Eit Deveney/BSC Physics PH438 E-fields in Matter: CH4

Note Title 12/1/2004 GRIFF

Electric fields in matter! GREAT PHYSICS!

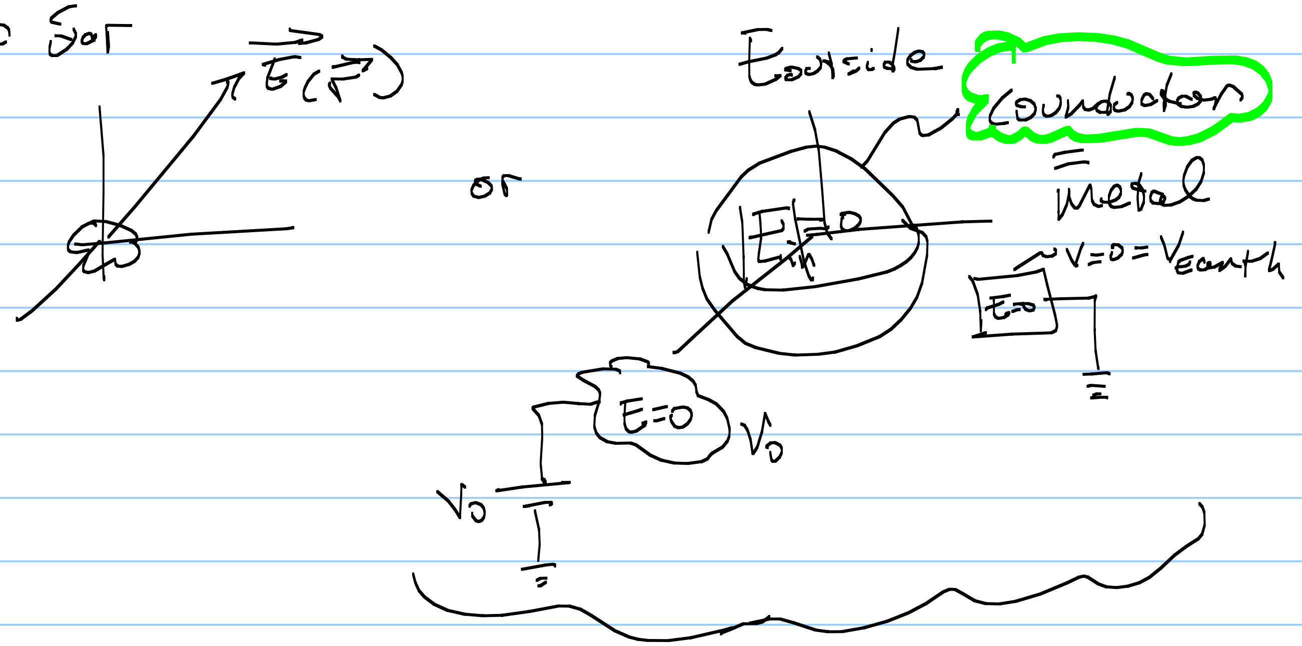
$\vec{E}$ is the thing! It's what we work to find!

source

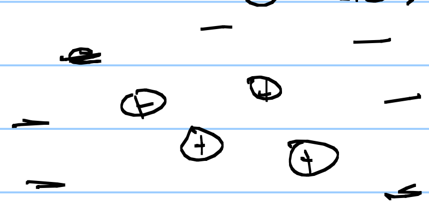
$\vec{E}(\vec{r})$ due to source

* Don't loose sight in all of this, we want & measure $\vec{E}$

So for



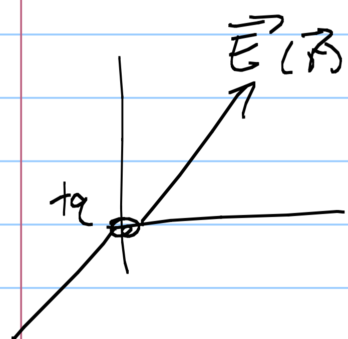
Big Deal was metals = metalic Bonds!



e^-'s ≅ FREE

But what about $\vec{E}$ in other material?

In fact



... In Vacuum? or air?

