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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Selected Books on Advanced Engineering Mathematics

Advanced Engineering Mathematics with MATLAB. Second Edition. *By Dean G. Duffy.* Chapman & Hall/CRC, Boca Raton, FL, 2003. $99.95. xvi+818 pp., hardcover. ISBN 1-58488-349-9.

Advanced Mathematics for Engineering and Science. *By C. F. Chan Man Fong, D. De Kee, and P. N. Kaloni.* World Scientific, River Edge, NJ, 2003. $58.00. xiv+880 pp., softcover. ISBN 981-238-292-5.

Mathematical Methods for Scientists and Engineers. *By Donald A. McQuarrie.* University Science Books, Sausalito, CA, 2003. $90.00. xiv+1161 pp., hardcover. ISBN 1-891389-24-6.

Advanced Mathematics and Mechanics Using MATLAB. Third Edition. *By Howard B. Wilson, Louis H. Turcotte, and David Halpern.* Chapman & Hall/CRC, Boca Raton, FL, 2003. $89.95. xiv+678 pp., hardcover. ISBN 1-58488-262-X.

Engineers generally complete their required mathematics courses in their first two years of college. Many departments of mathematical sciences also offer elective classes to teach more mathematics to juniors and seniors in these majors. The same or similar courses help prepare first- and second-year graduate students for masters- or doctoral-level study of engineering and science.

In the last year alone, quite a few mathematicians, scientists, and engineers have written textbooks designed for such courses. Reviewing some of them provides a springboard to re-evaluate

1. what subjects to include in a course on "advanced engineering mathematics" (AEM);
2. how to teach them, vis-à-vis organization of topics, level of detail, amount of mathematical rigor, degree of application, use of computer algebra systems (CAS); and
3. which textbook to use, if any.

The answers to these questions depend crucially on the purpose of and clientele for the course. One should cultivate mathematical sophistication in engineers and scientists by emphasizing how the subjects form a coherent structure. Alternatively, if AEM means a collection of various tools, the instructor should convey what broad purposes it serves. To tailor the content to his or her particular audience and to take advantage of all of the best resources available, the professor should interrelate material from various textbooks specializing in single subjects. The students can then view an AEM text as a reference for an assemblage of topics.

Publishers are invited to send books for review to Book Reviews Editor, SIAM, 3600 *University City Science Center, Philadelphia, PA* 19104-2688.

Table 1 *Curriculum proposals.*

Duffy A	Laplace transform / separation of variables in PDEs / vector calculus or linear algebra
Duffy B	Sturm–Liouville problems / separation of variables in PDEs / vector calculus or linear algebra
Duffy C	Complex variables / Fourier series / Fourier, Laplace, z-, and Hilbert transforms
Chan Man Fong et al.	Series solutions and special functions / vector and tensor analysis / PDEs / numerical methods
Kreyszig, 1999	Sequence of courses: (1) ODEs, (2) linear algebra and vector analysis, (3) complex analysis, (4) numerical methods
Kreyszig, 1962	Sequence of courses: (1) ODEs, (2) vectors and matrices (3) Fourier series and PDEs, (4) complex analysis
Hildebrand, 1949	Laplace transform / numerical ODEs / series solutions and special functions / boundary-value problems and orthogonal functions / vector analysis / PDEs / complex variables
Gross A	Complex numbers and functions / linear analysis / systems of ODEs / divergence and Stokes's theorems
Gross B	Series solutions and special functions / method of dominant balance / Sturm–Liouville theory / Fourier series and transform / PDEs

1. What Should We Teach in AEM?

The question of what to cover in a *course* on AEM differs from the question of what to include in a *textbook* on AEM. Indeed, some texts contain much elementary material from calculus and ordinary differential equations (ODEs), as well as an array of advanced topics far too extensive to address in a single course. The book may benefit the student more as an enduring reference than as a guide to careful study during one or two semesters.

Two of the volumes under review suggest curricula explicitly. Dean G. Duffy gives the more detailed guidance, proposing three courses in AEM for which his book can serve. (See Table 1.) His Course A assumes familiarity with basic differential equations, perhaps in the context of an expanded calculus class. This version of AEM includes the Laplace transform and separation of variables for the heat, wave, and/or Laplace equation. His more advanced Course B has a course on differential equations as a prerequisite. Course B teaches (or "reteaches") separation of variables from the point of view of Sturm–Liouville theory and eigenfunction expansions. Both Courses A and B conclude with a chapter on either vector calculus or linear algebra. The latter ends with a five-page section on systems of linear differential equations.

