E.F. Deveney/BSC Physics TRIG → Unit circle, → Identities, → Euler: Quantum → Classical transition!

Note Title 9/18/2004

* Note: don't Forget your Unit Circle


$|r| = 1$

~ dist $y \equiv \sin\theta$

distance $x \equiv \cos\theta$

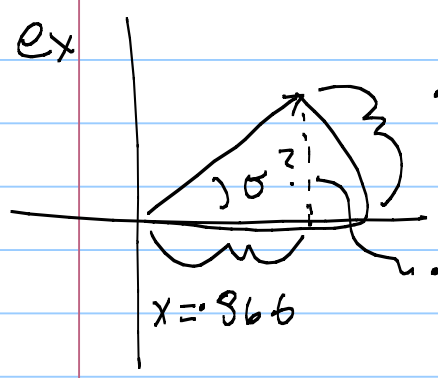
$\theta \equiv$ Arc length on unit circle (radians)

& $(\sin^2\theta + \cos^2\theta)^{\frac{1}{2}} = 1$

ex: $30^\circ$, $\cos 30 = .866$, $\sin 30 = .50$

radians: $\frac{30}{180}\pi = \frac{\pi}{6} = \sim .52$

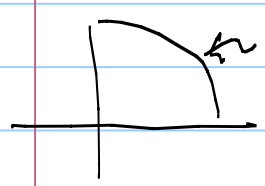
ex




$.524 = \frac{\pi}{6} = 30^\circ$

& $y/x = .578$

&



$1.57 = \frac{\pi}{2} = 90$

$x = 0$, $y = 1$

* so can see that simply build a table of x, y, y/x & can relate back to either deg or rad

table

| Arc length | x = (cosθ) | y = (sinθ) | x/y = (tanθ) |
|---|---|---|---|
| radians = π/6 = .524, or 30° | .866 | .50 | .5 ?? |
| rad = π/2 = 1.57, or 90° | 0 | 1 | 0 |

really all bad! to arc length or position on unit circle!

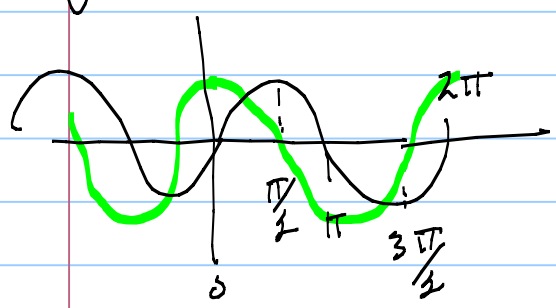
# Trig Identities

Clearly from (UNIT=1) circle,

$$x^2+y^2=1$$

new $\sin^2\sigma+\cos^2\sigma=1$

more




$\sin(-x) = (-1)\sin(x)$

ie: ODD

$\cos(-x) = (+1)\cos(x)$

ie: even

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\text{odd}}{\text{even}} = \text{odd}$$

$\tan(-x) = (-1)\tan(x)$

Clearly (?) check!

$\sin(x+\pi) = (-1)\sin(x)$

$\sin(x-\pi/2) = (+1)\cos(x)$

$\cos(x+\pi) = (-1)\cos(x)$

$\cos(x-\pi/2) = (-1)\sin(x)$

# Trig Addit Identities

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\sin\beta \pm \cos\alpha\cos\beta$$

From which Trig Double angle ....

$\alpha = \beta$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$
$$\sin 2\alpha = 2\sin\alpha\sin\beta$$

used in Q.M. alot!