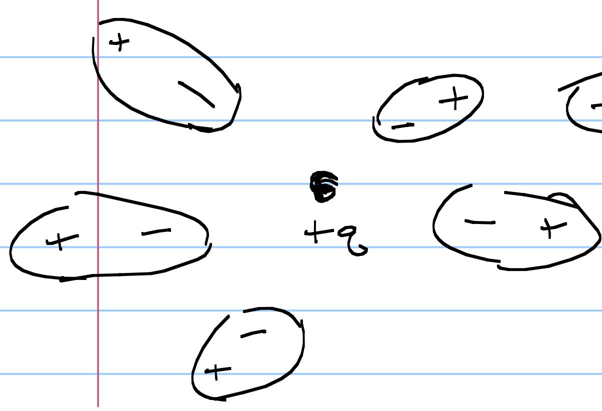
So we've been skirting this issue.

air = stuff = material

O = Group IV = $1s^2 2s^2 2p^6$

⇒ Messy
78% N_2

& ..... My Favorite .... even is in "pure" vacume

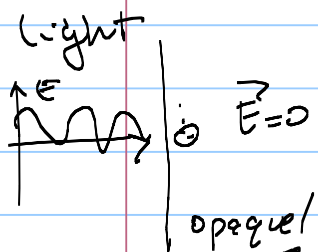


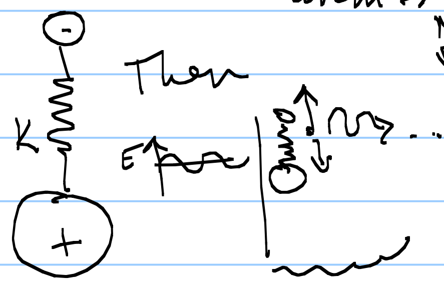
Vacuum =
particle + particle pairs
spring in & out of
Vacuum = STUFF!

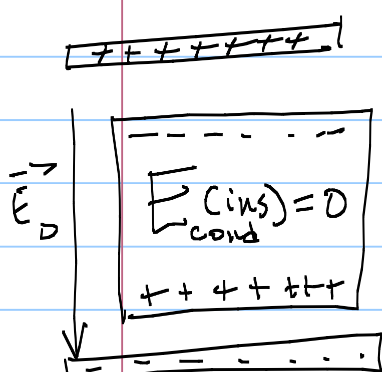
So $\vec{E}$ in materials!

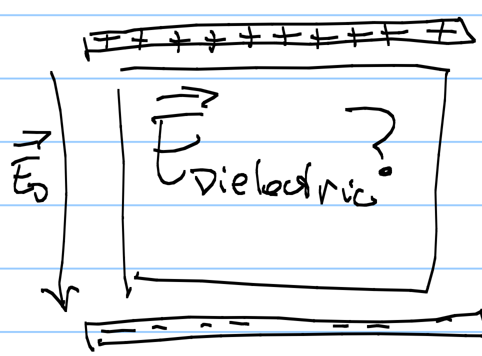
All about atomic-molec physics, Bonds .... AND Then SOME!

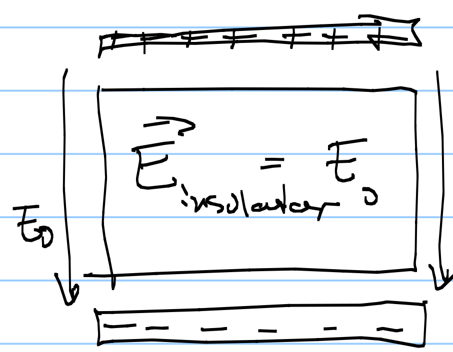
Conductors	Dielectrics	Insulators
metalic Bonds	Covalent Ionic	
$e \cong$ FREE! (no specific atom!)	e's Loosely (Bound to specific atom ⇒ OK Not really)	e's Tightly Bound





