Math, PH438, E.F. Deveney/BSC Physics

Note Title 9/9/2004

QFT model = E&M

need to know Everything about E&M!

So Classical E&M described by

I.) Maxwell's equations

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

II.) Better, "Covarient" 4-vectors

$F^{\mu\nu}$ = Field Tensor

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$$

$$\frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0$$

=

note if you take 'curl' of

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \right)$$

&

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

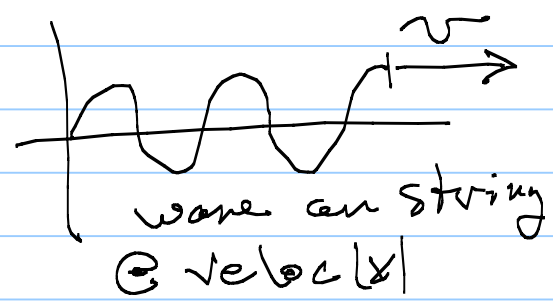
get

1.) $$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

2.) $$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

recognize

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \xi}{\partial t^2} \Rightarrow$$



$\therefore$ 1 & 2 $\Rightarrow$ E & B = waves @

$$|v| = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

Since 1 & 2 do not depend on absolute x_0, t_0, v_0

$\Rightarrow$ is you should get c from all inertial frames!

ie special relativity!

So Math! Laws of Physics =

inherently mathematical

$\neq$ #'s

But the "Forms" of

physical law!

Idea is that "True" laws of

physics will look the same to

everyone! Forms of Laws =

invariant to translations

to other frames!

Indeed Max's Equations are invariant
to transformations that <u>mix</u>
space & time $= (\vec{x}, t)$
= Lorentz transformation.

that $\Rightarrow$ when you observe moving frames

1.) time slows down, $\Delta t \rightarrow$ Bigger

2.) length dialites, $\Delta t \rightarrow$ smaller

Finally Math.

Real #s ($\mathbb{R}$), $-5, -4 \ldots 0, 1, 2 \ldots$

Rational #s, $\frac{\mathbb{R}\#}{\mathbb{R}\#} \Rightarrow$ repeating decimal

$.50000\overline{0} = \frac{5}{10}$

Irrational #s

ask $x^2 = 9$, $x = \pm 3$

$x^2 = 2$ NO Rational # soln!

invent $\sqrt{2}$, so that

$(\sqrt{2})^2 = 2$

$\sqrt{2}$ = irrational # NO repeating decimal equiv

ex: π

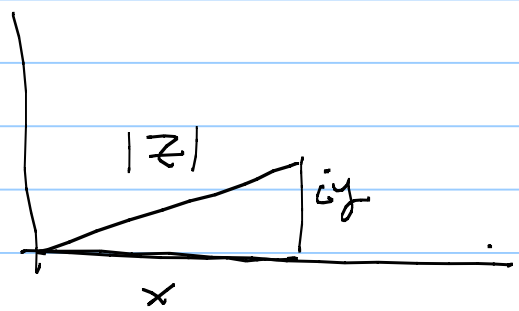
Complex #'s

ask $x^2 = -1$ No rational or irrational so again "invent" soln

$$i^2 = -1$$

complex #'s $\cong$ vectors

$$z = x + iy$$



$$|z| = \sqrt{z^* z} = \sqrt{(x-iy)\cdot(x+iy)}$$

$$= \sqrt{x^2+y^2}$$