Duffy has used the text himself recently in a Course C for electrical, communications, and systems engineers. It opens with a unit on complex variables, followed by Fourier series and Fourier, Laplace, and z-transforms. In this second edition, he has added a new chapter on the Hilbert transform as a concluding unit for just this audience. Two of the transform chapters distinguish the contents of Duffy's book from the contents of the other three under review, as Table 2 indicates.

Duffy has added another two chapters to this new edition, as well. He says they permit the book to serve as a text for a course in differential equations. Also unlike the 1998 version, this edition integrates MATLAB routines into the exposition.

Table 2 *Chapters included in only one of the four texts.*

Duffy	Z-transform, Hilbert transform
Chan Man Fong et al.	Tensor analysis, "special topics" (in advanced classical mechanics, plus statistical mechanics)
McQuarrie	Functions defined as integrals, probability, statistics
Wilson et al.	Use of MATLAB, bending analysis of beams of general cross section, nonlinear optimization applications

C. F. Chan Man Fong, D. De Kee, and P. N. Kaloni suggest an undergraduate course that starts with a unit on series solutions and special functions, followed by topics in vector and tensor analysis, partial differential equations (PDEs), and numerical methods, including numerical ODEs. They do not include in this outline their chapters on numerical PDEs, complex variables, or calculus of variations. The book also contains a review of calculus and ODEs, as well as a collection of short sections on "special topics" (in advanced classical mechanics and statistical mechanics). A couple of the subjects do not appear in the other texts under review, as Table 2 shows. In addition, Chan Man Fong et al. devote a consolidated section (about 20% of their book) to numerical methods, whereas Duffy interweaves numerical overviews into sections on differential equations and transforms. Chan Man Fong et al. also include some more advanced topics in PDEs, like Mellin transforms, similarity solutions, and a systematic treatment of Green's functions, unlike the authors of the other books under review.

Donald A. McQuarrie wrote his book for "students who have had $1\frac{1}{2}$ or 2 years of calculus and little else." He introduces a wide variety of topics, hoping to inspire the student to later deeper study, suggesting references at the end of each chapter. Indeed, of the texts under review, only his lacks the word "advanced" in the title. The volume opens with over 100 pages of calculus, four times as many as Chan Man Fong et al. McQuarrie also provides a chapter that gives an elementary introduction to vectors (before covering the divergence theorem and Stokes's theorem, as do the other authors, in a later chapter). He leaves to the instructor's discretion which of the remaining 19 chapters—each around 50 pages long—to cover. Table 2 lists those chapters that include material not included in the other three texts. Unlike the others, McQuarrie's book teaches no numerical methods.

Speaking of numerics, the book by Howard B. Wilson, Louis H. Turcotte, and David Halpern has a very different flavor from the other three. As the title suggests, simulations of mechanical systems receive a great deal of attention. The authors recommend their text as a "reference or supplementary text in computationally oriented courses emphasizing applications." Students in such courses must have backgrounds in Euclidean geometry, Newtonian mechanics, and "some mathematics beyond calculus." As a secondary text, the book does not impose a particular curriculum. Also, it emphasizes examples over mathematics and methods. Still, it does rather come to a consensus with the other authors on which topics to include under the title "advanced mathematics."

As one would expect, however, it treats those topics totally differently. For example, the chapter entitled "Summary of Concepts from Linear Algebra" spends a total of two pages on an introduction, vectors, norms, linear independence, and rank. Two subsequent sections include about fifteen pages each of examples of least squares

Table 3 *Topics commonly included in AEM texts.*

Most AEM texts	Many AEM texts
Linear algebra	Numerical methods
Vector calculus	Calculus of variations
ODEs	Probability
PDEs	Statistics
Complex variables	Optimization

fits and eigenvalue problems, respectively. After a few pages on computing natural frequencies for a rectangular membrane, the chapter closes with only two pages on column space, null space, orthonormal bases, and singular-value decomposition.

This third edition has integrated time-dependent solutions of linear PDEs and corresponding animations into the same chapter outline as the previous edition. Perhaps David Halpern, who did not coauthor the previous two editions, contributed this new material. See Table 2 for a list of chapters particular to this book.