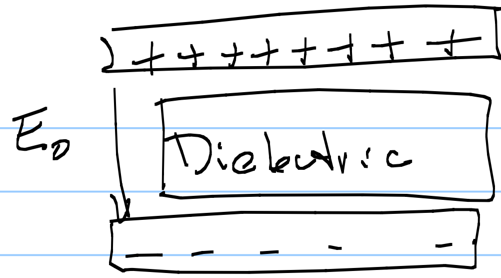






Turns out

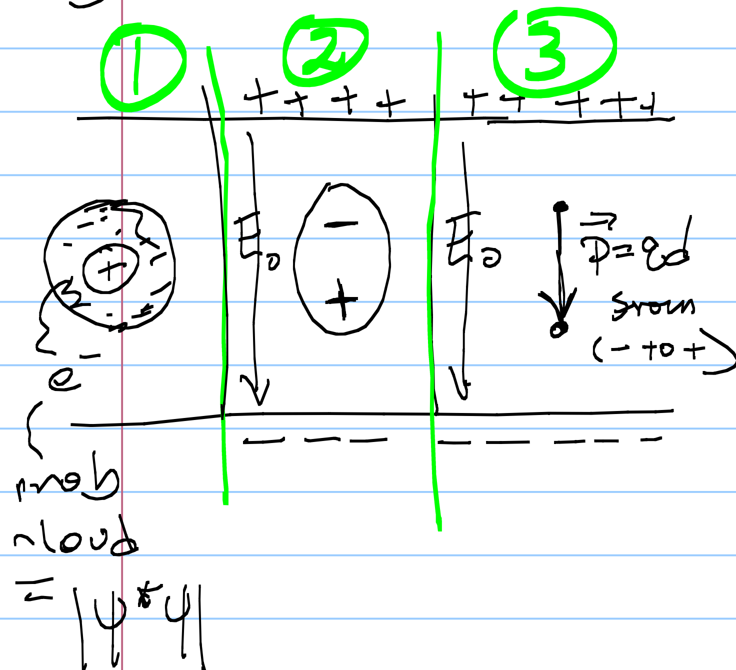
Dielectric $\cong$ poor conductor,
Can't Quite make $\vec{E}=0$

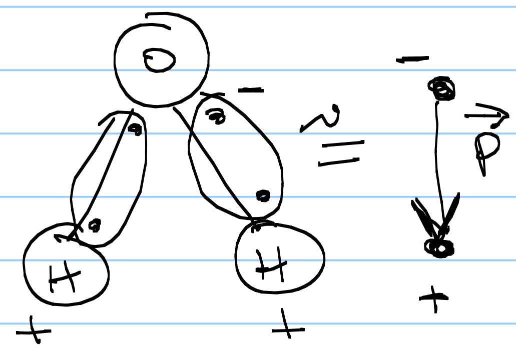


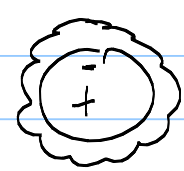
Dielectrics

I.) Neutral Atoms

II.) "Already" polarized molecules = H_2O





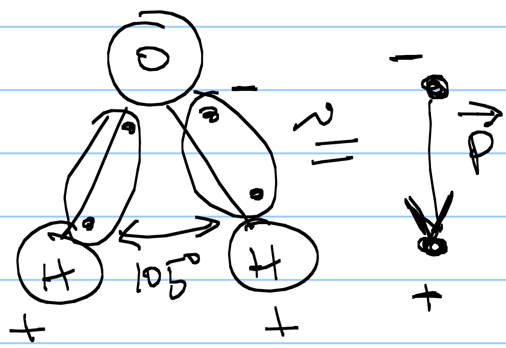
Neutral atoms 

$$\sum \vec{F}_{ext} = m\vec{a}$$
$$0 \quad = 0$$

So no Translation, BUT

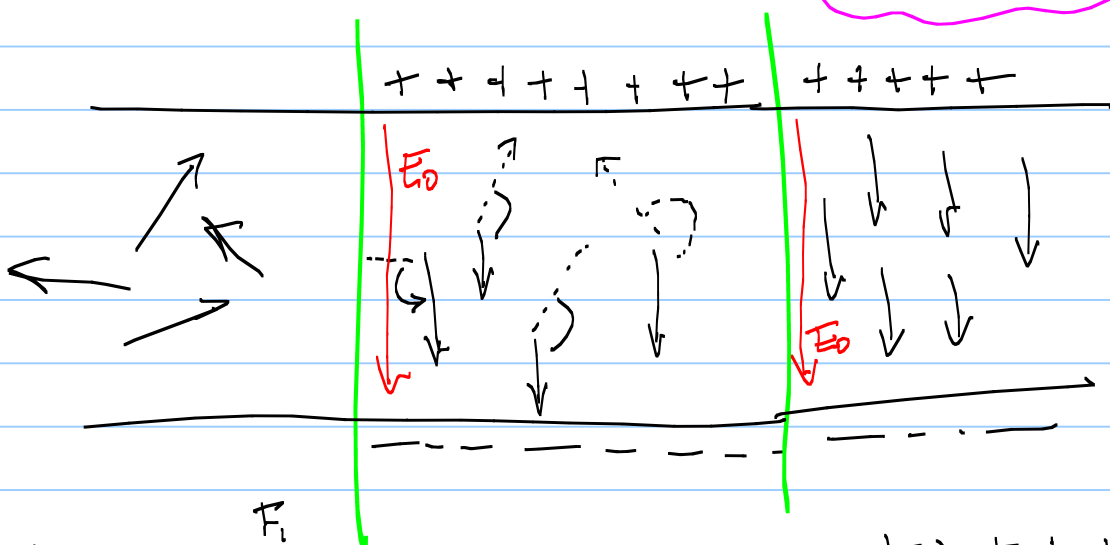
neutral atoms stretch $\Rightarrow$ form Electric Dipoles

