Conductors ⇒ think metals.

what makes a metal?

Metallic bonds! → heavier atoms

recall

G I G VII

$Na\cdot \rightarrow \cdot\ddot{Cl}:$

$(Na^+) \rightarrow \leftarrow (^-Cl)$

ionic Bond

atomic orbital (AO)

$H\cdot \quad \cdot H$

$H(:)H$

cavalent

Molecular orbital (MO)

Pg 325 chem

- ion-dipole
- dipole-dipole
- London Dispersive
- Hydrogen

} all $\ominus \rightarrow \leftarrow \oplus$

so

e−

Lots of closed shells

; $m_{nuc} \gg m_{e^-}$

$\cong 2000\, m_{e^-}$

so get

⊕ ⊕

⊕

⊕

$\cong$ fixed nuclei w/ $\cong$ free e^-'s

So e^-'s not confined to any 1 atom, they are essentially shared throughout

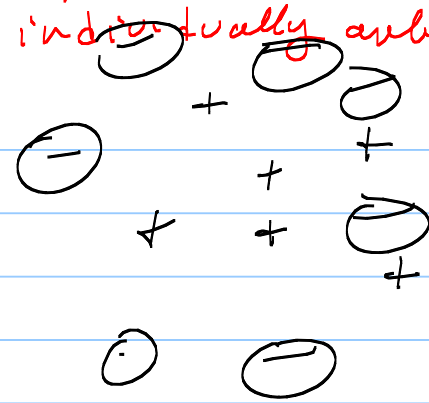
If you solve $i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \hat{H}\, \Psi(x,t)$

∿∿∿∿ ⇐ potential

e^-'s overlap w/ essentially

all atoms in metal, so as a whole, hold metal together but individually only weakly bound.

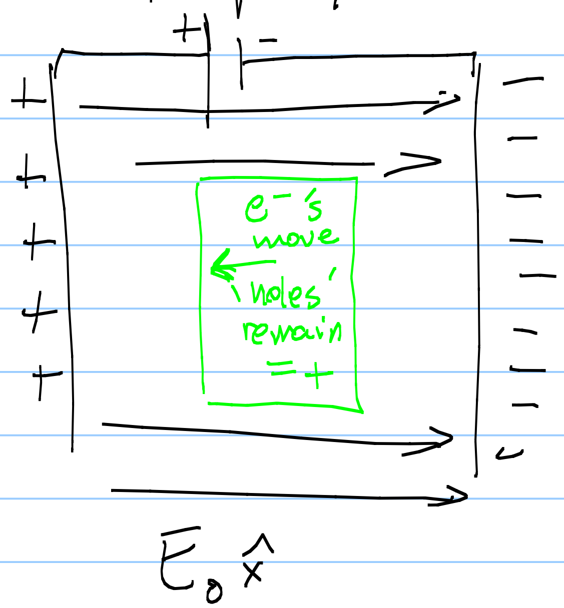
metal $\cong$



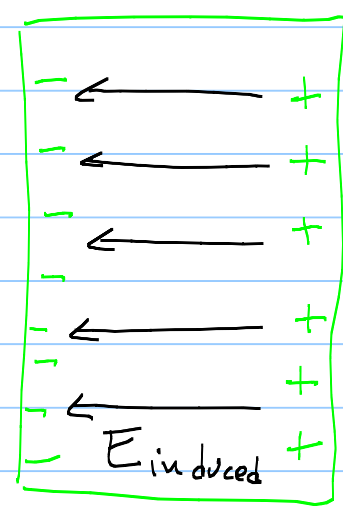
e^-'s Bind all the (+) cores together

Result: metals = Fixed (+) cores (atoms)
Free (−) (e^-'s)

So, now play w/ conductors:



so e^-'s move



e^-'s move until when?

in metal, e^-'s have essentially free ride to 'move' → 'respond' to any applied Electric field

$\sum \vec{F} = m\vec{a}$

$\vec{F}_{net} = m\vec{a}$

So e^-'s will accelerate w/ net Force until $\vec{F}_{net} = 0$

$$\sum \vec{F}_{ext} = m\vec{a}$$

$$\underbrace{-qE_i + qE_o}_{=0} \Rightarrow \vec{a} = 0$$ & e^-'s stop accelerating

Now $|E_i| \propto$ how many e^-'s
How many move?

well # of e^-s $> 10^{23}$ so lots &

recall $\left|\dfrac{F_{coulomb}}{F_{gravity}}\right| > 10^{34}$ so pretty strong

$\therefore$ as many e^-'s move as "required" to set up an $E_{induced} \leftarrow$
to counter $E_0 \rightarrow$

when that point is reached, (e^-'s) stop

Thats the big point, e^-'s are free so will always move just right until $\vec{E}_{induced}$ cancels E_0 ($E_{applied}$)

