E. F. Deveney/BCS Physics

TRIG IDENTITIES

Unit Circle

Note: Don't forget your Unit Circle. \(|r|=1\)

distance \(= \cos \theta\)

\[ x = \cos \theta \]

\[ y = \sin \theta \]

\[ \theta = \frac{\text{Arc length}}{\text{Radian}} \]

\[ \text{Radians} = \frac{\text{Degrees}}{180} \]

Ex: \(30^\circ, \sin 30^\circ = 0.50\)

\[ 0.50 = y \]

\[ \frac{y}{x} = 0.50 \]

Ex: \(90^\circ, \cos 90^\circ \approx 0\)

<table>
<thead>
<tr>
<th>(\text{Radians})</th>
<th>(\cos)</th>
<th>(\text{Degrees})</th>
<th>(\sin)</th>
<th>(\tan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.524</td>
<td>0.866</td>
<td>30°</td>
<td>0.50</td>
<td>0.87</td>
</tr>
<tr>
<td>1.57 (\pi/2)</td>
<td>0.00</td>
<td>90°</td>
<td>1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

So can see that simply build a table of \(x, y, \frac{x}{y}\) and relate back to a unit deg or rad.

Arc length: \(x = (\cos \theta)\), \(y = (\sin \theta)\)

\[ x = (\cos \theta) \]

\[ y = (\sin \theta) \]

\[ \frac{x}{y} = (\tan \theta) \]

Really all back to arc length and position on unit circle!
Trig Identities

Clearly from (UNIT = 1) circle,

\[ x^2 + y^2 = 1 \]

\[ \sin^2 \sigma + \cos^2 \sigma = 1 \]

\[ \sin (-x) = (-1) \sin (x) \]

ie: odd

\[ \cos (-x) = \cos (x) \]

ie: even

\[ \tan (x) = \frac{\sin (x)}{\cos (x)} = \frac{\text{odd}}{\text{even}} = \text{odd} \]

\[ \tan (-x) = (-1) \tan (x) \]

Clearly (??) check!

\[ \sin (x + \pi) = (-1) \sin (x) \]
\[ \sin (x - \pi) = \sin (x) \]

\[ \cos (x + \pi) = \cos (x) \]
\[ \cos (x - \pi) = (-1) \cos (x) \]
Trig Addit Identities

\[ \cos(x + y) = \cos x \cos y - \sin x \sin y \]

\[ \sin(x + y) = \sin x \cos y + \cos x \sin y \]

from which Trig Double angle -----

\[ \alpha = \beta \]

\[ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \]

\[ = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha \]

\[ \sin 2\alpha = 2\sin \alpha \cos \alpha \]

\[ \Rightarrow \text{ If } \alpha = \frac{\beta}{2} \]

\[ \cos \alpha = 2\cos^2 \frac{\alpha}{2} - 1 = 1 - 2\sin^2 \frac{\alpha}{2} \]

so

\[ \sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \]

\[ \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2} \]
Euler's Relation!

\[ e^\pi i = \cos \pi + i \sin \pi = -1 \]

\[ e^{\pi i} = -1 \]

all algebra \( \rightarrow \) geometric representation

\[ y = mx + b \rightarrow \](Ref: Feynman lectures)

\[ c = x^2 + y^2 \rightarrow \]

What is \( e^{\pi i} = ? \) Source according to Feynman

\[ e^{\pi i} = \text{VECTOR} \]

\[ e^{\pi i(\theta)} = \text{VECTOR on clock hand} \]

As in Q.M. the universe is invariant to transformations

\[ \Psi' = e^{\pi i} \Psi = \bigcirc \Psi \]

Can show \( \delta = S = S \int L \, dt = \) action

Thus define

\[ \text{Prob of } A \text{ to } B = |A_1 + A_2 + \cdots + A_n|^2 \]
These interfere!

So wavelike phenomena (interference) can be described with $e^{i\sigma}$ phases!

Note

$$r = re^{i\sigma} = \sqrt{\cos^2 + \sin^2} = \sqrt{r^2 \cos^2 + \sin^2} = \sqrt{r^2 (\cos^2 + \sin^2)} = r$$

If $\sigma = \sigma(t)$, then $\sigma(t) = \omega t$

So

$$e^{i\sigma} = e^{i\omega t}$$

$\sqrt{\omega t}$ sweeps $= \text{CLOCK}$.

Note QM.

$$\Psi = e^{-i\int x(t) dt}$$ looks like clock hand.

Note $|\int x(t) dt|^2$ =
\[ -i \left( \frac{S_{\text{Ld}}}{k} \right) \cdot \mathbf{\hat{x}} \equiv 10^{-34} \frac{\text{S}}{\text{J}} \cdot \mathbf{\hat{z}} \]

so

\[ \mathbf{\hat{e}} = -i \omega \mathbf{\hat{e}} \]

\[ \mathbf{\hat{e}} \Rightarrow \text{HUGE Baseballs} \]

\[ \mathbf{\hat{e}} = \sum \text{not so Big Son } e^{-i} \]

\[ \Rightarrow \text{Destructive interference for all but paths near the Minimum of } S_{\text{Ld}} \]

which is for paths described by

\[ \frac{d}{de} \left( \frac{1}{2} \frac{d}{dt} \mathbf{\hat{e}} \right) = 0 = \text{Euler-Lagrange Condition} \]

For which all of classical mechanics is derived.

While for \( \mathbf{\hat{e}} \), all paths even away
from the minimum of \( f \) contribute

ie Q.M.