2\pi\varepsilon_0} \hat{z} \right) \text{ but } \varepsilon_r = 1 \text{ for vacuum or air} \]
So once you get outside of dielectric, it is as if the dielectric is not even there, just free charges.

\[ \begin{array}{ccccccc}
+ & + & + & + & + & + \\
\end{array} \]

But no, new change can't!

So looking from outside, just see free.

\& not \& band, which must add to zero.

Cause don't make 'new' changes!
Now huge use of dielectrics is in capacitors:

```
  \[ C = \frac{\varphi}{\Delta V} \]
```

so the \( \varphi \) is what happens to the voltage inside cause \( \varphi \) is fixed by the battery.

Dwell since know \( E_{\text{in}} = \frac{E_{\text{tot}}}{E_r} \)

Then

\[
(V_+ - V_-)_{\text{dial}} = -\int S^+ E_{\text{inside}} \cdot d\ell
\]

\[
(V_+ - V_-)_{\text{dial}} = -\int S^+ \frac{E_{\text{tot}}}{E_r} \cdot d\ell = \frac{1}{E_r} \left( -\int S^+ E_{\text{tot}} \cdot d\ell \right)
\]

\[
(V_+ - V_-)_{\text{dial}} = \frac{\Delta V_{\text{tot}}}{E_r} \frac{\Delta V}{E_r}
\]

or \( \Delta V_{\text{dial}} \) is also reduced by \( \frac{1}{E_r} \)
So
\[ C = \frac{Q}{(\Delta V)_{\text{diele}}} = \frac{Q}{\Delta V} = \varepsilon_r \frac{Q}{\Delta V} \]

or
\[ C_{\text{dielec}} = \varepsilon_r C_{\text{vacuum}} \]

you↑ the capacitance!
Finally, let's talk about what really matters... Q.F.T.

Imagine a single $\gamma$ of zero size (0-D) in a vacuum = no atoms or molecules.

$$E = \frac{1}{r^2}$$

Normally, we say

$$E = \frac{1}{4 \pi \varepsilon_0} \frac{q^2}{r^2}$$

But $|E| \to \infty$ so $\gamma \to 0$, $|E| \to \infty$.

This means the virtual $\gamma$ fields can have $>1$ MeV energy to create $e^-e^+$ pairs. So

$\gamma = $ changeless.

So the Q.F. field vacuum is all of part + anti part pairs. Dielectric.

So $E$ we measure is actually reduced.
so the real measure is

\[ |E| = \frac{1}{\epsilon_r} \frac{\frac{q e^{-}}{4\pi \epsilon_0 \epsilon_r^2}}{r^2} \]

so what we actually see for \( \epsilon \)

is not the true \( \epsilon = \text{BARE} \)

it is

\[ \frac{1}{4\pi \epsilon_0 \epsilon_r^2} \left( \frac{q e^{-}}{3 \epsilon_r} \right) \]

we see "clothed" or screened charge, \( q \), of the \( e^- \) \( \epsilon \) never see the real \( q \) of \( e^- \).

Now \( \epsilon_r \to \infty \) \to radius of \( r \) gets close \( \to 0 \)

\[ \Rightarrow q e^- \text{ must } \to \infty \]

true, bare charge of \( e^- \to \infty \)
In QCD you get quite a different effect

QED, vacuum = dielectric screening changes reduced $|E|$ so $E$ sells off!

QCD

Quarks carry color charge

Gluons carry color charge

But gluons themselves carry color charge

So $g \gamma_5 b$, for example

The $r$ & $b$ colors create more gluons!
Now this effect of making more 'color' changes
by virtue of making so many
\textit{color} gluons

Causes, in essence, \underline{MORE} \underline{not less as}
\textit{in the case of QED \& charges photons, giving
reduced E fields} \underline{Color change \& \underline{\& \& \&}}

The \underline{quark} \underline{Color change}!
The net affect is
Ant. screening
& why
The force between quarks is not 1/s as you try to
separate!

In fact, 1/s color charge is strong sense
so much you CANNOT even make or
find an isolated quark!

= QUARK (constituant)

(other end, close, quarks look
sure = Asymptotic freedom)

2004 Nobel Prize in Physics
For using QCD on these 2 limits
1) David Gross UC Santa Barbara
2) Frank Wilczek CMU
3) David Politzer C*Tech