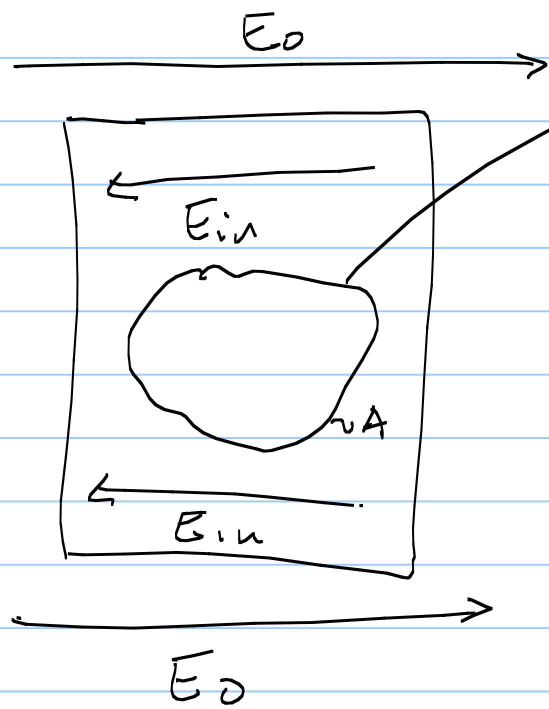
AND it all happens @ $\cong$ the speed of the $\vec{E}$ field wich is $= c$

So, almost instantly, e^-'s adjust to make $\vec{E}_{ind}$ cancel $\vec{E}_{ext}$

* If not enough e^-'s or $\vec{E}_{ext}$ too big, the metal would start to ionize

→ "Electrostatic Equilibrium" is set up quickly

So lets look @ the consequences of EE



$\vec{E}_{inside} = E_0 \rightarrow + E_{in} \leftarrow$

1) conductor $\vec{E}_{inside} = 0$

2.) Inside $\underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{use symm}} = \frac{q_{in}}{\varepsilon_0}$

$0 = q_{in}/\varepsilon_0$

∴ Net $q_{inside\ conductor} = 0$

or ρ_{net} = net charge density $= 0$ all +'s cancel

3.) where are the e^-'s? Well they repel each other

while they are pushed by the $\vec{E}$ fields.
So they will make up $\vec{E}_{induced}$ and be as far away from each other as can be.

So Combined w, $\rho_{inside} = 0$, and

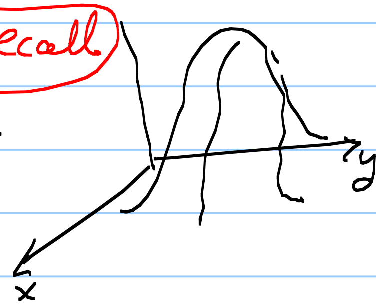
all charge on conductor resides on the surface of the conductor!

4.) ⇒ BIGGIE!

A conductor is an equipotential! of Electric Potential V

Recall HUGE Recall

mountain, $h(x,y) =$



& we argued $\vec{\nabla} h = \frac{\partial h}{\partial x}\hat{i} + \frac{\partial h}{\partial y}\hat{j} = \vec{grad}\, h =$

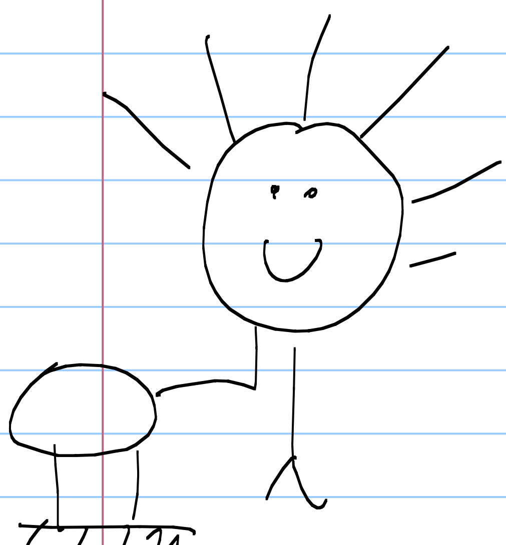
max space rate of change vector $\begin{cases} \to 1.)\ \text{mag} \\ \to 2.)\ \text{dir} \end{cases}$

& when we investigated the dir of $\vec{grad}\, h$, $\vec{\nabla} h$, we argue that is was everywhere ⊥ 'contours' of constant h

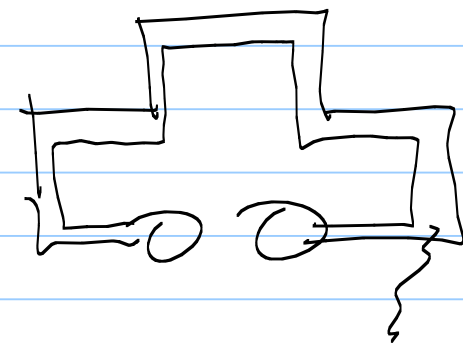
* we looked @ $\vec{\nabla} h \cdot d\vec{l} = |\vec{\nabla} h||d\vec{l}|\cos\theta = dh$

, for, $dh = 0$ but $d\vec{l} \neq 0$, $|d\vec{l}| = 1$, then $\vec{\nabla} h$ must be ⊥ to $d\vec{l}$ along $dh = 0$

Charges repel so move as far away as can be from each other!