II.) Already polarized molecules = H_2O

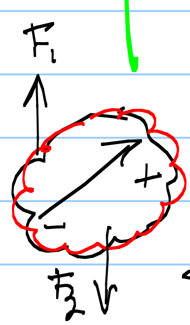


Now $\vec{p}_{H_2O} = 6.1 \times 10^{-30}$ Coul·m

which is BIG and accounts for effectiveness (in part) as a SOLVENT



Again



$\sum F_{ext} = m\vec{a}$

$/ \; 0 \; \backslash = \vec{a}$

so no net translation of molecule

But definitely rotation

$\sum \vec{\tau} = I\alpha$

$-\vec{r}_+ \times \vec{F}_1 - \vec{r}_2 \times \vec{F}_2 = I\alpha$

so get

$\tau_{NET} = \vec{p} \times \vec{E}$

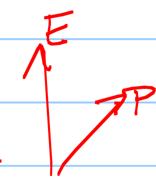
So perm $\vec{E}$-dipoles Align w/ $\vec{E}_{ext}$

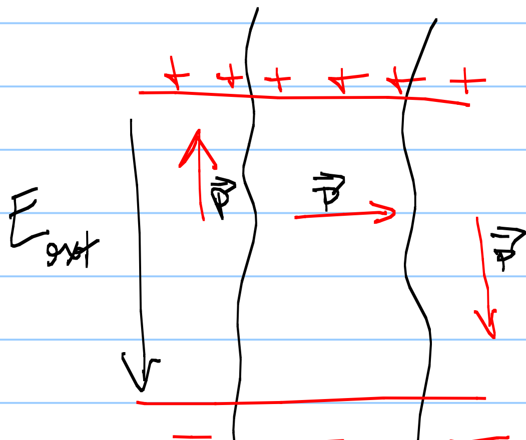
Further,

Energy of $\vec{E}$-dipole

$$E = -\vec{p}\cdot\vec{E}_{ext} = |p||E|\cos\theta$$

θ =





$E_1 = \uparrow \cdot \downarrow$ $= (-)|p||E|$ $+|p||E|$ $\theta = 180$

$E_2 = \rightarrow \cdot \downarrow$ $= 0$ 0 $\theta = 90$

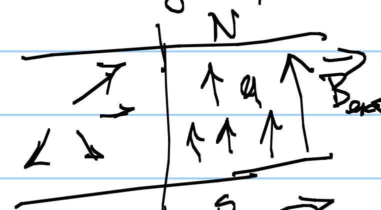
$E_3 = \downarrow \cdot \downarrow$ $= (+)|p||E|$ $= -|p||E|$ $\theta = 0$

Later:

$\vec{\mu}$ = magnetic dipole

$\vec{\mu}$ I

same story, in $\vec{B}_{ext}$ $\vec{\mu}$'s rotate to align w/ $\vec{B}$

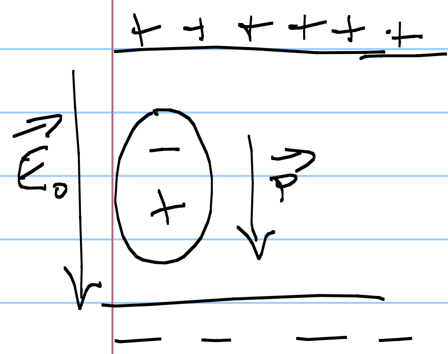


Energy $= -\vec{\mu}\cdot\vec{B}$

So Energy of E-dipole in E_{ext} field

These two effects allow fundamental studies!!
They let you align & control!
eEDM & Quantum computer = dipole-dipole interaction

Let's look @ Induced Dipoles from neutral atoms.....