Upon confronting her 15 pounds of new books on AEM, some applied mathematicians asked this reviewer, "Why use something other than Kreyszig?" Looking at this question from the curriculum standpoint, one observes that Erwin Kreyszig's text [8] contains the topics in Duffy's Courses A and B, as listed in Table 1. His book does not address some of the other entries in that table, however. It lacks the z- and Hilbert transforms of Duffy's Course C and the tensor analysis that Chan Man Fong et al. advocate.

In the preface to the eighth edition of his AEM text, Kreyszig asserted that matrix algebra, numerical methods, statistics, and graph theory play larger and larger roles in engineering applications. He does not suggest how to integrate these elements into a course on AEM, however. Instead, he proposes the four-semester sequence in Table 1, namely, (1) ODEs, (2) linear algebra and vector analysis, (3) complex analysis, and (4) numerical methods.

As another alternative to a multitopic course on AEM, Kreyszig suggests several one-semester courses with sharper focus, e.g., (1) numerical linear algebra, (2) (linear) optimization, and (3) graphs and combinatorial optimization. He also includes a class on Fourier series and PDEs in the list. As Table 1 shows, Kreyszig's first edition included that course in the four-semester sequence. Later, numerical methods replaced it.

Like Kreyszig's first edition in 1962, many popular AEM texts have crystallized around the subjects presented in Table 3, namely, linear algebra, vector calculus, ODEs, PDEs, and complex variables. Some, like Kreyszig's current edition, also include numerical methods, probability, statistics, and optimization. The textbooks under review largely fit this framework, laid out in Francis B. Hildebrand's *Advanced Calculus for Engineers* [6] in 1949 (except that the latter includes no linear algebra).

Hildebrand chose the textbook's title to represent areas of mathematics "based upon a sound working knowledge of elementary calculus and ... important in a number of fields of application" (p. v). In fact, the second edition (1976), still in print, bears the title *Advanced Calculus for Applications*. Hildebrand presents the contents as a course curriculum, suggesting one may cover complex variables at the end or at the beginning and that some may consider the chapter on linear ODEs as a review. He promises an "integrated treatment" with PDEs of mathematical physics as the end point and the remaining chapters, including the Laplace transform, numerical ODEs, series solutions and special functions, boundary-value problems and orthogonal functions, and vector analysis, as steps upon the way. (See Table 1.)

The tables of contents of textbooks and the ideas listed in Table 1 can only provide starting points for the design of a successful curriculum in AEM. To teach an appropriate course, one must *know one's audience.* In 1990, Garfunkel and Young [4] estimated that 170,000 students in the United States enroll annually in mathematics courses beyond calculus *outside of mathematics departments.* Their study shows that "mathematics faculty and curricula are often seen as at best irrelevant and at worst counterproductive." One hopes that mathematicians today communicate with the markets their courses serve to ensure that our colleagues outside mathematics view AEM as not only relevant, but also valuable, as the instructors must intend.

This reviewer has recently engaged in such discussions with engineers and scientists inside and outside of her home institution and inside and outside of academia. She suggests that doing similarly will help other mathematicians carry out the following steps:

1. *Identify the Purpose of AEM.* For example, if the course targets undergraduates going directly to industry, some practicing engineers suggest that topics include complex numbers and functions, numerical root-finding, linear algebra and numerical linear algebra, qualitative and numerical methods for solving differential equations, and/or the real and complex Fourier series and Fourier transforms. One could integrate technological tools widely used in the workplace, like graphing calculators and MATLAB. This reviewer has some of these elements in her Course A at The University of Akron. (See Table 1.) If, on the other hand, the course prepares students for graduate study, particularly of mechanical or chemical engineering, then consider teaching series solutions and special functions, Sturm–Liouville theory, and PDEs (a subset of Hildebrand's curriculum in Table 1). This reviewer's Course B follows this outline. (See Table 1.) One of the former undergraduates in the class sent an unsolicited e-mail saying, "It is quite a luxury to be this well prepared for a class [conduction heat transfer]" in his graduate program. Colleagues have received similar feedback when teaching Course B, suggesting that a pioneering AEM curriculum from 1949 remains relevant today.
2. *Identify the Students' Backgrounds.* For undergraduates, set as the prerequisites the requirements for their majors. If graduate students have passed mathematics qualifying exams, identify the content. Find out whether the students have access to and know how to use a CAS or at least a graphing calculator.
   This reviewer's Courses A and B (Table 1) reflect that the engineering and science undergraduates who enroll have previously taken introductory courses in differential equations and linear algebra. They have strong academic records and often minor in applied mathematics. Engineering graduate students in the process of preparing for mathematics qualifying exams also take her courses, cross-listed as both undergraduate and graduate courses.
3. *Extend the Students' Knowledge of Prerequisite Fields.* One could easily fit a one-semester course (or more) under this heading.
   This reviewer chooses to unite familiar material from linear algebra and elementary ODEs via an integrated discussion of the following:
   1. Vector spaces
   2. Linear operators and linear operator equations
   3. Basis and dimension