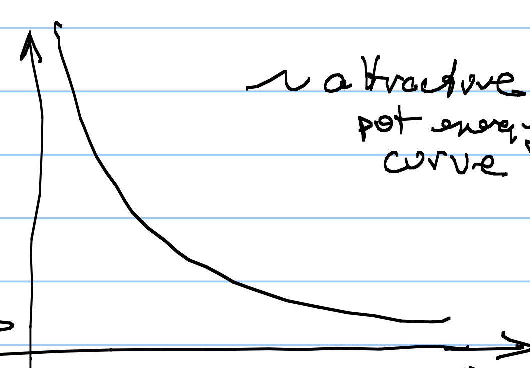


Van de Graass generator of electric potential via accumulation of charges



Car made of metal.

put 2 free e^-'s on it an they will see position to be further apart & to minimize potential energy



far min →

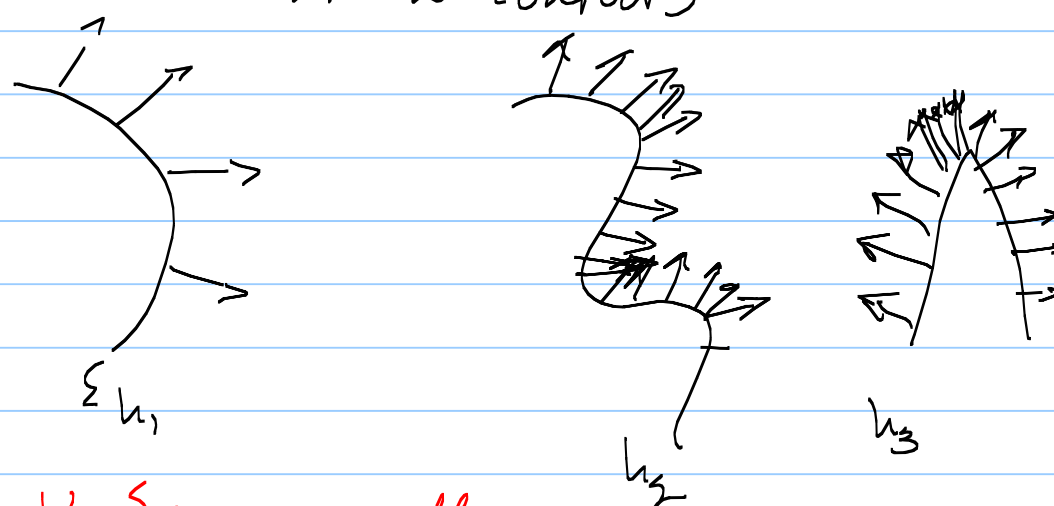
$\ominus \xleftrightarrow{r} \ominus$ $|r| = \max!$ as possible

So $\vec{\nabla} h = \perp$ to 'contours' of constant /

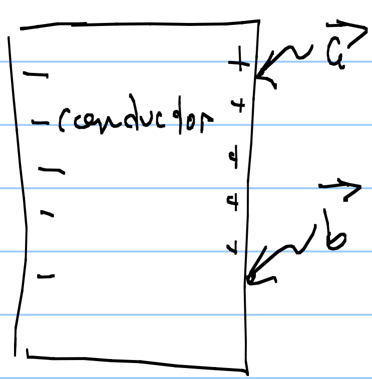
$|\vec{\nabla} h| \propto$ # lines/Area in stream line interpretation

$|\vec{\nabla} h| \propto$ mag of space rate of change ~ so can look & see how quickly space derivs are changing

So constant 'h' contours



OK... back from recall...



Since "Electrostatic Equi" e^-'s @ REST!

∴ $-\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot \underline{d\vec{l}}$ *KEY along surface

= must = 0! IS not then would ⇒ $\vec{E}_{surface} \neq 0$ and e^-'s would move.

$$\text{So} \quad -\int_{\vec{a}\text{ on surf}}^{\vec{b}\text{ on surf}} \vec{E}(\text{surface}) \cdot \underbrace{d\vec{l}}_{\substack{\text{stays} \\ \text{on} \\ \text{surface} \\ \text{whatever} \\ \text{it is}}} = -\int_{\vec{a}}^{\vec{b}} E_{\parallel}\, dl = 0$$

1.) $E_{\parallel}$ to surf $= 0$

2. $$V(\vec{b}) - V(\vec{a}) = \int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}_{surf} = 0$$

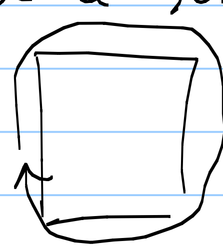
so

$$V(\vec{b}) = V(\vec{a}) \qquad \vec{a}\ \&\ \vec{b} \text{ on surface.}$$

$\therefore$ Everywhere on the surface of a conductor is @ same V, Electrical Potential, Voltage

There is no Δ Potential for charges to move on a surface of a conductor

so q_+



can move around freely

(5.) $\vec{E}$ @ surface is
1.) $\parallel$ to surface $= 0$
2.) could have something $\perp$ to surface

$$-\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot d\vec{l} = 0$$

$$\int_{\vec{a}}^{\vec{b}} \underbrace{(\vec{\nabla} V)\cdot d\vec{l}} = 0$$

$$\vec{\nabla} V \cdot d\vec{l} = 0$$ Same as Hill problem!

This $\Rightarrow$'s $\vec{\nabla} V$ ($\vec{E} = -$) is everywhere $\perp$ to contours of constant V

* e^-'s don't move $\perp$ to surface cause they are still confined by the metal

What are the contours of constant V, ie where

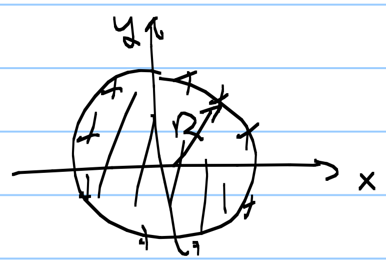
$V(\vec{b}) - V(\vec{a}) = 0$ --- The surface of whatever your conductor is!