* NOTE!

$\vec{p}$ = tiny dipole

$\vec{P}$ = total dipole (moment)

Clearly

$$\underbrace{\vec{p} \propto \vec{E}}$$

$\Rightarrow$'s

$$\underbrace{\vec{p} \text{ is } \parallel \text{ to } \vec{E}}$$

generally true $\Rightarrow$ Linear Dielectrics!

Not all as we will see!

$\Downarrow$

investigate

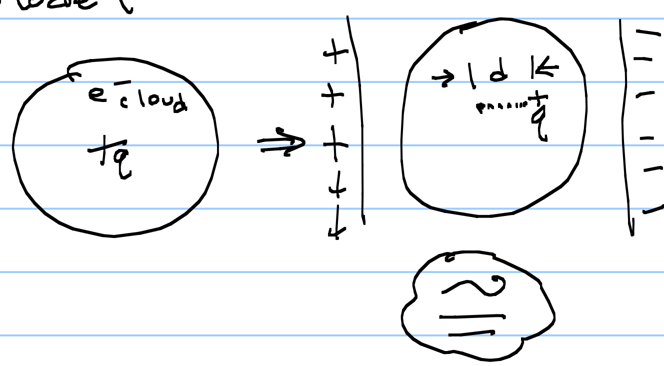
Linear materials

Linear polarization

$$\vec{p} = \alpha \vec{E}$$ atomic polarizability!

Model

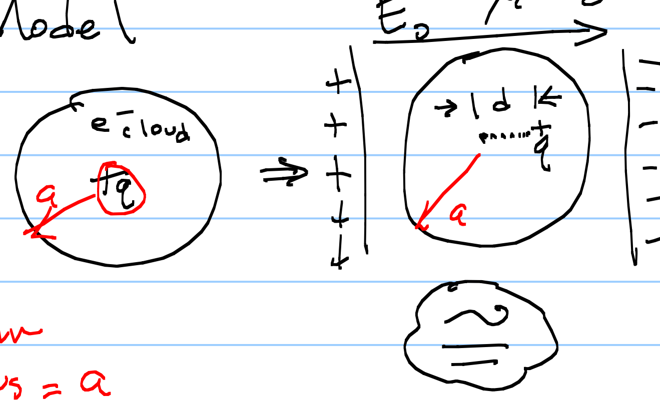
say +q shifts by d



Linear polarization

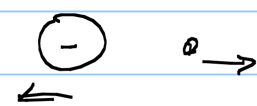
$\vec{p} = \alpha \vec{E}$ atomic polarizability!

Model



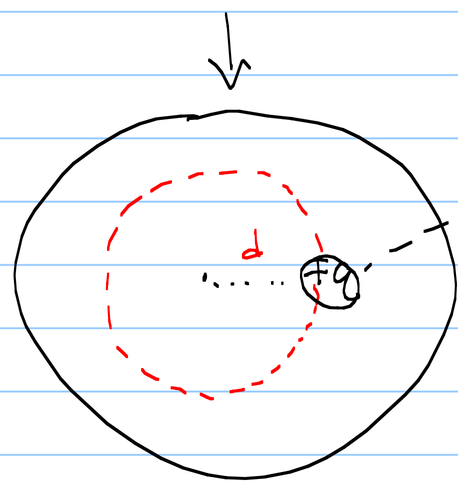
say $+q$ shifts by d

* actually

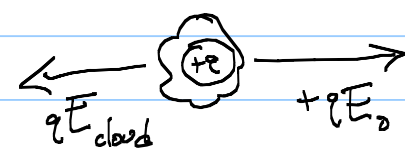


But shifts so small let e^- cloud $\cong$ spherical

atom radius $= a$



This is the nucleus



$\Sigma \vec{F} = ma$

$\left(-qE_{cloud} + qE_0 \right) = 0$ ← equilib.