4. Inner products, orthogonality, induced norms, and orthonormality
5. Eigenvalues and eigenvectors

Table 1 summarizes these as "linear analysis," in the spirit of Donald L. Kreider et al., who wrote their book [7] for courses in advanced engineering mathematics. The topics above constitute a highly condensed and interwoven treatment of material distributed in six of their chapters.[1] However, this reviewer's Course A addresses the items in the list above in the context of vector spaces over the complex numbers, having opened the course with introductory material in complex variables (Table 1).

A sophomore-level course in ODEs usually includes only a couple of the following units:

1. Laplace transform
2. Series solutions and special functions
3. Linear and nonlinear systems and phase plane analysis
4. Fourier series and separation of variables for solving PDEs
5. Sturm–Liouville problems, eigenfunction expansions, and separation of variables for solving PDEs
6. Further methods for solving PDEs
7. Numerical methods
8. Asymptotic methods

Teach some of the remaining items. This reviewer covers all of these areas in an introduction to differential equations, together with Courses A and B in AEM. (For the latter, see Table 1.)

Consider picking up vector analysis where the calculus course ended. This reviewer's Course A covers the divergence theorem and Stokes's theorem and uses them to reformulate integral conservation laws in differential form. (See Table 1.)

4. *Introduce the Students to Subjects New to Them.* Include linear algebra if they haven't taken it. The Mathematics Working Group of the European Society for Engineering Education (SEFI) specifically recommends linear transforms and least-squares fitting [10] as part of a core curriculum for engineers.

   The group also considers introductory probability and statistics as core areas, as well as such discrete math topics as logic, sets, induction, recursion, and graphs [10].

   Choose from among countless other candidates. Table 3 includes some. This reviewer teaches complex numbers and functions in her Course A (Table 1).

   Cover topics of particular interest to the audience, e.g., discrete transforms for electrical engineers, residue theory for control theorists, or tensor analysis, group theory, or Green's functions for physicists.

2. How Should We Teach AEM?

Kreysig [8] wrote:

> It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead it is important that students become familiar with ways to think mathematically, recognize the need for applying mathematical methods to engineering problems, realize that

[1] The chapters cover (1) real vector spaces, (2) linear transformations and matrices, (3) the general theory of linear differential equations, (7) Euclidean spaces, (8) convergence in Euclidean spaces, and (12) boundary-value problems for ODEs.

> mathematics is a systematic science built on relatively few basic concepts and involving powerful unifying principles, and get a firm grasp for the interrelation between theory, computing, and experiment.

One can debate whether mathematics in fact contains relatively few basic concepts, but nobody can deny that AEM courses often contain too many narrow methods, obscuring the powerful unifying principles. However, in the same preface quoted above, Kreyszig described the chapters in his book as quite independent, accommodating a variety of courses. The same holds true for most AEM texts, so bringing out the connections falls heavily on the instructor.

This reviewer tells students that the word "advanced" appears in the course title not only because they will study subjects beyond their basic requirements, but also because they will acquire a more sophisticated understanding of material they have studied already. They should see the connections among subjects they previously viewed as disjoint. Such an appreciation will serve them well in future encounters with mathematics in their fields.

For example, the one-semester Course A that this reviewer proposes (Table 1) introduces complex numbers and functions to lead into vector spaces over the complex numbers. The unit on linear analysis outlined above (section 1) integrates the students' prior knowledge of linear algebra and elementary ODEs. Eigenvalues provide a point of transition to exact and qualitative treatments of linear and nonlinear ODE systems, including phase plane analysis of nonlinear equations. The discussion of direction fields will serve as a new application of the vector fields the students first encountered in multivariable calculus. They can then review the operations of divergence and curl on vector fields, together with the gradient. Because third-semester calculus often cannot cover them thoroughly, the divergence theorem and Stokes's theorem come next. Using them to derive PDE conservation laws ends the class with a preview of further work to do in PDEs.