Wow, thats alot!

yes!

For example:

say you have a conductor



V = constant on the surface

so what are the 'contours' of equipotential?

$$x^2+y^2-R^2=0$$

For this R, $V(R) = \text{const} = V_0$

So

$V = V(x,y)$ in general

but $\underbrace{V(x,y) = y^2+y^2-R^2 = \text{const}}$

Therefore

$V(x,y) = x^2+y^2-R^2$

= equipotentials!

$\therefore$

$$\vec{E} = -\nabla V = -\frac{\partial}{\partial x}V(x,y) + -\frac{\partial}{\partial y}V(x,y)$$

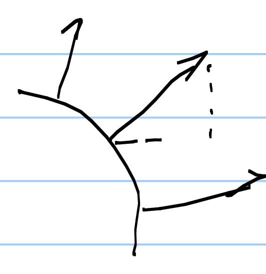
$$= 2x\hat{i} + 2y\hat{j}$$

at

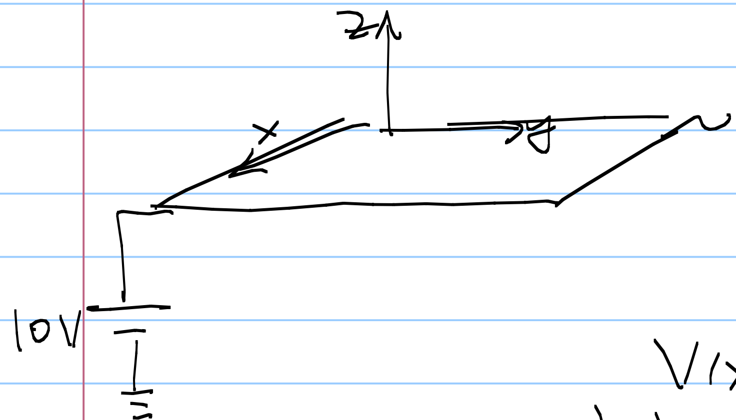
$$\hat{E} = \frac{2x\hat{i}+2y\hat{j}}{[(2,2)\cdot(2,2)]^{1/2}} = \frac{1}{2\sqrt{2}}(2x\hat{i}+2y\hat{j}) =$$

$$\hat{E} = \frac{1}{\sqrt{2}}x\hat{i} + \frac{1}{\sqrt{2}}y\hat{j}$$

$$= \hat{r}$$



So can say



~ sheet of constant potential

$V(x,y,z)$ in general

but

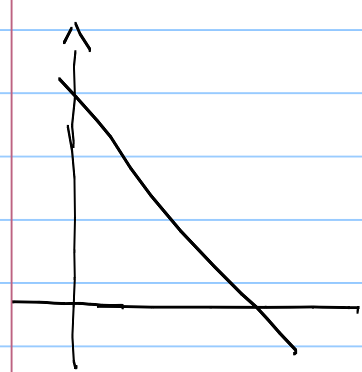
$V(x,y,z) = z = \text{constant}$
= surface or contour of constant potential.

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} + -\frac{\partial V}{\partial z}\hat{k}$$

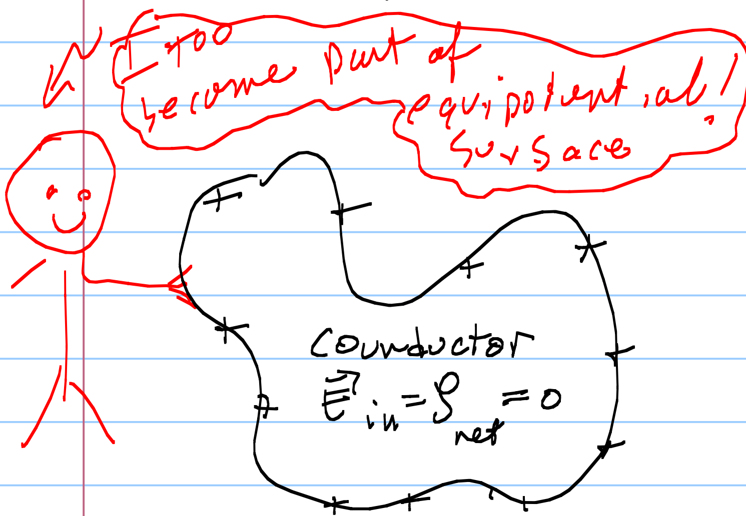
$$\vec{E} = -\hat{k}$$

$$|E| = 1$$

$$\text{div} = \hat{k}$$



all together!