IF $\alpha = \alpha/2$

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = 1 - 2\sin^2\alpha/2$$

so

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

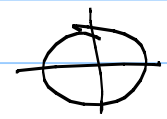
Feynman QED

(Ref: Feynman's lectures)

# Euler's Relation!

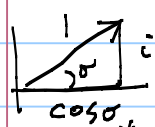
all algebra $\rightleftarrows$ geometric representation

$y = mx + b$ $\rightleftarrows$ 

$c = x^2 + y^2$ $\rightleftarrows$ 

what is $e^{i\sigma} = ?$ Jewel according to Feynman

$$\boxed{e^{\pm i\sigma} = \cos\sigma \pm i\sin\sigma}$$

$=$ 


$$\boxed{|e^{i\sigma}| = (e^{i\sigma})^* e^{i\sigma} = e^{-i\sigma} e^{i\sigma} = 1}$$

$e^{i\sigma} =$ ARROW ---- I.E. VECTOR

$e^{i\sigma(t)} =$ ARROW on Clock hand --- I.e. rotating vector!

As in Q.M., the universe is invariant to transformations

$$\Psi' = e^{i\sigma}\Psi = \circlearrowleft * \Psi$$

Can show $\sigma = \frac{S}{\hbar} = \frac{\int_{path} \mathcal{L}\, dt}{\hbar}$ = action integral

thus

$$A \longrightarrow \boxed{\begin{matrix}\text{All paths} \\ |A_1 + \cdots|\end{matrix}} \longrightarrow B$$

define

prob Amp$_1$ $\equiv a_1 = A_1$

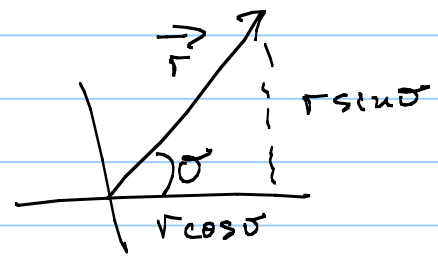
prob of A to B $= |A_1 + A_2 \cdots A_\infty|^2$

& these interfere!

So wavelike phenomena (interference) can be described w/ $e^{i\sigma} \Leftarrow$ phases!

Note

$\vec{r} = re^{i\sigma} = r\cos\sigma + ir\sin\sigma =$




$|\vec{r}| = \sqrt{r^* r} = \sqrt{re^{-i\sigma} re^{i\sigma}} = \sqrt{r^2} = r$

or

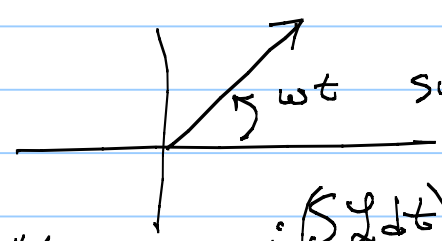
$\sqrt{(r\cos\sigma - ir\sin\sigma)(r\cos\sigma + ir\sin\sigma)} = \sqrt{r^2(\cos^2\sigma + \sin^2\sigma)}$

$= r$

If $\sigma = \sigma(t)$, then $\sigma(t) = \omega t$

so

$e^{i\sigma} = e^{i\omega t}$




sweeps = CLOCK!

Note Q.M. $\bar{\psi} = e^{-i\left(\frac{\int \mathcal{L}\,dt}{\hbar}\right)} \bar{\psi}$ looks like clock hand

Note $\left|\frac{\int \mathcal{L}\,dt}{\hbar}\right| =$

$$e^{-i\left|\frac{\int \mathcal{L}\,dt}{\hbar}\right|} \; ; \; \hbar \cong 10^{-34}\ \text{J}\cdot\text{s}$$

$$\cong e^{-i\omega t}$$

so

$$\omega = \begin{cases} \rightarrow \text{HUGE Baseballs} \\ \rightarrow \text{not so Big for } e^-\text{'s} \end{cases}$$

$\therefore$

$$\vec{A} \longrightarrow \boxed{\begin{matrix}\text{All paths} \\ |A_1 + A_2 + \cdots A_n|^2\end{matrix}} \longrightarrow \vec{B}$$

get DESTRUCTIVE interference for all but paths near the Minimum of $\int \mathcal{L}\,dt$

which is for paths described by $\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = 0$ = Euler-Lagrange 'Condition' on $\mathcal{L}$

For which <u>all</u> of classical mechanics is derived

while for $e^-$, all paths even away

From the minimum of $\mathcal{L}$
contribute

ie Q.M.