This reviewer does some of the further work in Course B (Table 1), designed to share Hildebrand's framework discussed above (and listed in Table 1). In particular, the heat, wave, and Laplace equations constitute the goal, with the ODE topics as steps toward that end. Series solutions lead to the derivation of Legendre polynomials and Bessel functions. To complement expansions about ordinary and regular singular points, the curriculum also includes the method of balance at irregular singular points, in particular at infinity. Carl M. Bender and Steven A. Orszag [2] covered these topics in a thorough chapter on local analysis.

In the next unit, Sturm–Liouville theory contrasts with local analysis; one can emphasize that a partial sum of an eigenfunction expansion of the solution to an inhomogeneous Sturm–Liouville problem approximates the solution well on the whole problem domain. The Fourier series becomes a special case of eigenfunction expansion. The course next introduces the Fourier transform. The last third of the course involves solving PDEs via separation of variables, the Laplace and Fourier transforms, d'Alembert's method, and the method of characteristics.

The Courses A and B that this reviewer teaches (Table 1) have the titles AEM I and AEM II. Part II does not depend on Part I. In fact, Hildebrand's book shows that Course B need not rely even on linear algebra. One can make many connections informally with the linear analysis of Part I, however, to enrich the experience of students taking both semesters.

AEM should give a big (probably rough) picture of the wide topics (such as those in Table 3) that it addresses, even if it can only include a small slice in detail. Students need to develop the ability to tackle basic problems and interpret solutions (sometimes qualitatively), see the limitations of the methods they study, and have some idea of how to transcend these limitations.

For example, students learning about systems of ODEs can reinforce their perspectives on ODEs as a whole, e.g., that many linear problems yield to known methods (with solutions to constant-coefficient problems especially accessible) and that solutions to inhomogeneous problems build on the homogeneous. They will easily classify differential equations and give examples of corresponding physical problems. They will know that one nth-order equation can be re-expressed as a system of n first-order equations. Students will use phase plane analysis to show qualitative behaviors clearly, recognizing that in many physical systems varying a parameter can produce an abrupt change in equilibrium solution behavior. They will know when linearization gives reliable information and how much error the process accrues. They will know what numerical methods are, when to turn to them, and that software can execute some of them quite directly.

In connecting different subjects and giving a wide perspective, as this reviewer advocates, one must guard against covering material too theoretically. Certainly AEM should use proof very judiciously. Contrary to an instructor's plans, a high degree of rigor can interfere both with unifying different subjects and with providing a broad perspective. Hildebrand [6] took the following approach:

> It is not infrequently true that the natural way of *discovering* a particular technique or relationship is by no means the most efficient way of *establishing its validity*. The compromise adopted here, in such cases, usually consists in showing in as direct a way as possible that the desired result is plausible, and in then stating conditions under which the result can be rigorously established. Thus, the engineering reader may proceed in the direction along which he might have discovered a given fact for himself. . . .

In Garfunkel and Young's study, those outside mathematics departments commonly complained that "[m]athematics faculty teach mathematics as an art with full abstraction, not as a tool" [4]. They wanted applications, in contrast to "overly generalized presentations that are not (usually) presented as being useful or interesting to a practical person" [4].

One must use physical context to motivate students. On the other hand, one must not devote so much time and effort to one setting as to make a technique appear too narrowly applicable. Such an approach would contribute to the impression of many of Garfunkel and Young's respondents that "[m]athematics courses do not give students the knowledge or the mathematical maturity for further work" [4]. P. C. Cretchley, A. J. Roberts, and C. J. Harman [3] "note that where applications are used to motivate learning, hasty treatment of enough material only for the task in hand does not invite the longer-term higher-order reflection that ensures deeper secure learning." They say, "Given the dangers of fragmentation of content and shallow treatment of core topics, we argue for coordinated consolidation of core skills" [3].

