

E.F. Deveney/BSC Physics: Integration Repertoire

Note Title

1) des/ines

2) know

3) sum

9/18/2004

No Schawns or Maple
until exhausted These!
⇒ Physicists!

4) sol

5) by parts

6) diss under int

7) TRIG
Substitution!

I.) Definite vs Indefinite Integrals

$$\int f'(x) dx = f(x) + C \Rightarrow \text{indefinite limits}$$

integration
constant

Can determine if you know
the B.C.'s (initial)

* need as many B.C.'s as order
of integral: 2nd order,
need 2

Definite

$$\int_{x_i}^{x_s} f'(x) dx = f(x_s) - f(x_i)$$

II.) gotta know them!

$$\int \frac{1}{x} dx = \ln x + C$$

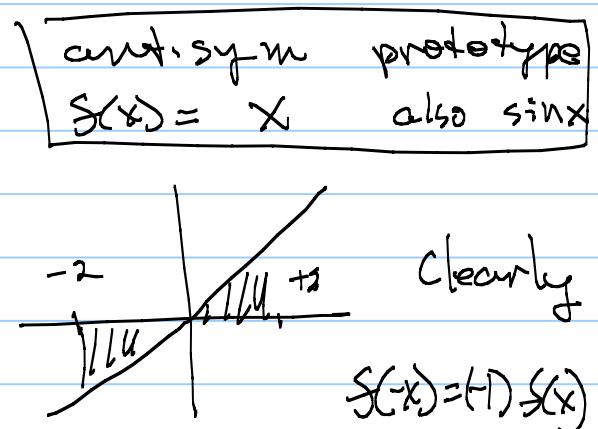
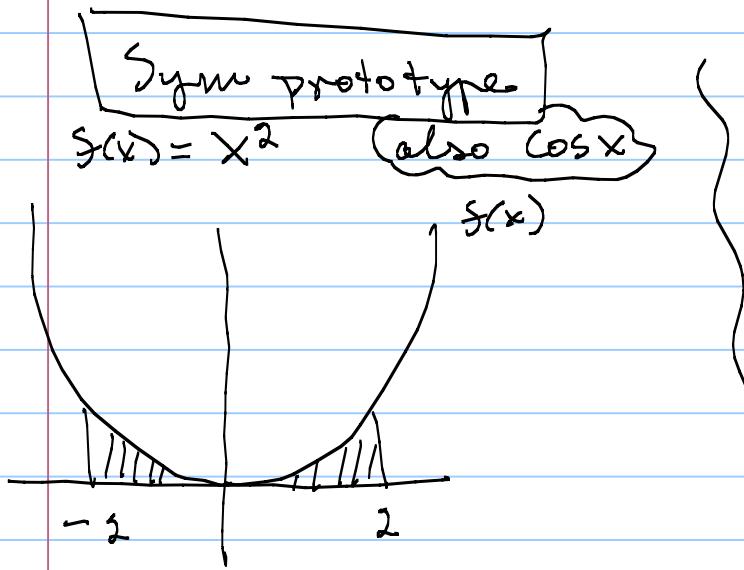
$$\int \sin(x) dx = -\cos x + C$$

$$\int \cos x dx = \sin x$$

III Symmetry \Rightarrow HUGE!

define $s(-x) = +1 s(x)$ = sym (even)
 $s(-x) = -1 s(x)$ = antisym (odd)

otherwise \neq not symmetric



Now consider
integral over sym limits

$$\int_{-a}^{+a} s(x) dx = \text{ } \uparrow$$

Easy!

Clearly
 $s(-x) = + s(x)$
consider integral over sym limits

Now +_a

$$\int_{-a}^{+a} s(x) dx =$$

$$2 \int_0^a s(x) dx$$

III sym cont... The next step is
to use it!

$$\int_{-a}^{+a} \frac{\cos(\omega_1 x) \sin(\omega_2 x)}{x^3} dx$$

$$= \int_{-a}^{+a} \frac{(\text{even})(\text{odd})}{(\text{odd})} dx = \int_{-a}^{+a} (\text{odd}) = 0$$

DONE!

No Shocks!