By Gauss's law,
$\vec{E}_{cloud}$ (d on q) due to all charge inside sphere radius d (outside don't contribute)

$$\oint \vec{E}_{cloud} \cdot d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

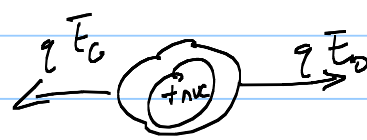
$$= \int_0^d \frac{\rho \, d\tau}{\varepsilon_0}$$ where $\rho = \text{uniform} = \dfrac{q}{\frac{4}{3}\pi a^3}$

$$\oint \vec{E}_{cloud} \cdot d\vec{A} = \frac{q}{\frac{4}{3}\pi a^3}\frac{1}{\varepsilon_0}\int_0^d dV = \frac{q\,\cancel{\tfrac{4}{3}\pi} d^3}{\cancel{\tfrac{4}{3}\pi}\, a^3 \varepsilon_0} = \frac{q d^3}{\varepsilon_0 a^3}$$

$$|E_c|\, 4\pi d^2 = \frac{q d^3}{\varepsilon_0 a^3}$$

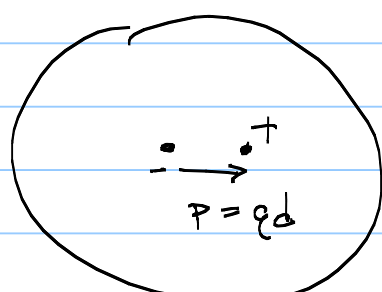
$$E_{cloud} = \frac{1}{4\pi\varepsilon_0}\frac{q d}{a^3} = k\, q\, d / a^3$$

OK: Now



$$E_c = E_0$$

$$\therefore \quad E_0 \text{ or } E_{ext} = k \frac{q d}{a^3}$$



so $E_{ext} = k \dfrac{p}{a^3}$

or

$$\vec{p} = k a^3 \vec{E}_{ex}$$

$$\vec{p} = \alpha \vec{E}_{ex}$$

yeah very simple approx atomic polarizability for linear atoms.

table 4.1 = Experimentally determined atomic polarizabilities

I	II	III	IV	VI	VIII ~ Nobels
H	He	Li	Be	C	Ne
.667	.205	24.3	5.6	1.76	.396
Na 24.1					Ar 1.64
K 43.4					
Cs 59.6					

Trends
1.) Bigger atoms
Bigger α
2.) group I's ↑ 1 s e⁻
3.) VIII = nobels = ↓
closed shells

$$\alpha = 4\pi\varepsilon_0 a^3$$

$$\alpha = 3\varepsilon_0 V_{atom}$$

Units

$$\vec{p} = \alpha\vec{E}$$

$$qd = [\alpha]\frac{F}{C} = [\alpha]\frac{N}{C} = [\alpha]\frac{kg\frac{m}{s^2}}{C}$$

$$C\,m = \frac{[\alpha]\,kg\frac{m}{s^2}}{C}$$

$$\alpha = \frac{C^2 s^2}{kg}$$

$$F = k\frac{q_1 q_2}{r}$$

$$N = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{1}{N}\frac{C^2}{m^2}$$

$$= \frac{C^2}{\frac{kg\,m}{s^2}\cdot m^2}$$

$$\varepsilon_0 = \frac{C^2 s^2}{kg}$$

So above #'s in units of $\frac{\alpha}{4\pi\varepsilon_0}$

Now linear is nice ... but not always the case!

Linear apply $\vec{E}_{ext}$

induce $\vec{p} = \alpha \vec{E}_{ext}$ ie $\vec{p} \parallel$ to $\vec{E}_{ext}$

but in general not the case (molecules in matter = more complicated)

apply $\vec{E}_{ext}$ & $\vec{p} \not\parallel$ to $\vec{E}$

so need

$$\vec{p} = \underbrace{\alpha}_{\text{polarizability TENSOR!}} \vec{E}_{ext}$$

"mixes" x, y & z's

$$\vec{p} = p_x\hat{i} + p_y\hat{j} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\vec{E}_{ext} = E_0\hat{i} + 0\hat{j} = \begin{bmatrix} E_0 \\ 0 \end{bmatrix}$$

Then $\alpha_{ij} = \begin{bmatrix} \alpha_{11} & 0 \\ \alpha_{21} & 0 \end{bmatrix}$

i.e. intermolecular forces $\neq$ equal so pull

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} \alpha_{11} & 0 \\ \alpha_{21} & 0 \end{bmatrix} \begin{bmatrix} E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_{11} E_0 \\ \alpha_{21} E_0 \end{bmatrix}$$

ex: $CO_2 \uparrow$ $\alpha = 4.5 \times 10^{-40} \frac{C^2 m}{N}$

& $CO_2 \rightarrow$ $\alpha = 2 \times 10^{-40}$

so even though $\vec{E}_0 = E_0 \hat{i}$

get $\vec{p} = p_x \hat{i} + p_y \hat{j} = \alpha_{11} E_0 \hat{i} + \alpha_{21} E_0 \hat{j}$

clearly

$\vec{p} \neq \parallel$ to $\vec{E}_0$ external

In general:

$$\vec{p} = \underline{\alpha} \vec{E}_{ext}$$

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z \\ \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z \\ \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z \end{pmatrix}$$