In disagreement with this reviewer, one could argue that consolidating core skills in a field of mathematics requires taking an entire course in it, and that coherence comes from taking such courses sequentially or in some carefully prescribed pattern. However, this paradigm applies only to the mathematical sciences major. Indeed, many departments teach their own advanced mathematics courses expressly because (as quoted by Garfunkel and Young [4]) "[t]here is not room . . . for every student to take the separate courses in differential equations (ordinary and partial), vector and tensor analysis, complex variables, Fourier series, probability.... [A]ll these are covered in our one-year course by omitting the detailed proofs and generalizations." In such a setting, another way to unify the topics is through the field of application itself. After identifying the purpose of the course, as outlined in the previous section, this approach may prove appealing.

Regardless, one must integrate applications into the course. Although the name AEM shows that the subject matter is mathematics, the word "engineering" appears, too. The instructor must communicate the reason: how the material arises in physical problems. A biomedical engineer in this reviewer's class said he enjoyed the realization that certain techniques apply not only to certain problems in a given field, but arise in many fields. One can make this point without spending time on the details.

Returning to the quotation that opened this section, one would like to show students the relationships among theory, computation, and experiment. Including experiment in a course on AEM may prove a tall order, but one can easily emphasize physical interpretations of models and solutions. Hildebrand recognized the importance of computation in his 1949 book [6], including a chapter on numerical methods for solving ODEs. Such material constitutes a vital part of the curriculum today: Engineers use complicated models that generally demand numerical treatment. The SEFI report (p. 48) noted "considerable benefit from the integration of numerical methods with their analytical counterparts" [10]. It noted that a few steps using a simple (even simplistic) method like Euler's can teach much about the "nature" of numerical solutions and that "much of this would probably be overlooked if all the students did was learn the appropriate syntax to invoke a differential equation solver in a commercial software package" [10].

Like applications, the CAS in AEM should provide a clearer and more motivating way of addressing certain topics, not serve as an end in itself. As an example of an appropriate use of technology, showing direction fields with Maple gives much immediate and useful information about solutions to initial-value problems, particularly in nonlinear systems of ODEs.

3. From Which Textbook Should We Teach AEM?

First, consider the four texts under review. Duffy has highlighted integral and discrete transforms. An instructor whose syllabus reflects Course C in Table 1 should use Duffy's textbook over other AEM texts, which commonly omit z- and Hilbert transforms.

In their book, Chan Man Fong et al. pay attention to the needs of physicists who take AEM. (See Table 2.) Most AEM texts will not appeal to a professor who shares their interest in tensor analysis, but many established volumes on mathematics for physics fit the bill, such as that of George Arfken (and now his coauthor Hans Weber) [1]. If the teacher wants that chapter under the same cover as a fairly extensive catalog of numerical methods, the pool of candidates shrinks, and he or she should consider Chan Man Fong et al.

Recall that McQuarrie aims to give a first exposure to many topics, to pique the student's interest for further study. Although the material matches the other textbooks quite closely, the book has a somewhat more informal tone. The chatty approach tends to embed definitions in the text. The author sets apart some theorems (not generally labeled as such) in italics, while he includes others within the discussion like any other sentence. This works well for the casual read, bringing out a few key points. Perhaps some arts-and-sciences readers will enjoy the style of exposition more than engineering students. The latter group often favors a thorough accounting of boxed facts.

Instructors who incorporate *Mathematica* into their classes will appreciate that McQuarrie sprinkles *Mathematica* statements here and there. The problems that require a CAS, however, explicitly permit the use of whichever one the student prefers. (Some of these are just as easy to solve by hand, for example, finding the gradient of a simple function.)

The biographies, which the author's wife Carole wrote, make clear connections to the technical content. Also, unlike biographies in many mathematics textbooks, they include plenty of facts likely to hold the typical student reader's interest.

McQuarrie's 22 chapters have a nice organization overall. For instance, the early introduction to complex variables has advantages. However, some scrambling occurs, as well. For example, the author expresses too much willingness to fill the chapter on matrices and eigenvalue problems with applications from linear systems of ODEs, which the subsequent chapter addresses—in a sixth section (with section 1 covering ODEs of first order and first degree).

A first edition will tend to have errors (as will likely any edition of a thousand-plus pages). An egregious one jumped out at this reviewer in an example on page 495: The list of eigenvectors includes the zero vector, leading to the conclusion that the double eigenvalue does not have linearly independent eigenvectors.