IV integration by substitution

which \Rightarrow 's getting your limits right!

$$\int_{x_i}^{x_s} \cos(\omega_B w x) dx = ?$$

problem is I know $\int_{x_i}^{x_s} \cos x dx$ but confused here!

so use substitutions to make it look like what you know

let

$$a = \omega_B w x$$

$$\frac{da}{dx} = \omega_B w$$

$$\begin{aligned} a &= \omega_B w x \\ a_i &= \omega_B w x_i \\ a_s &= \omega_B w x_s \end{aligned}$$

$$\text{so } \frac{1}{\omega_B w} da = dx$$

great

$$= \int_{\omega_B w x_i}^{\omega_B w x_s} \cos(a) \frac{da}{\omega_B w} = \frac{1}{\omega_B w} \int_{\omega_B w x_i}^{\omega_B w x_s} \cos(a) da$$

another example

$$\int_{x_i}^{x_s} \frac{x dx}{(x+2x^2)^{\frac{3}{2}}} = ?$$

let $a = x + 2x^2$

$$\frac{da}{dx} = 4x$$

$$\frac{da}{4} = x dx$$

→ Note: Helps!

Sometimes Hurts!

i.e. $\int dx$

$$= \int_{x_i^2}^{x_s^2} \frac{1}{4} \frac{da}{a^{\frac{3}{2}}} = \frac{1}{4} \int_{x_i^2}^{x_s^2} a^{-\frac{3}{2}} da$$

easy!

VI Integration by Parts! Ref: Grissom!

Idea is $\int [\text{mess}] [\text{nice}] dx = \int (\text{mess}) \left(\frac{d(\text{nice})}{dx} \right) dx$

↑ ↑
would go away easy to integrate
is card
differentiate
it ie x piece $\{$
or x^2 by
repeated uses! don't get new
garbage
when you do it!
ie
 $\int e^x dx = e^x$

$$\int \cos x dx = \sin x$$

For example ..

$$\int x e^x dx \text{ or } \int x^2 \sin x dx$$

Then can use this nice trick!

$$\int (\text{mess})(\text{nice}) dx = \int s(x) \left(\frac{dg}{dx} \right) dx$$

\Rightarrow check this

$$\int_a^b \frac{d}{dx} (sg) dx = \int_a^b s \left(\frac{dg}{dx} \right) dx + g \left(\frac{ds}{dx} \right) dx$$

$$\int_a^b \frac{d}{dx} (sg) dx = \int_a^b s \left(\frac{dg}{dx} \right) dx + \int_a^b g \left(\frac{ds}{dx} \right) dx$$

$$\int_a^b d(sg)$$

$$sg \Big|_a^b = \int_a^b s \left(\frac{dg}{dx} \right) dx + \int_a^b g \left(\frac{ds}{dx} \right) dx$$

$$\int_a^b s \left(\frac{dg}{dx} \right) dx = sg \Big|_a^b - \int_a^b g \left(\frac{ds}{dx} \right) dx$$

NOTE The limits!

$$\text{ex: } \int_0^\infty x e^{-x} dx = \int_0^\infty s\left(\frac{dg}{dx}\right) dx$$

messes
things good
up but to different. order

$$s=x \therefore \frac{ds}{dx}=1$$

$$\frac{dg}{dx} = -x \therefore g = \int e^{-x} dx$$

$$g = -e^{-x}$$

$$= sg \Big|_0^\infty - \int_0^\infty g \left(\frac{ds}{dx} \right) dx$$

$$= (-e^{-x})(x) \Big|_0^\infty - \int_0^\infty (-e^{-x})(1) dx$$

$$= - \frac{1}{e^x} \Big|_0^\infty + \int_0^\infty e^{-x} dx$$

$$- e^{-x} \Big|_0^\infty = - \frac{1}{e^x} \Big|_0^\infty = - \left(\frac{1}{\infty} - 1 \right) = +1$$

(I b P)

But Great use of Int by Parts is Multiple applications!

$$\int (\text{mess})(\text{nicer}) dx = \int_a^b g \left(\frac{dg}{dx} \right) dx = g \Big|_a^b - \int_a^b g \left(\frac{dg}{dx} \right) dx$$

what happens is
each time you apply Int by Parts
I b P

you
'reduce' the mess by 1 order

so

$$\int x^4 \cos x dx = g \Big|_a^b - \int_a^b g (4x^3) dx$$

$\underbrace{\hspace{10em}}$

↳ apply again

$$- \left[\int_2 g \right]_a^b - \int_a^b g (12x^2) dx$$

$\underbrace{\hspace{10em}}$

↳ Eventually
will get
there!

apply again
↓
and again

Thus IS what a physicist does!