0.) Electrostatic equi ⇒ quickly

1.) $\vec{E}_{in} = \rho_{net\ in} = 0$

2.) charge is entirely on surface

3.) $\vec{E}_{surface}$ $\begin{cases} \parallel \text{ to } d\vec{l} \text{ on surf} = 0 \\ \perp \text{ to } d\vec{l} \text{ on surf} \neq 0 \end{cases}$

4.) Everywhere on entire surface =

equipotential surface $\Big\{$ @ same electrical potential, V

($V(\vec{a}) - V(\vec{b})) = 0$, with $\vec{a}$, $\vec{b}$ on surface

5.) $\vec{E}_{surface} = \perp$ to surface

$= -\nabla V$

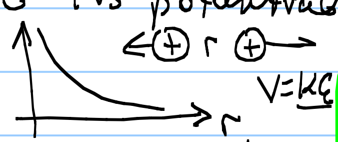
so $\vec{E} = -\frac{\partial V}{\partial x}\hat{x} + -\frac{\partial V}{\partial y}\hat{y} + -\frac{\partial V}{\partial z}\hat{z}$

$|E|$ = max space rate of change

dir = $\perp$ to Equipotential which means $\perp$ to surface of conductor!

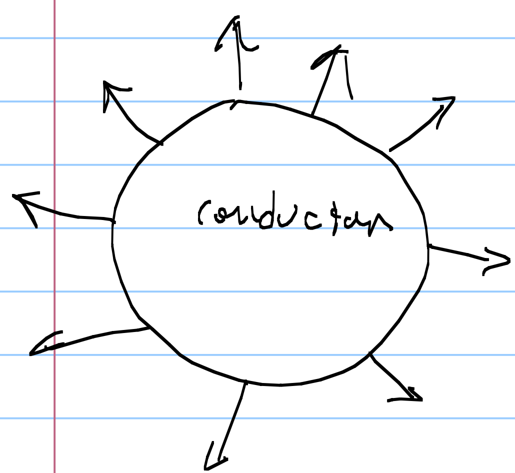
* All of this could be just as well argued from energy considerations

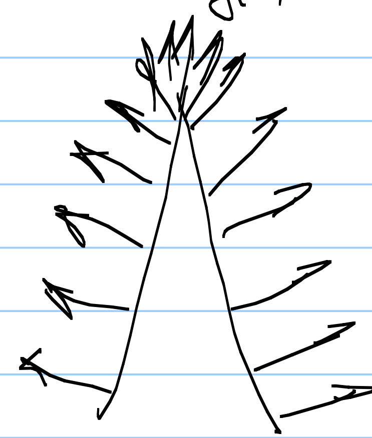
The system "seeks" configuration to minimize its potential energy.



which is minimized w/ all charge distributed uniformly on the surface! to be as far away as possible

Therefore





$|E|$ = constant for all θ & ϕ @ $|r|$

dir = $\hat{r}$

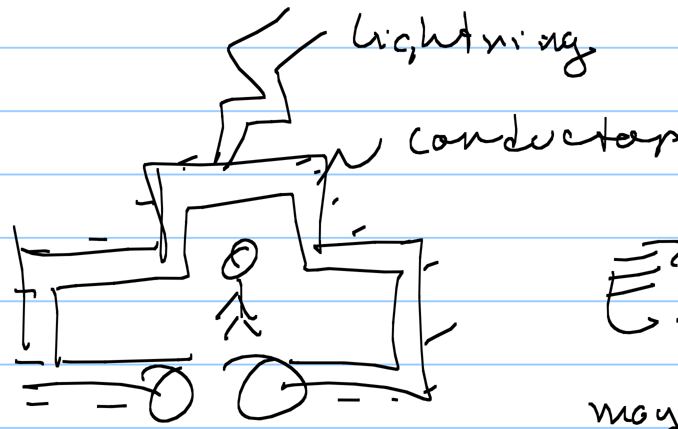
Clearly

$|E|$ mag = space rate of change so max where $\frac{\partial}{\partial x}$ = max

ie $|mag| \propto$ # lines / space

dir = $\perp$ to surface!

This is why "sharp" conductors "attract" lightning: the $|E|$ is max at the max space rate of changes (points) where air gets ionized 1st and back to



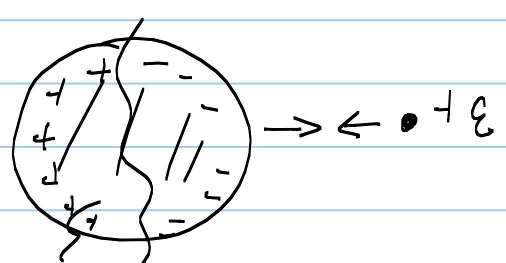
$\vec{E}_{inside} = 0$!

may get cooked from heat of lightning but won't be electrocuted

Implications!

Clearly $\vec{E}_{cavity} \neq 0$
but $\vec{E}_{conductor} = 0$

lets see how

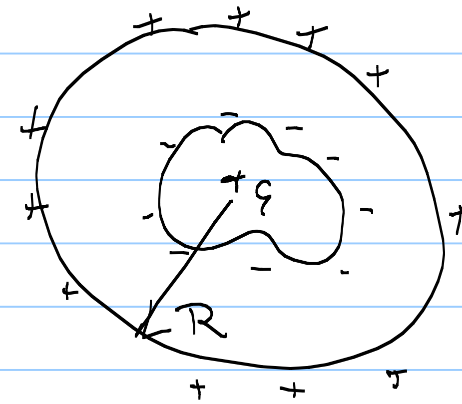


There attract!
But note induced charge is built to neutralize $\vec{E}_{TOT}$ in the conductor

note +q induced } −q induced

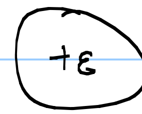
TOT = 0

Note here, $\int$ induced while uniform specially is not nec the same so now



so use Gauss

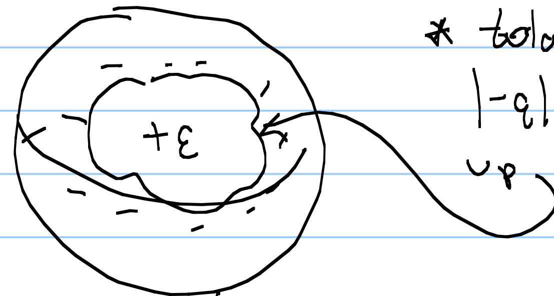
1.) Cavity



$$\oint \vec{E}_{cavity} \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$\vec{E}_{cav} \neq 0$$