Material in the book by Wilson et al. can serve as a valuable resource for the AEM course that provides a brief exposure to numerics. It can also supplement the more heavily numerical course. In the former case, one could focus, for example, on the chapter on integration of nonlinear initial-value problems. The instructor could conveniently insert the chapter, with its accessible succinct sections, into the curriculum for a qualitative or somewhat more detailed overview. He or she could also connect it easily (perhaps loosely) with other topics.

The first section heading contains the somewhat peculiar terminology "nonlinear matrix differential equations" and opens with the usual (for this book) evangelical statements on user-friendly software. The student may notice that most of the text-books cited date from decades ago, but the authors make the point that products like MATLAB make the methods much easier to use.

The chapter's brief introduction makes for good reading for engineering students, bringing out many key broad ideas in applied mathematics and numerical methods: It addresses appropriate levels of complexity in mathematical models and the sensitivity of nonlinear systems to small changes in physical parameters. It explains nicely that analytical techniques cannot produce solutions to most differential equations and that many approximate methods derive from series expansions.

The two-page summary of Runge–Kutta methods gives a nice bird's eye view to the student who has studied the subject in a little more detail and to the novice who reads carefully and patiently. (Both groups will notice a typographical error, which also appeared in the previous edition: "Reducing the step-size by h [sic] reduces the truncation error by about a factor of $(1/2)^3 = 1/8$.")

The text emphasizes the importance of understanding the error inherent in any numerical method. A short section derives a stability condition quite intuitively for an nth-order Runge–Kutta method on the example $y'(t) = \lambda y$ for λ complex. It shows a little MATLAB program to plot the stability boundary in the complex plane, using an intrinsic function that finds complex roots, and gives a clear interpretation of the resulting graphs.

The chapter on integration of nonlinear initial-value problems also discusses the trade-offs and limitations of reducing step size and gives a good and strictly descriptive explanation of variable step-size algorithms.

The remainder of the chapter discusses examples of forced oscillation. These include an inverted pendulum, dynamics of a spinning top, motion of a projectile, and others.

Wilson et al. include the MATLAB programs in their entirety. Good explanations bracket many of them, and they include their purposes in comments. Nevertheless, key commands remain uncommented. The reasonably well trained eye can decipher them, but some will certainly remain opaque even to the thorough student. Downloading and running many of these routines will produce output from a black box. The philosophy of the methods and the interpretation of the results can serve a pedagogical purpose, but the students might gain only limited insight from experimenting with the codes themselves.

Having pondered some pros and cons associated with these new books, many readers will remain committed to their preferred books on AEM. A few will like them enough to teach multiple courses from them, resulting in financial savings for the students. (When this reviewer used Peter V. O'Neil's book [11], she learned that one of her students had used an earlier edition for a two-semester sequence on differential equations as a freshman in mechanical engineering at National Cheng Kung University in Taiwan. He liked picking the book up again in AEM.) For example, the book costs for Kreyszig's four semesters in Table 1 would average \$31.99 per term, much less than the average cost for a book in the single subject.

Generally, however, teaching multiple courses from the same book poses a variety of challenges, first and foremost that most students don't have the luxury of enrolling in three courses beyond differential equations. Those students will pay too much for their very heavy differential equations textbook (e.g., \$127.95 and five pounds for Kreyszig).

One colleague told this reviewer that publishers should ship AEM texts to physical education departments, where students could carry them to earn credit in weight lifting. Luckily, professors can request a pruned version of a text. A special division of the publishing company excludes chapters irrelevant to a course and includes the instructor's own material.

For instance, this reviewer's Course B (Table 1) could use Kreszig's chapter on series solutions and special functions, as well as the two chapters on Fourier analysis and PDEs, cutting 20 chapters superfluous to the curriculum. The savings would allow ample room to add sections on inhomogeneous Sturm–Liouville and some treatment of asymptotic methods.

Copyrights needed for any new components affect the price of such a custom product. Most often the book costs less than the original. However, departments need to commit to purchasing a minimum order of several hundred copies over a couple of years.

Readers who have not yet identified an AEM text they especially like should realize that most texts will address the subjects in Table 3. Few do so in a way that differs radically from the others.

The occasional AEM text has a special way of interrelating subject matter. For example, Gilbert Strang's book [13], which serves a one-year course in "applied mathematics and advanced calculus and engineering mathematics" (p. viii), integrates the continuous with the discrete, emphasizing linear algebra and numerical methods throughout and showcasing the finite element method, the fast Fourier transform, combinatorics, and optimization. It de-emphasizes series solutions and "avoid[s] a total and fatal immersion into vector calculus—which has too frequently replaced applied mathematics, and taken all the fun out of it" (p. viii).