H.W.

$$\int_a^b x^2 \sin(\alpha^2 x) dx$$

$\alpha = \text{constant.}$

IV Differentiating Under the integral ref: Feynman

Start w/

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

so why not diff w/ rep to α
as long as do it to both
sides!

$$\frac{d}{d\alpha} \left[\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\pi} \alpha^{-\frac{1}{2}} \right]$$

$$\int_{-\infty}^{+\infty} -x^2 e^{-\alpha x^2} dx = -\frac{1}{2} \alpha^{-\frac{3}{2}} \sqrt{\pi}$$

or

$$\int_{-\infty}^{+\infty} x^2 e^{-\alpha x^2} dx = -\frac{1}{2} \sqrt{\pi} \alpha^{-\frac{3}{2}}$$

Cool! Can you get $\int_{-\infty}^{+\infty} x^4 e^{-\alpha x^2} dx = ?$

* Int by trig sub: pg 454 Anton

For integrands ~

$$\sqrt{a^2 - x^2} \rightarrow x = a \sin \theta$$

$$\sqrt{a^2 + x^2} \rightarrow x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \rightarrow x = a \sec \theta$$

Show what

$$\text{ex: } \int_{x_1}^{x_2} \frac{dx}{\sqrt{x^2 + a^2}}$$

$$x = a \tan \theta \quad \begin{cases} \text{atan } x_2 \\ \text{atan } x_1 \end{cases}$$

$$\frac{dx}{d\theta} = a \sec^2 \theta$$

$$\sqrt{a^2 - a^2 \sin^2 \theta}$$

$$\sqrt{a^2(1 - \cos^2 \theta)}$$

$$= a |\cos \theta|$$

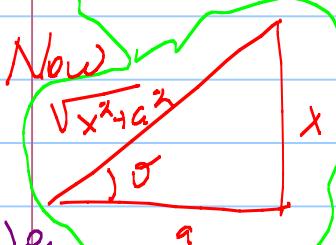
$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

From this relation just make up
atan x_2 $\frac{a \sec^2 \theta d\theta}{\sqrt{a^2 \tan^2 \theta + a^2}} = \int \frac{a \sec^2 \theta d\theta}{a |\sec(\theta)|} = \int \frac{a \sec^2 \theta}{a \sec \theta} d\theta$

$$\int_{x_1}^{x_2} \frac{dx}{\sqrt{x^2 + a^2}} = \int_{\text{atan } x_1}^{\text{atan } x_2} \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 \tan^2 \theta + a^2}} = \int_{\text{atan } x_1}^{\text{atan } x_2} \frac{a \sec^2 \theta d\theta}{a |\sec(\theta)|} = \int_{\text{atan } x_1}^{\text{atan } x_2} \frac{a \sec^2 \theta}{a \sec \theta} d\theta$$

Since $\sec \theta > 0$
 $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$= \int \sec \theta d\theta = \left[\ln |\sec \theta + \tan \theta| \right]_{\text{atan } x_1}^{\text{atan } x_2}$$



Made up!

$$= \ln \left| \frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right| \Big|_{x_1}^{x_2}$$

$$= \left(\ln \left| \sqrt{x^2 + a^2} + x \right| \Big|_{x_1}^{x_2} - \ln(a) \right)$$

$$= \left(\ln \left| \sqrt{x_2^2 + a^2} + x_2 \right| - \ln(a) \right) - \left(\ln \left| \sqrt{x_1^2 + a^2} + x_1 \right| - \ln(a) \right)$$

$$= \ln \left| \sqrt{x^2 + a^2} + x \right| \Big|_{x_1}^{x_2}$$

So in end, a & theta
 you just made up &
 keep w/ restrictions, they cancel

