Let's see how to use a t test to decide whether gas isolated from air is "significantly" heavier than nitrogen isolated from chemical sources. In this case, we have two sets of measurements, each with its own uncertainty and no "known" value. We assume that the population standard deviation (σ) for each method is essentially the same.

For two sets of data consisting of n_1 and n_2 measurements (with averages $\bar{x}_1$ and $\bar{x}_2$), we calculate a value of t with the formula

$$t_{\text{calculated}} = \frac{|\bar{x}_1 - \bar{x}_2|}{s_{\text{pooled}}}\sqrt{\frac{n_1 n_2}{n_1 + n_2}} \tag{4-8}$$

where $|\bar{x}_1 - \bar{x}_2|$ is the absolute value of the difference (a positive number) and s_{pooled} is a *pooled* standard deviation making use of both sets of data:

$$s_{\text{pooled}} = \sqrt{\frac{\sum\limits_{\text{set 1}}(x_i - \bar{x}_1)^2 + \sum\limits_{\text{set 2}}(x_j - \bar{x}_2)^2}{n_1 + n_2 - 2}} = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}} \tag{4-9}$$

$t_{\text{calculated}}$ from Equation 4-8 is compared with t in Table 4-2 for $n_1 + n_2 - 2$ degrees of freedom. *If $t_{calculated}$ is greater than t_{table} at the 95% confidence level, the two results are considered to be different.* There is less than a 5% chance that the two sets of data were drawn from populations with the same population mean.

If $t_{\text{calculated}} > t_{\text{table}}$ (95%), the difference is significant.

Equations 4-8 and 4-9 assume that the population standard deviation is the same for both sets of measurements. If this is not true, then we use the equations[4]

$$t_{\text{calculated}} = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \tag{4-8a}$$

$$\text{Degrees of freedom} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} \tag{4-9a}$$

For Rayleigh's data in Figure 4-7, we suspect that the population standard deviation from air is smaller than that from chemical sources. Using Equations 4-8a and 4-9a, we find that $t_{\text{calculated}} = 21.7$ and degrees of freedom $= 7.17 \approx 7$. This value of $t_{\text{calculated}}$ still far exceeds values in Table 4-2 for 7 degrees of freedom at 95% or 99.9% confidence.

TABLE 4-4 Critical values of $F = s_1^2/s_2^2$ at 95% confidence level

Degrees of freedom for s_2	Degrees of freedom for s_1													
	2	3	4	5	6	7	8	9	10	12	15	20	30	∞
2	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5
3	9.55	9.28	9.12	9.01	8.94	8.89	8.84	8.81	8.79	8.74	8.70	8.66	8.62	8.53
4	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.63
5	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.36
6	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.67
7	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.58	3.51	3.44	3.38	3.23
8	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	2.93
9	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.71
10	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.70	2.54
11	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57	2.40
12	3.88	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.30
13	3.81	3.41	3.18	3.02	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.38	2.21
14	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.31	2.13
15	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.07
16	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.19	2.01
17	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.15	1.96
18	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.11	1.92
19	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.07	1.88
20	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.84
30	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.62
∞	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.00

Critical values of F for a one-tailed test of the hypothesis that $s_1 > s_2$. There is a 5% probability of observing F above the tabulated value.

You can compute F for a chosen level of confidence with the Excel function FINV(probability,deg_freedom1,deg_freedom2). The statement "=FINV(0.05,7,6)" reproduces the value F = 4.21 in this table. The statement "=FINV(0.1,7,6)" gives F = 3.01 for 90% confidence.

The ***F* test** tells us whether two standard deviations are "significantly" different from each other. *F* is the quotient of the squares of the standard deviations:

$$F_{\text{calculated}} = \frac{s_1^2}{s_2^2} \qquad \text{(4-12)}$$

We always put the larger standard deviation in the numerator so that $F \geq 1$. We test the hypothesis that $s_1 > s_2$ by using the one-tailed *F* test in Table 4-4. If $F_{\text{calculated}} > F_{\text{table}}$, then the difference is significant.

Use the *F* test for Case 2 in comparison of means in Section 4-3.

If $F_{\text{calculated}} < F_{\text{table}}$, use Equation 4-8.
If $F_{\text{calculated}} > F_{\text{table}}$, use Equation 4-8a.

The square of the standard deviation is called the *variance*.

EXAMPLE **Is the Standard Deviation from Chemical Decomposition Significantly Greater Than the Standard Deviation from Air in Rayleigh's Data?**

In Table 4-3, the standard deviation from chemical decomposition is $s_1 = 0.001\,38$ ($n_1 = 8$ measurements) and the standard deviation from air is $s_2 = 0.000\,14_3$ ($n_2 = 7$ measurements).

Solution To answer the question, find *F* with Equation 4-12:

$$F_{\text{calculated}} = \frac{s_1^2}{s_2^2} = \frac{(0.001\,38)^2}{(0.000\,14_3)^2} = 93._1$$

In Table 4-4, look for F_{table} in the column with 7 degrees of freedom for s_1 (because degrees of freedom $= n - 1$) and the row with 6 degrees of freedom for s_2. *Because $F_{calculated}$ (= 93.1) > F_{table} (= 4.21), we accept the hypothesis that $s_1 > s_2$ above the 95% confidence level.* The obvious difference in scatter of the two data sets in Figure 4-7 is highly significant.

Test Yourself If two variances were $s_1^2 = (0.002\,00)^2$ (7 degrees of freedom) and $s_2^2 = (0.001\,00)^2$ (6 degrees of freedom), is the difference significant? (***Answer:*** No. $F_{\text{calculated}} = 4.00 < F_{\text{table}} = 4.21$)