Bender and Orszag's text [2] corresponds to a course on qualitative methods for ODEs and difference equations for advanced undergraduates and beginning graduate students. Originally published in 1978 by McGraw-Hill and republished in 1999, the book presents an excellent organization of a subject that can often seem like a hodgepodge of surprising tricks. The four parts include fundamentals (ODEs and difference equations), local analysis (approximate solutions of linear differential equations, nonlinear differential equations, and difference equations, plus asymptotic expansion of integrals), perturbation methods, and global analysis (boundary-layer theory, WKB theory, and multiple-scale analysis).

V. V. Mitin, D. A. Romanov, and M. P. Polis organized their text [9] around the concept of "mappings, their properties and manifestations," claiming to avoid what they see as the overemphasis on operators and on discrete mathematics. As such, their AEM book opens unconventionally with chapters on set theory, relations and mappings, mathematical logic, and algebraic structures. After discussing linear mappings and matrices, they address metrics and topological properties, then Banach and Hilbert spaces. The final chapters cover orthonormal bases, operator equations, Fourier and Laplace transforms, and PDEs.

Nearly all AEM texts, however, will still require picking which ideas to cover and how to relate them. Instructors will often find a weakness in the way an author addresses at least one of the desired topics, tempting them to choose material not from among AEM chapters, but from among chapters in single-subject books with applied flavors. Everyone has his or her favorites on the fields listed in Table 3. Frequently, AEM chapters on the same subjects read like poor cousins, right down to less interesting problems and applications.

To take an example at random, Michael D. Greenberg [5] included a mostly solid set of 14 exercises on "generalized vector spaces." However, he has covered enough material to have included more variety: like proving that the set of two-by-two matrices or the set of solutions to some linear homogeneous differential equation is a vector space. (One problem errs in asking the student to prove that the set of nth-degree polynomials is a vector space.) Only three of the problems deal with inner products. He also gives no practical context for vector spaces, which engineering students always find mysterious and abstract. He does mention that he will revisit them (which he does nicely) many chapters later when he covers Fourier methods and PDEs. On the other hand, in his linear algebra textbook [14], Gareth Williams has a good mix of accessible problems on inner products, in addition to a compelling elementary discussion of the representation of space-time as a vector in $\mathcal{R}^4$. He describes the quest for an inner product that would produce the appropriate geometry for capturing experiments in relativity (and explains Minkowski's pseudo-inner product).

Although the multiple-textbook scheme seems like it would cause even more injury to the student's pocketbook and biceps, most institutions can combine book chapters into a single course packet and sell copies, given enough lead time for copyright clearance. Schaum's outline [12] then provides a cheap, light, and reasonably thorough companion for reference. This reviewer plans to use this approach the next time she teaches AEM.

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An Introduction to Numerical Analysis. *By E. Süli and D. Mayers.* Cambridge University Press, Cambridge, UK, 2003. $35.00. x+433 pp., softcover. ISBN 0-521-81026-4.

The present book has grown out of printed notes that have accompanied lectures given to undergraduate mathematicians at Oxford University over many years. Quoting from the back cover, "Based on a successful course at Oxford University, this book covers a wide range of ... problems from the approximation of functions and integrals to the approximate solution of algebraic, transcendental differential and integral equations. Throughout the book, particular attention is paid to the essential qualities of a numerical algorithm-stability, accuracy, reliability and efficiency.

"The authors go further than simply providing recipes for solving computational problems. They carefully analyse the reasons why methods might fail to give accurate answers, or why one method might return an answer in seconds while another would take billions of years. This book is ideal as a text for students in the second year of a university mathematics course. It combines practicality regarding applications with consistently high standards of rigor."

The reviewer finds that the authors have, indeed, largely met these aims. The book is well written and offers a good level of rigor for the intended readership. The expected level of background would be more in keeping with that of upper level undergraduates of a typical U.S. university. The exercises are sensible and tractable. An email address is given for instructors to obtain a LaTeX file for solutions to the exercises. The intention of this book is not to be a handbook on the topic of numerical analysis, but rather to give mathematics students the concepts for numerically solving a number of mathematical problems. "Some knowledge of matrix