2.) Conductor



* total of $|-q| = +q$ builds up interior surface

Gaussian sphere

so $$\oint E_{conductor} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{+q+q}{\varepsilon_0}$$

$$|E_{cond}| = 0$$

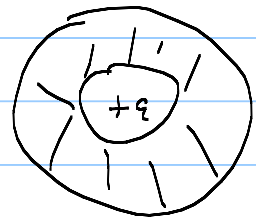
3.) Outside $r > R$

$$\oint \vec{E}_{outside} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{+q+-q+q}{\varepsilon_0} = \frac{+q}{\varepsilon_0}$$

$$\boxed{\vec{E}_{outside} = K\frac{q}{\varepsilon_0}\hat{r}}$$

As if conductor not even there for $r>R$

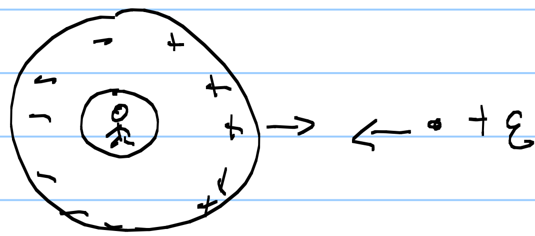
So



$\vec{E}$ looks like pt charge @ o

the conductor "communicates" inside signals to outside

but



But $\vec{E}$ do I feel? Zero!

So (conductor) protects you from $\vec{E}$ externals!

So, say you have important equipment

put it in a conductor & it protects it from stray "bad" fields

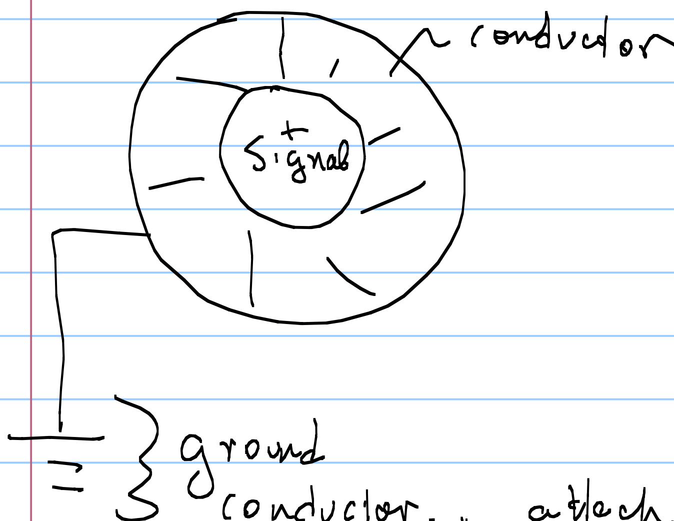
Further

cable signal

conductor "shield"

protects import signal from noise!

Try this ...



} ground conductor .. attach to earth!

$= \infty$ sink of e^-'s
or
∞ source of e^-'s

so
$e^- \rightarrow$ or $e^- \leftarrow$

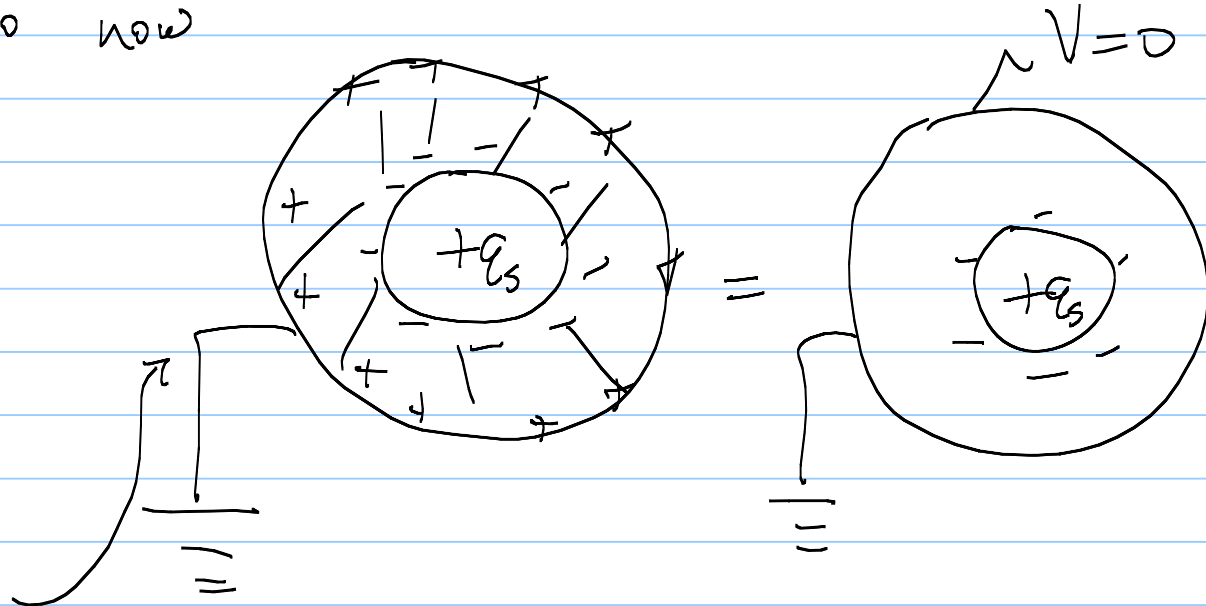
such that conductor become's same pot as Earth.....