In general $\underline{\alpha}$ depends on axis you choose

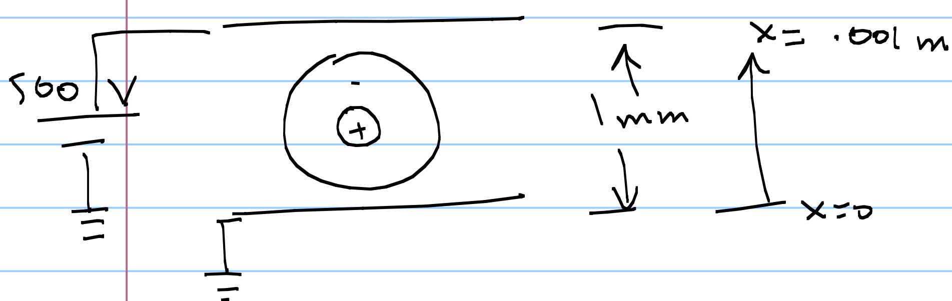
* HOWEVER: possible to choose 'principal' axis so off diag terms drop out leaving

$$\underline{\alpha} = \begin{pmatrix} \alpha_{xx} & 0 & 0 \\ 0 & \alpha_{yy} & 0 \\ 0 & 0 & \alpha_{zz} \end{pmatrix}$$

Ch. 38 H.W. 4.1

H-atom, $r = 0.5\,\text{Å}$

No Calculators Allowed!



1.) what fraction of atomic radius does separation d amount to

2.) what voltage do you need to ionize?

1.) since from our crude model

$$E_{ext} = K\frac{e\,d}{a^3} \; ; \quad d = \frac{|E_{ext}|\,a^3}{K\,e}$$

what is $E_{ext} = E_{inside}$ Capacitor?

recall in cap: $\nabla^2 V = 0 \Rightarrow$'s $\quad V(x) = \frac{V_0}{h}x + 0$

Then $|\vec{E}| = |-\nabla V| = \dfrac{V_0}{h}$

$$d = \frac{4\pi\varepsilon_0 \left|\frac{V_0}{h}\right| a^3}{q} = \frac{\left(4\pi\, 8.85\times10^{-12} \frac{\cancel{C}^2}{\cancel{N m^2}}\right) * \left(.5^3 * 10^{-30}\ \cancel{m^3}\right)\left(\frac{500\ \cancel{N\cdot m}/\cancel{C}}{.001\ \cancel{m}}\right)}{1.6\times10^{-19}\ \cancel{C}}$$

m

$$\cong \frac{(\sqrt{12})(9)(.125)(5\times10^5)(10^{-12})(10^{-30})}{(2)\times10^{-19}}$$

$$\cong \frac{45}{2} \times \frac{10^{-37}}{10^{-19}} \cong 20\times10^{-18}$$

Griss: 2×10^{-16} m

or $2\times10^{-17}\,m * \frac{1\ \text{Fermi}}{10^{-15}\,m} = 2\times10^{-2}\ F$

or $\frac{1}{50}$ diameter of a nucleus!

B.) To ionize: presumable would have to get

$d \geq$ Bohr radius $= .5\times10^{-10}$ m

Solving for V_0: $V_0 = \frac{d\,q\,h}{4\pi\varepsilon_0 a^3} = \frac{(.5\times10^{-10}\,m)(1.6\times10^{-19}\,C)(.001\,m)}{\left(4\pi\, 8.85\times10^{-12} \frac{C^2}{N m^2}\right)(.5^3\times10^{-30}\,m^3)}$

$$= \frac{1.6\times10^{-32}}{(\sqrt{12})(9)\times10^{-12}(.25)\times10^{-30}}\left(\frac{N}{C}\right) \cong \frac{8\times10^{-32}}{10^{-40}} = 8\times10^{8}\ V$$

YIKES!

Griss 10^{8} Volts