Call it $\boxed{0V}$

Essentially attaching the "ground" means you are connecting the equ-potential of the earth (whatever it is) to the object conductor: Then

So Earth looks like HUGE CONDUCTOR

So now

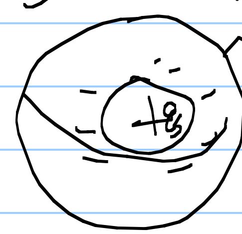


So that

1.) in Cavity

$\vec{E}_{cav} \neq 0$, ie = signal

2.) in meat of conductor



Gauss

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_s - q_s}{\varepsilon_0}$$

$$\vec{E}_{inside} = 0$$

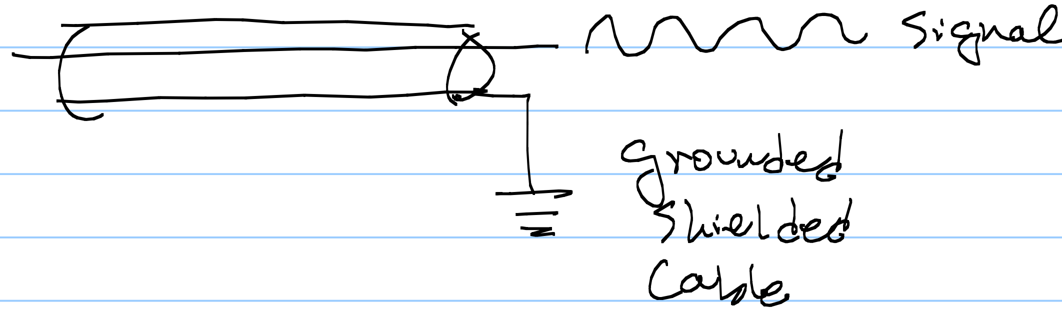
3.) outside $r > R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{+q_s - q_s = 0}{\varepsilon_0}$$

$\vec{E}_{out} = 0$ now

wow! A grounded conductor "shields" your inside signal now from getting outside!

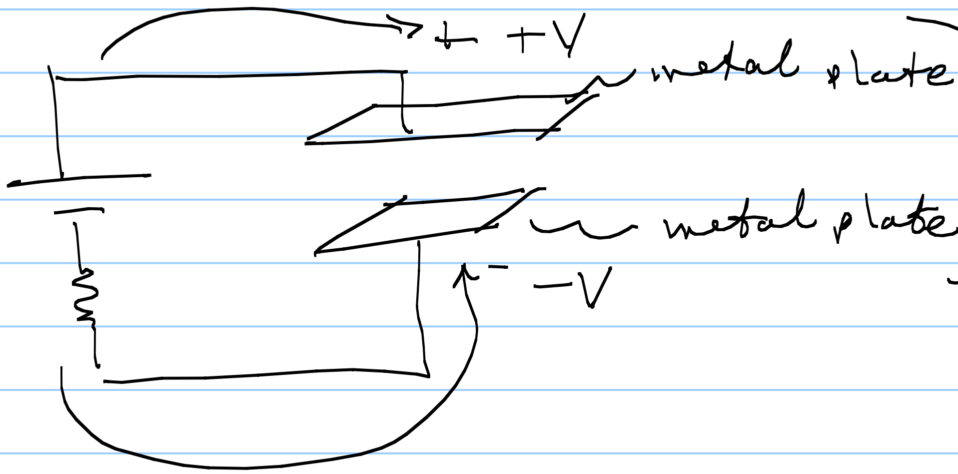
So Coax Cable



1.) protects noise from getting into signal wire

2.) Shields from signal getting out of cable!

Finally … w/ Conductors can make useful thing called a capacitor!



each "is" an equipotential surface because they are conductors

But could just as well be

2 conductors

What is the potential difference between the 2 conductors?

$$\underbrace{V = V_+ - V_-}_{\text{same def as before}} = -\int_-^+ \vec{E}\cdot d\vec{l}$$

same def as before

* cool

But in this sense using (V_-) as ref $\vec{0}$

Know we argue that

$$\vec{E} \propto Q$$

$$\& \ V \propto Q$$

so define $\frac{|E|}{V} \propto \frac{Q}{V}$ = call it his proportionality constant

C = Capacitance

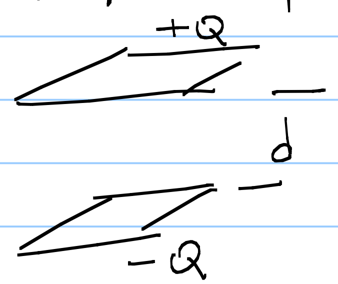
units = $\frac{\text{Coulomb}}{\text{Volt}}$ = F = farad

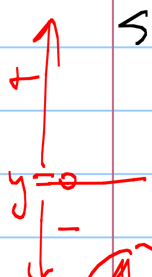
* incidently 1 farad is HUGE, usually buy micro $\times 10^{-6}$ or pico $\times 10^{-9}$ F

Definitions, $Q \equiv Q$ of + conductor & $V = \int_-^+$

Helps

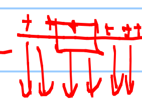
$\parallel$-plate capacitor

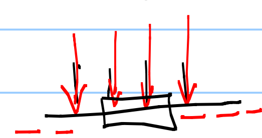




So $|\sigma| = Q/A$

could have done





$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$|E|A = \frac{\sigma A}{\varepsilon} = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{1}{\varepsilon_0}\frac{Q}{A} = \text{constant}$$

+ side

$$\text{so } V = -\int_0^d -\frac{1}{\varepsilon_0}\frac{Q}{A}\,dx$$

$$V = +\frac{Q}{\varepsilon_0 A} d$$

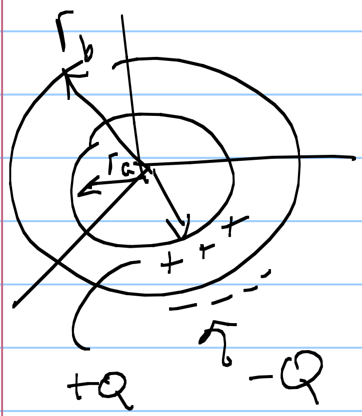
$$\text{So } C = \frac{Q}{V} = \frac{Q}{Q/\varepsilon_0 A \, d} = \frac{A\varepsilon_0}{d}$$

ex: $A = 1 \times 1$ cm & $d = 1$ mm

$C = 9 \times 10^{-13}$ F

So clearly...

2 concentric metal spheres, radii a & b = Capacitor



$$V = -\int_{-}^{+} \vec{E} \cdot d\vec{l}$$

$$\vec{E} \text{ by Gauss} = \frac{+KQ}{r^2}\hat{r}$$

$$V = -\int_{r_b \,(-\text{@})}^{r_a \,(+\text{@})} KQ \frac{dr}{r^2} = +\frac{KQ}{r}\Bigg|_{r_b}^{r_a}$$

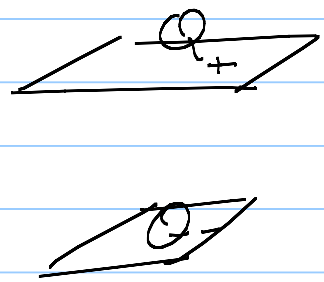
$$= +KQ\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{KQ(r_b - r_a)}{r_a r_b}$$

so

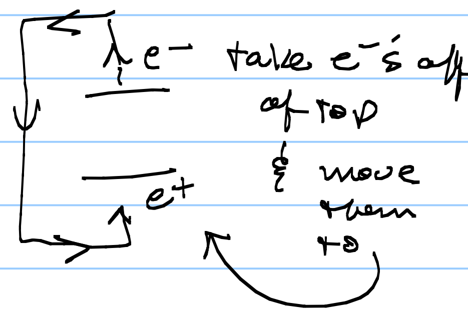
$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{K\cancel{Q}(r_b - r_a)}{r_a r_b}}$$

$$= \frac{4\pi\epsilon_0\, r_a r_b}{(r_b - r_a)}$$

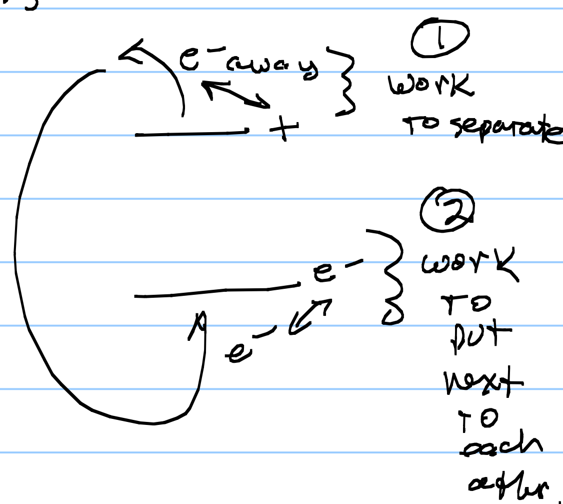
to make a capacitor,



you effectively



Since each e^- you deliver to the bottom means



Well, what is total amount work to overcome these 2?

is @ some pt q is taken from top, so it is $+q$

so that $C = \frac{Q}{V}$, $V = \frac{Q}{C}$

here $V = \frac{q}{C}$

then $dW = V\,dq$ } recall $V = \frac{\text{Joule}}{C}$

so $\left(\frac{q}{C}\right) dq$

$\therefore$ The total work nec to go from $q = 0$ to $q = Q$

is

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

& since $C = \frac{Q}{V}$; $Q = CV$

$$\boxed{W_{TOT} = \frac{1}{2} C V^2}$$

here C = Capac of Cap

& V = Voltage you would like to obtain!

Of course this work is usually provided by the battery!